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Reversal in the nonlocal large-scale $\alpha\Omega$-dynamo

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Abstract

Inversion of the magnetic field in a large-scale model of $\alpha\Omega$-dynamo with nonlocal α-effect is under the investigation. The model allows us to reproduce the main features of the geomagnetic field reversals. It was established that the polarity intervals in the model are distributed according to the power law. Model magnetic polarity time scale is fractal. Its dimension is consistent with the dimension of the real geomagnetic polarity time scale.

1 Introduction

The existence of large-scale magnetic fields of planets, stars and galaxies is usually attributed to the action of the dynamo mechanism (Zeldovich et al., 1983). Magnetohydrodynamic equations are symmetric with respect to the change of sign of the magnetic field, which leads to a potential reversal in the dynamo system. These reversals are observed in real space dynamo systems. For example, the reversal of the magnetic field of the Sun occurs approximately every 11 years, Stix (2002). We get the information on the geomagnetic field reversals from paleomagnetic records, on the basis of which the geomagnetic polarity time scale is constructed. The sequence of moments of geomagnetic field reversals is a non-periodic random sequence (Merril et al., 1996). Thus, the statistical reversals of magnetic fields of the Sun and Earth are very different. However, concerning geomagnetic reversals, we mean the transition between stable states of a geomagnetic dipole, averaged over a few thousands of years (Merril et al., 1996). Therefore, the difference in the reversals of the magnetic fields of the Sun and the Earth is the difference in the processes at absolutely different time scales.

It is known that different scales of geomagnetic polarity form a self-similar fractal structures (Ermushev et al., 1992; Ivanov, 1993; Pechersky et al., 1997). Intervals between the reversals (polarity intervals) differ by several orders of magnitude, there are long intervals without reversals, superchrons (Gaffin, 1989; Merril et al., 1996). A large scatter of the interval lengths does not allow us to use such characteristics as mean or variance correctly. It is known that the random variables with the properties such as self-similarity of the set of realizations, the range, the infinity of points may be well described by power law distributions (Sornette, 2006).

Of course, one can not rely on the construction of a geodynamo model, which would fully reproduce the real paleomagnetic scale. It is only possible to get similar statistical characteristics. Different models of geodynamo allow us to obtain random sequence reversals, the properties of which are very different. In some models the solutions are periodic or quasiperiodic (Hejda and Anufriev, 2003; Rikitake, 1965), in others they exhibit fractal properties (Anufriev and Sokoloff, 1994; Hollerbach et al., 1992).

In this paper, we consider the bi-modal model of a large-scale $\alpha\Omega$-dynamo in which there is the perturbation of α-effect of a Non-Markovian random pulse process. Physically, this process may be interpreted as the effect of rejected modes of mean-field. According to the authors, non-Markovian character of the process is of principle, because it expresses the temporal nonlocality (hereditarity) of the model, the response values of α-effect on the change in the magnetic field depends not only on the present, but also on the previous values of the field.

The mechanism of $\alpha\Omega$-dynamo was proposed by Parker (Parker, 1955). This kind of dynamo is typical for astrophysical objects (planets, stars, galactic disks) and suggests differential rotation of the object and turbulence in the character of motion of a conducting medium in this object.

The essence of such a dynamo is as follows. During the initial moment, the existence of a poloidal field of dipole type is supposed. During the differential rotation, the magnetic field lines of a highly conducting medium curl around the axis of rotation, this leads to the appearance of the toroidal field in the convective zone of a star or a planet liquid core.

To close the cycle, it is necessary to get a new poloidal field from this toroidal one. It is assumed that this is due to the breaking of mirror symmetry flows in the convection zone. Turbulent mirror-asymmetric flow generates effective EMF in the direction of the

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toroidal field (α-effect), which leads to the excitation of a new poloidal field. The theory of α-effect was developed by Steenbeck, Krause and Rädler (Steenbek et al., 1966; Steenbek and Krause, 1969). A detailed description of the mean-field theory is given in the books Krause and Rädler (1980), Moffat (1978), and Zeldovich et al. (1983).

Induction equation for the magnetic field in a conducting medium is the following:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \mathbf{B}) + \nu_{\mathrm{m}} \Delta \mathbf{B},$$
$$\nabla \mathbf{B} = 0, \tag{1}$$

where $\boldsymbol{v}$ is the velocity field of the medium, and ν_{m} is the magnetic viscosity.

