

1 Reply to Reviewer 1

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3 Dear Prof. Perez-Munuzuri, and Dear Referee 1

4 Again I have to thank the reviewer for his criticism; actually the paragraph that
5 tried to explain the new Fig. 4 of the paper was not that clear. I changed some
6 parts of Section 3.1 where Fig.4 was described and I added there the results of
7 new simulations that support the idea that the imposed vertical stratification can
8 change the implicit diffusivity of the model and consequently the thermal energy
9 distribution of the model. I made also a few corrections in other parts of the
10 paper. A marked-up paper version is attached below.

11 Sincerely

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13 Nazario Tartaglione

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15 **Equilibrium temperature distribution and Hadley circulation**
16 **in an axisymmetric model.**

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24

25 **Abstract**

1

2 The impact of the equilibrium temperature distribution, θ_E , on the Hadley circulation
3 simulated by an axisymmetric model is studied. The θ_E distributions that drive the model are
4 modulated here by two parameters, n and k , the former controlling the horizontal broadness
5 and the latter controlling the vertical stratification of θ_E . In the present study, variations of the
6 θ_E distribution mimic changes of the energy input of the atmospheric system leaving as an
7 almost invariant the equator-poles θ_E difference. Both equinoctial and time-dependent Hadley
8 circulations are simulated and results compared. The results give evidence that concentrated
9 θ_E distributions enhance the meridional circulation and jet wind speed intensities even with a
10 lower energy input. The meridional circulation and the subtropical jet stream widths are
11 controlled by the broadness of horizontal θ_E rather than the vertical stratification, which is
12 important only when θ_E distribution is concentrated at the equator. The jet stream position
13 does not show any dependence with n and k , except when the θ_E distribution is very wide
14 ($n=3$) and in such a case the jet is located at the mid-latitudes and the model temperature
15 clamps to forcing θ_E . Using $n=2$ and $k=1$ we have the formulation of the potential temperature
16 adopted in classical literature. A comparison with other works is performed and our results
17 show that the model running in different configurations (equinoctial, solstitial and time-
18 dependent) yields results similar to one another.

2

1 Introduction

2 The earth's atmosphere is driven by differential heating of the earth's surface. At the
3 equator, where the heating is larger than that at other latitudes, air rises and diverges poleward
4 in the upper troposphere, descending more or less at 30° latitude. This meridional circulation
5 is known as Hadley cell. Two subtropical jets at the poleward edges of the Hadley form
6 because of earth rotation and the conservation of the angular momentum. A poleward shift
7 (Fu and Lin, 2011) and an enhanced wind speed of these jets (Strong and Davis, 2007) are
8 associated with a possible Hadley cell widening and strengthening, which has been observed
9 in the last decades (Fu et al., 2006; Hu and Fu, 2007; Seidel et al., 2008; Johanson and Fu,
10 2009; Nguyen et al., 2013).

11 There are a few studies suggesting possible causes of these phenomena. One of the
12 theories postulates global warming as a possible cause of Hadley cell widening (Lu et al.,
13 2009). However, the atmosphere is a complex system containing many subsystems interacting
14 with one another and the global warming might not be the only cause that is suggested to
15 explain the widening. Ozone depletion (Lu et al., 2009; Polvani et al., 2011), SST warming
16 (Chen et al., 2013; Staten et al., 2011) and aerosol (Allen et al., 2012) have also been invoked
17 to explain the Hadley cell widening.

18 Climate models vary to some extent in their response and the relationship between global
19 warming and Hadley cell is not straightforward. For instance, Lu et al. (2007) found a smaller
20 widening than the observed one. Gitelman et al. (1997) showed that the meridional
21 temperature gradient decreases with increasing global mean temperature and the same result
22 can be found in recent modeling studies (Schaller et al., 2013).

1 Much of our understanding on the Hadley cell comes from theories using simple models
2 (Schneider 1977, Schneider and Lindzen 1977 and Held and Hou 1980, hereafter HH80) and
3 such a simple model will be adopted here in order to understand how temperature
4 distributions can change the Hadley circulation. How much temperature change impacts the
5 real Hadley circulation is not clear yet, perhaps because of discrepancies between
6 observations, reanalysis (Waliser et al., 1999) and climate model outputs, although these
7 differences are becoming less marked because of newer observational datasets or correction
8 of the older ones (Sherwood 2008, Titchner et al., 2008, Santer et al., 2008). Hence, it is
9 critical to understand the possible mechanisms behind the cell expansion starting from a
10 simple model.