If the velocity field is defined, then the Eq. (1) is linear and defines the kinematic dynamo problem. However, the magnetic field affects the flow of a medium by the Lorentz force. The effect of this force in the equations of motion of the medium is quadratic in the magnetic field, so in the case of small magnetic fields, we can be restricted to kinematic approximation. The formal criterion of the non-applicability of the kinematic approximation is the satisfaction of the ratio $E_{\mathrm{K}} \lesssim E_B$, where E_{K} and E_B are the kinetic energy of the moving medium and the energy of the magnetic field, respectively.

In this case, it is necessary either to solve Eq. (1) together with the equations of motion, or to enter a modeling approach, where $\boldsymbol{v}$ is the given functional of $\mathbf{B}$. In any case the solved equations become nonlinear.

In the mean-field theory the expansion of fields $\boldsymbol{v}$ and $\mathbf{B}$ in the large-scale $\overline{\mathbf{U}}$ and $\overline{\mathbf{B}}$ and fluctuations $\boldsymbol{u}$ and $\boldsymbol{b}$ are introduced. We do not assume the smallness of fluctuations. Then from the Eq. (1) we obtain the equation for the mean-field generation (Zeldovich et al., 1983):

$$\frac{\partial \overline{\mathbf{B}}}{\partial t} = \nabla \times \left(\overline{\mathbf{U}} \times \overline{\mathbf{B}} + \alpha \overline{\mathbf{B}} \right) + \beta \Delta \overline{\mathbf{B}},$$
$$\nabla \overline{\mathbf{B}} = 0. \tag{2}$$

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Here α and β are, in general, second-rank tensors, depending on the velocity and magnetic field. To determine the form of these curves is the main task of mean-field theory. Convolution of $\alpha\overline{\mathbf{B}}$ determines the turbulent EMF (α-effect), and $\beta\Delta\overline{\mathbf{B}}$ gives the diffusion of the magnetic field, which consists of molecular and turbulent diffusions.

We will further consider the isotropic case of scalar α and β; β is assumed to be the constant.

2 Equations of the large-scale $\alpha\Omega$-dynamo

We suppose that the spatial structure of the mean-field is simple and confine ourselves to a single-mode approximation for the toroidal and poloidal components. Then these components may be described by the scalar functions $B^{\mathrm{T}}(t)$ and $B^{\mathrm{P}}(t)$, respectively. We also assume that the average flow $\overline{\mathbf{U}}$ is of differential rotation nature.

Taking into the account the above assumptions, on the basis of Eq. (2) the dynamo cycle stages may be written in the form of the following equations:

$$\frac{\mathrm{d}B^{\mathrm{T}}}{\mathrm{d}t} = GB^{\mathrm{P}} - \beta L^{-2}B^{\mathrm{T}},$$
$$\frac{\mathrm{d}B^{\mathrm{P}}}{\mathrm{d}t} = L^{-1}\alpha B^{\mathrm{T}} - \beta L^{-2}B^{\mathrm{P}}, \quad (3)$$

where $G > 0$ is the characteristic value of the differential rotation, α is the value of the alpha-effect, L is the characteristic linear dimension of the region. The first of Eq. (3) describes the Ω-stage, and the second is for the α-stage cycle.

Note that G is not the angular velocity of the field in these equations but just a measure of differential nature of the middle course. For example, if r is the distance to the rotation axis and the Ω is the angular velocity, then $G \sim |r\partial_r\Omega|$.

It is convenient to make the system dimensionless on the characteristic time of the magnetic diffusivity $L^2\beta^{-1}$ and the characteristic value of the field B_0. As a result, we

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obtain the following system of dimensionless variables:

$$\frac{\mathrm{d}B^{\mathrm{T}}}{\mathrm{d}t} = R_{\Omega} B^{\mathrm{P}} - B^{\mathrm{T}},$$
$$\frac{\mathrm{d}B^{\mathrm{P}}}{\mathrm{d}t} = R_{\alpha} B^{\mathrm{T}} - B^{\mathrm{P}}. \tag{4}$$

Dimensionless characteristics of the stages of the dynamo cycle R_{Ω} and R_{α} are GL^2/β and $\alpha L/\beta$, respectively.

In the assumption of the constancy of R_{Ω} and R_{α}, field generation, i.e. the growth of small fluctuations of $B = \sqrt{|B^{\mathrm{T}}|^2 + |B^{\mathrm{P}}|^2}$ occurs at $R_{\alpha} > R_{\Omega}^{-1}$. The field increases indefinitely at an exponential rate. If $R_{\alpha} < R_{\Omega}^{-1}$, the field is attenuated. Limited-largest nonvanishing solution can occur only if $R_{\alpha} R_{\Omega} = 1$. Thus, $D = R_{\alpha} R_{\Omega}$ is the dynamo-number. When $D = 1$, except for the zero steady-state solution, a lot of stationary regimes of the form $B^{\mathrm{T}} = R_{\Omega} B^{\mathrm{P}}$ appear in the system (Eq. 4), forming a straight line in an asymptotically stable phase plane.

Limited nonvanishing solutions of Eq. (2) are obtained by taking into account the feedback that is the change of the turbulent flow characteristics by the magnetic field in the result of the Lorentz force. In the models of Eq. (4) type this mechanism is implemented in the form of the prescribed dependence R_{α} on B. In the simplest case, functional dependencies of the form $R_{\alpha} = f(B(t))$ are introduced. Such type models are known as algebraic quenching models and the α-effect value depends on the field current value, i.e. a response to the changes in the field of turbulence instant. The simplest version of this dependence is given in Zeldovich et al. (1983). More complex variants, based on the representation of α as the differences between the kinetic helicity and corrent helicity, were studied, for example, in Field and Blackman (2002) and Brandenburg and Sandin (2004).

It is more realistic, however, that the restructuring of turbulence takes some time. Thus, it is interesting to note the results of Frick et al. (2006), the authors of which

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investigated the multiscale model dynamo. In this model, the equations of large-scale dynamo and the equations of shell-model of MHD turbulence were integrated. In the large-scale part of the model, the authors used the α^2-dynamo when a toroidal field is generated from a poloidal one by α-effect. The α-effect values were calculated by the variables of shell-model.

Having calculated the cross-correlation between the model variations of B and R_α, the authors of Frick et al. (2006) found that simultaneous values of B and R_α are uncorrelated. Moreover, if the response of B to the change of R_α is fast, the inverse response occurs with a noticeable delay, and the corresponding to the response cross-correlation decline is slow. As a result, the authors came to the conclusion that the response of R_α to B is essentially dynamic in nature and may not be described in terms of algebraic quenching.

This behavior indicates the presence of "memory" (hereditarity) or nonlocality in time. We can consider two ways to introduce nonlocality in the model (Eq. 4). In the first case, R_α is not a function, but a functional of B, i.e. α-effect value depends not only on the current state of the field, but also on all its previous states. In the second case, R_α is a function of B and a non-Markovian randomly process $\xi(t)$. Physically, this process may be comprehended as a contribution to the α-effect of discarded modes of mean-fields $\overline{\mathbf{U}}$ and $\overline{\mathbf{B}}$. The dependence of R_α on previous values B will be implemented through the "memory" of the process $\xi(t)$. These two variants of nonlocality will be further called the dynamic and randomly nonlocalities, respectively. Of course, combination of these two types of nonlocalities is also possible.

Further, the simplest variant of the algebraic quenching will be used as the original form of the feedback

$$R_\alpha(t) = R_\Omega^{-1}\left[1+\varepsilon\left(1-B^2(t)\right)\right], \tag{5}$$

where $\varepsilon > 0$ is the model parameter, which determines the efficiency of the feedback. A similar form of the dependence was considered in Zeldovich et al. (1983).

For the model (Eqs. 4 and 5), there are three stationary points. First and foremost is the zero point, which is unstable, which provides generation of the field. In addition, there are rest points of the form $B^{\mathrm{T}} = \pm R_\Omega (1+R_\Omega^2)^{-1/2}$, $B^{\mathrm{P}} = \pm(1+R_\Omega^2)^{-1/2}$. It is easy to show that these points are asymptotically stable. Thus, the model (Eqs. 4 and 5) gives field generation with the output to the characteristic value of $B = 1$. In this case, R_Ω determines the ratio of characteristic values of the toroidal and poloidal components. Therefore, during model calculations we'll always assume that $R_\Omega = 1$.

Introduction of nonlocality of dynamic type requires the following modification of the formula (Eq. 5):

$$R_\alpha = R_\Omega^{-1}\left[1+\varepsilon\left(1-\int\limits_0^t h(t-\tau)B^2(\tau)\mathrm{d}\tau\right)\right], \qquad (6)$$

where $h(t) \geq 0$ defines the hereditarily of the system.

The nonlocal model (Eqs. 4 and 6) has the same equilibrium points as Eqs. (4) and (5), and the computational experiments have shown that the nature of their stability remains the same.

The asymptotic stability of the points $B^{\mathrm{T}} = \pm R_\Omega(1+R_\Omega^2)^{-1/2}$, $B^{\mathrm{P}} = \pm(1+R_\Omega^2)^{-1/2}$ for this nonlocal model gives no possibility to reversals.

We return to the Eq. (4) with the constants R_α and R_Ω and consider their solutions more carefully. When $R_\alpha R_\Omega > 1$, the solution increases indefinitely without oscillations, and the characteristic time of the increase is $(-1+\sqrt{R_\alpha R_\Omega})^{-1}$. When $0 \leq R_\alpha R_\Omega < 1$, the solution decays without oscillations for the characteristic time of ~ 1. If $R_\alpha R_\Omega < 0$, the solution oscillates with the frequency $\sqrt{|R_\alpha R_\Omega|}$ and decays for the characteristic time of ~ 1.

Supposing now that R_α is a variable, we can say that negative peaks of this value are required for the occurrence of reversals, since during negative R_α in the linear case, oscillations appear. They must be strong enough, so that the oscillation period

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would be less than the characteristic decay time, and rare enough, so that a feedback mechanism would recover the field, decreasing during the reversal.

3 Model with random nonlocality

Peaks in the value of R_α, necessary for the formation of reversals may be obtained by the introduction of a random nonlocality into the model in the following form:

$$R_\alpha(t) = R_\Omega^{-1}\left[1+\varepsilon\left(1-B^2(t)\right)+\xi(t)\right], \quad (7)$$

where $\xi(t)$ is some non-Markov random pulse process with zero mean value.

In order to analyze the effect of pulses in $\xi(t)$ on the field, we first made some calculations in the model (Eqs. 4 and 7) for a case of non-random and regular process $\xi(t)$, which is a sequence of pulses with alternating signs of the type $\pm A\,e^{-t}, A \geq 0$. The interval between the pulses was 50 time units, the value $\varepsilon = 0.5$. The initial conditions were given as $B^{\mathrm{T}}(0) = 0$, $B^{\mathrm{P}}(0) = 10^{-2}$. The results of these calculations are shown in Fig. 1.

It is clear that positive pulses cause a sharp rise in the field, but are not accompanied by reversals. Field response on the negative peaks depends on the magnitude of these peaks. We see that for the small pulses the poloidal component does not change the sign ($A = 2$), then $B^{\mathrm{P}}(t)$ changes the sign for a short time and returns to its original value ($A = 4.2$). Such behavior of the field is well known in the paleomagnetic data and is called excursion (Merril et al., 1996). Then there is the reversal ($A = 10$). During the subsequent growth, the reversal is replaced by field excursion ($A = 15$) again. Then excursion combination appears with the subsequent reversal ($A = 30$), followed by two consecutive excursion ($A = 50$). The trend shown in Fig. 1 continues further, for example, when $A = 100$ the combination of two excursions and a reversal appears. However, such sharp peaks in R_α are difficult to admit in a real system. It is also clear that there are critical values of the amplitude A, separating the different types of field reversal. In particular, the critical value of A, separating the cases $A = 4.2$ and $A = 10$ from Fig. 1 is 4.455 ± 0.005.

We also see that for the chosen value of $\varepsilon = 0.5$ the time of t_B field transfer to a steady state is about 30 units. In general, as the numerical experiments showed, this dependence has a power law $t_B \sim \varepsilon^{-0.9}$.

We now define the random process $\xi(t)$ by the following formula

$$\xi(t) = \sum_{\theta_k \le t} \eta_k \exp\{-\lambda(t - \theta_k)\}. \tag{8}$$

Here θ_k is the increasing sequence of random instants of exponential pulses, η_k is random pulse amplitude, the constant $\lambda^{-1} > 0$ determines the pulse width.

We assume that the time intervals between pulses $\tau_k = \theta_k - \theta_{k-1}$ are independent and identically distributed with the probability density function (pdf) $p_\tau(t)$. Amplitudes η_k are assumed to be Gaussian random variables that are independent between each other and with the times of the pulses having zero mean and variance σ^2.

The important element of this model is the law of distribution of $p_\tau(t)$. If it is exponential, the pulse sequence forms a Poisson processes of events, and the process $\xi(t)$ turns to be a Markov one. Any other kind of law $p_\tau(t)$ leads to the fact, that the waiting time of the next pulse will depend on the time from the previous pulse. Thus, $\xi(t)$ will turn to be a non-Markov process.

We assume that the law $p_\tau(t)$ has a power asymptotic dependence $\sim 1/t^\gamma, \gamma > 1$. We will give a number of arguments in favor of this assumption.

Random intervals τ_k may be considered as the result of the joint effect of a large number of independent factors. If we suppose the additive character of the joint effect, than, according to the generalized central limit theorem, $p_\tau(t)$ should refer to the class of stable laws (Samorodnitsky and Taqqu, 1994). All such laws, except for the Gaussian one, have the power asymptotic dependence. Note, that for stable power laws $1 < \gamma < 3$.

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In addition, just the power distributions have the property of self-similarity, manifesting themselves in the reversals of geomagnetic field. Finally, the power of statistics are generally characteristic for turbulent phenomena, which include α-effect.

The explicit form of the pdf for the unilateral power stable laws is unknown, except for the distribution of Levi–Smirnov ($\gamma = 3/2$). It causes difficulties in obtaining their computer implementations.

Therefore, in the calculations we used the following expression for the pdf:

$$p_\tau(t) = \frac{\gamma - 1}{(1+t)^\gamma}, \quad t \geq 0, \quad 1 < \gamma < 3. \tag{9}$$

This form of the distribution law allow us to obtain the random variables τ_k easily.

The accepted distribution coincides with the stable one only asymptotically, and therefore for the distribution of polarity intervals, we will further be interested only in its asymptotics.

4 Simulation results

We consider the results of computational experiments in the model (Eqs. 4 and 7–9).

In the calculations, the values of the parameters $R_\Omega = 1$, $\lambda = 1$ were applied. SD of random amplitudes η_k of pulses in Eq. (7) is $\sigma = 6.6$. For this value of σ the reversal in the result of negative pulse in $\xi(t)$ occurs with the probability of 0.5. The initial values for the field components were chosen as $B^\mathrm{T}(0) = 0$, $B^\mathrm{P}(0) = 10^{-2}$.

We suppose the characteristic size of the Earth $L = 2.1 \times 10^6$ m (the thickness of the liquid core) and the turbulent magnetic diffusion $\beta = 10\,\mathrm{m^2\,s^{-1}}$. Then our dimensionless time $\sim 53\,700$ corresponds to the length of the longest scale of geomagnetic polarity (Pechersky, 1997) in 1700 Myear. Therefore, calculations in the model were carried out up to $t = 5 \times 10^4$.

Figure 2 shows an example of a segment of one of the magnetic field realizations and the toroidal and poloidal components for $\varepsilon = 5$.

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We calculated the different values of the parameters ε and γ. For ε parameter, the values 0.1, 0.5, 1.0, 5.0 were used, and for γ, the values from 1.1 to 2.9 with 0.2 step were used. For each parameter combination, histogram of the interval lengths of polarity and the fractal dimension of polarity model scale were estimated.

First, the obtained distribution of interval lengths of polarity ζ is considered. They are illustrated in Fig. 3.

It is clear that we may speak about the power asymptotic dependence of distribution of these intervals $\sim 1/\zeta^{\delta}$. More specifically, the power type for $\varepsilon = 0.1$ and $\varepsilon = 0.5$ appears from $\zeta = 10$, and for $\varepsilon = 1$ and $\varepsilon = 5$, from $\zeta = 30$. Deviation from the power law at low frequencies also occurs. Single events correspond to these frequencies. Therefore, these deviations may be explained by insufficient data.

We have calculated the value of the index δ on the straight section of the chart shown in Fig. 3 The obtained values and the correlation coefficients corresponding to the straight sections are shown in Table 1.

According to these values, it is easy to show that, for different ε, the correlation coefficient between γ and δ is more than 0.92. It means that γ and δ are linear, the coefficients of which depend on the parameter ε.

Integrability conditions of pdf for ζ implies that $\delta > 1$. These values are obtained from the above-mentioned linear relations for $\gamma > 2.1$.

Also note that Fig. 3 does not show the distributions for $\gamma < 1$. This is due to the fact that for such values of γ the rectilinear sections, corresponding to the power laws, do not occur on the graphs.

It may be concluded that the degree distribution of polarity interval occurs in the model at $\gamma > 2.1$.

Now consider the fractal dimension of the derived polarity scales. In the calculation, we followed the procedure proposed in Pechersky et al. (1997) for real geomagnetic polarity time scale.

The technique is as follows. On the scale of T length, some interval of Δ length is distinguished. $N(\Delta)$ is the number of intervals of Δ length on this scale, on which

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at least one reversal occurs. If $\Delta \ll T$ and the reversal are distributed uniformly, then $N(\Delta) \sim \Delta^{-1}$. If $\Delta \ll T$ and reversal are distributed unevenly, then we may expect the dependence of the form $N(\Delta) \sim \Delta^{-d}$. In this case, d is the Hausdorff dimension of the scale reversals and for $0 < d < 1$ the reversal series is fractal.

We made calculations in the model for the above mentioned values of γ and ε. The value of Δ decreased in the geometric progression from 5000 ($\Delta \ll 5 \times 10^4$) to ~ 10.

Graphs of the obtained dependencies are illustrated in Fig. 4 The dependence of $N(\Delta)$ accurately follows the power law. The figure legend shows the values of the Hausdorff dimension d. It is clear that in all the cases, $0 < d < 1$, and the reversal series is fractal, although there is a tendency to achieve the boundary of the fractal region when γ increases.

Note that, according to the data of Pechersky et al. (1997), Hausdorff dimensions for real geomagnetic polarity time scales for 170 Myear, 560 MYear, 1700 MYear are 0.88, 0.83 and 0.87, respectively.

5 Conclusions

The large-scale model of $\alpha\Omega$-dynamo with nonlocal α-effect for the modeling of geomagnetic field reversals was proposed. Nonlocality in time is provided by the pulse Markov process, which is supposed to be interpreted as the influence of rejected modes of mean-fields.

The power law was applied as the distribution law of the pulse waiting time. The reason for this was the power-law character of stable one-sided distributions, limiting distributions in the scheme of summation of independent random variables with slowly decaying pdf.

It was found out, that the power law of polarity interval distribution is asymptotically realized in this model, if the exponent γ in the distribution of the pulse waiting time is not less than 2.1. The exponent δ in the distribution of polarity interval is linearly

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related to the γ. The coefficients of this linear relation depend on the effectiveness of the feedback in α-effect.

It is shown that the model scale of geomagnetic polarity is a fractal set with Hausdorff dimension of $\gtrsim 0.7$. It is consistent with the actual Hausdorff dimension of geomagnetic scale according to the paper Pechersky et al. (1997).

Thus, it was established that the proposed large-scale dynamo model allows us to reproduce the main features of the process of geomagnetic field reversals.

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Table 1. Power in the distribution of polarity intervals.

γ	ε 0.1	0.5	1.0	5.0
2.1	1.05	0.97	1.04	1.12
2.3	1.11	1.28	1.36	1.49
2.5	1.53	1.63	1.5	1.93
2.7	1.93	1.67	1.91	2.04
2.9	2.22	2.12	1.94	3.53

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Reversal in nonlocal large-scale $\alpha\Omega$-dynamo

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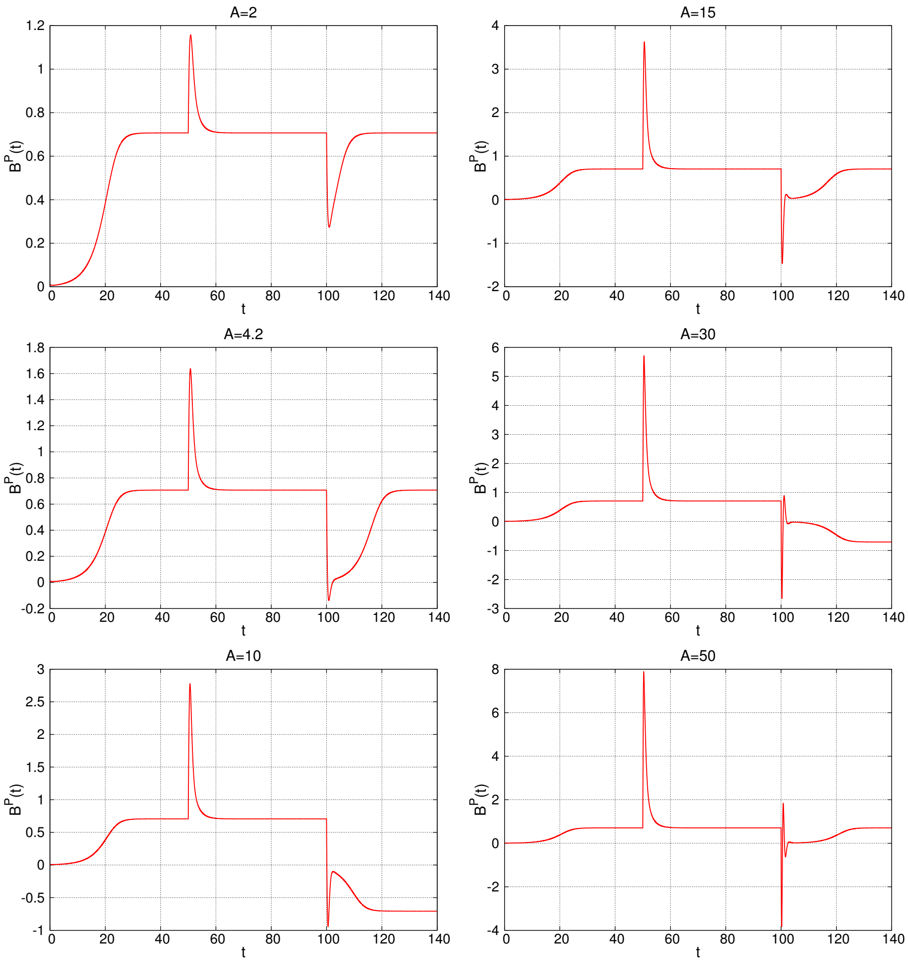

Figure 1. The response of the poloidal component $B^P(t)$ on the regular sequence of alternating pulses with different amplitudes A.

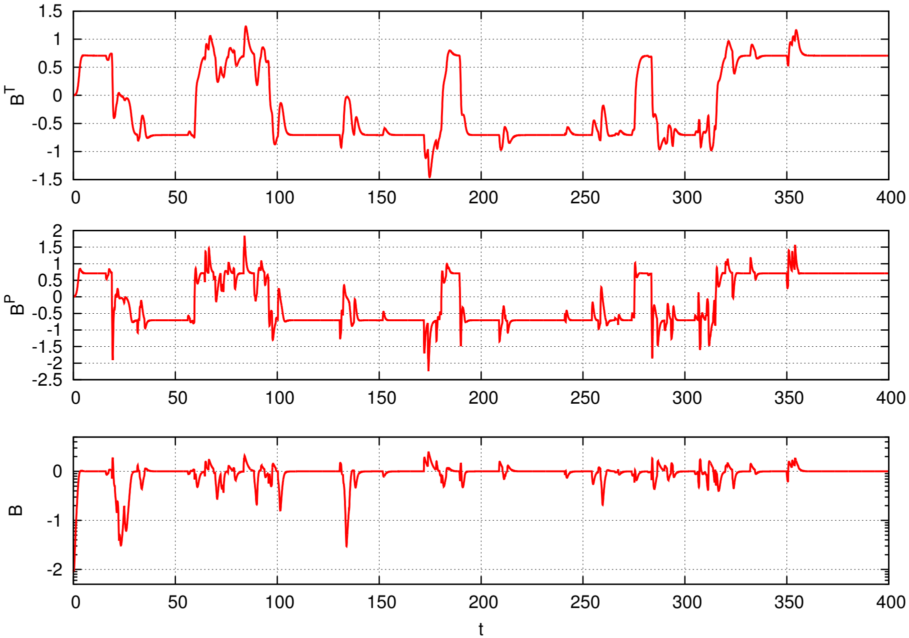

Figure 2. The segment of the magnetic field realization for $\varepsilon = 5$, $\gamma = 2.5$: the toroidal component of B^{T}, the poloidal component of B^{P}, field value of $B = \sqrt{|B^{\mathrm{T}}|^2 + |B^{\mathrm{P}}|^2}$.

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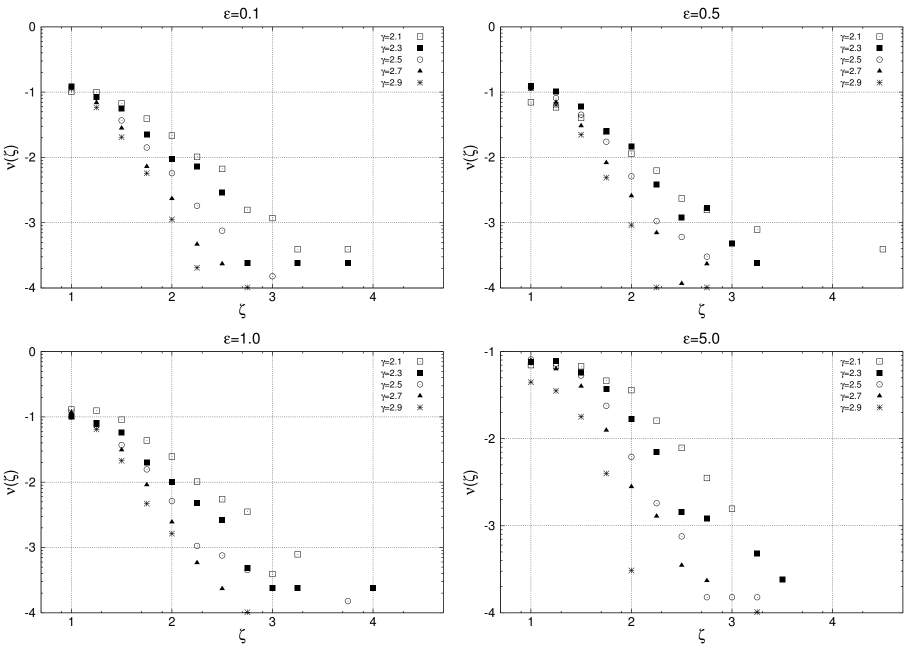

Figure 3. Distribution of relative frequencies ν of polarity intervals with the lengh ζ.

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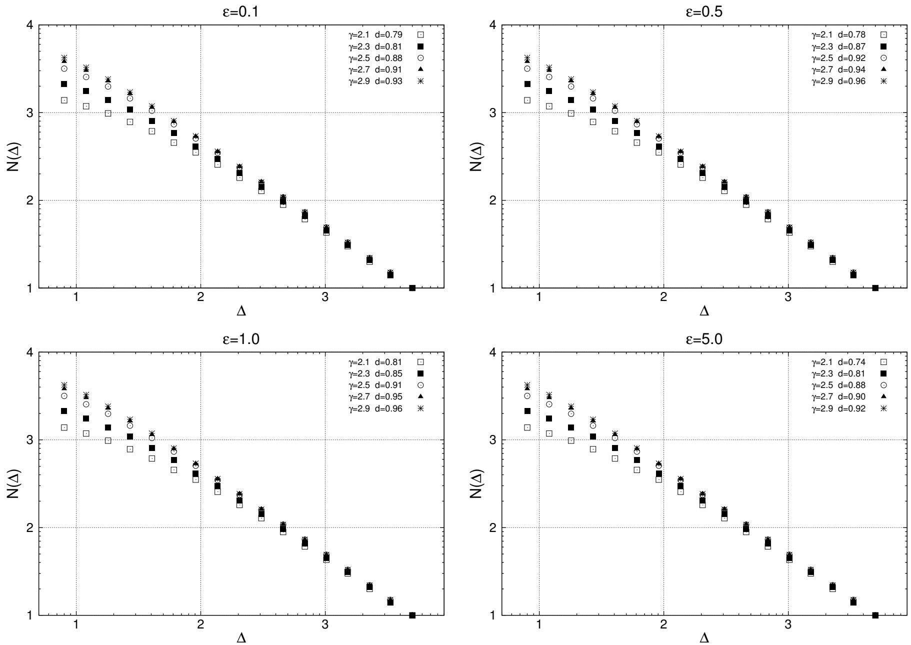

Figure 4. Number of $N(\Delta)$ intervals of Δ length, which contain at least one inversion.

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