11 The objective of this study is to analyze the sensitivity of a model of ~~the~~ symmetric
12 circulation to the radiative-convective equilibrium temperature distribution. Our point of
13 departure is the symmetric model used by Cessi (1998), which is a bidimensional model
14 considering atmosphere as a thin spherical shell. This model will be briefly described in Sect.
15 2. The model describes mainly a tropical atmosphere, hence it does not allow for eddies.
16 Although eddies may play a central role in controlling the strength and width of the Hadley
17 cell (e.g. Kim and Lee, 2001; Walker and Schneider, 2006), a symmetric circulation, driven
18 by latitudinal differential heating, can exist even without eddies and it is a robust feature of
19 the atmospheric system (Dima and Wallace, 2003). The temperature distributions used in this
20 study represent some paradigms of tropical atmospheres. Among the possible causes that can
21 change temperature distributions there are El Niño, global warming and change of solar
22 activity. We will show, in Sect. 3, that the energy input is not as important as the forcing
23 distribution. Our results are consistent with those obtained both by Hou and Lindzen (1992)

1 (hereafter HL92), and recently by Tandon et al. (2013) who performed experiments similar to
2 those described here. The conclusions will be drawn in Sect. 4.

3 **2 The model**

4 The model used in this study is a bidimensional model of the axis-symmetric atmospheric
5 circulation described in Cessi (1998). The horizontal coordinate is defined as $y = a \sin \phi$ from
6 which we have

$$7 \quad c(y) = \cos \phi = \sqrt{(1 - y^2/a^2)} \quad (1)$$

8 where a is the radius of a planet having a rotation rate Ω , the height of atmosphere is
9 prescribed to be H .

10 The model is similar to the Held and Hou model (HH80), but it prescribes a horizontal
11 ~~diffusion viscosity~~ ν_H other than the vertical ~~diffusion viscosity~~ ν_v . The prognostic variables are
12 the angular momentum M , defined as $M = \Omega a c^2 + u c$ where u represents the zonal velocity;
13 the zonal vorticity ψ_{zz} with the meridional stream function ψ defined by

$$14 \quad \begin{aligned} \frac{\partial_x \psi}{\partial_x \psi} &\equiv -w; & \partial_y \psi &\equiv w; \\ \partial_z \psi &\equiv -c v & \partial_z \psi &\equiv -c v \end{aligned} \quad (2)$$

16 and the potential temperature θ that is forced towards a radiative-convective equilibrium
17 temperature θ_E . Starting from the dimensional equations of the angular momentum, zonal
18 vorticity and potential temperature, we will obtain a set of dimensionless equations. The new
19 equations are non-dimensionalized using a scaling that follows Schneider and Lindzen (1977),
20 but the zonal velocity u is scaled with Ωa . A detailed description can be found in Cessi
21 (1998).

1 The non-dimensional model equations are:

$$2 \quad M_t = \frac{1}{R} \left\{ M_{zz} + \mu [c^4 (c^{-2} M)_y]_y \right\} - J(\psi, M) \quad (3a)$$

$$3 \quad \psi_{zzt} = \frac{1}{(R^2 E^2)} \gamma c^{-2} (M^2)_z - \frac{1}{c^{-2}} J(\psi, c^{-2} \psi_{zz}) + \frac{1}{(R E^2 c^{-2})} \theta_y + \frac{1}{(R c^{-2})} [c^{-2} \psi_{zzzz} + \mu \psi_{zzyy}] \quad (3b)$$

$$4 \quad \theta_t = \frac{1}{R} \left\{ \theta_{zz} + \mu [c^2 \theta_y]_y + \alpha [\theta_E(y, z) - \theta] \right\} - J(\psi, \theta) \quad (3c)$$

5 The term $J(A, B) = A_y B_z - A_z B_y$ is the Jacobian.

6 The thermal Rossby number R ; the Ekman number E , the ratio of the horizontal to the vertical

7 viscosity μ and the parameter α are defined as

$$8 \quad \left\{ \begin{array}{l} R \equiv gH \Delta_H / (\Omega^2 a^2); \quad E \equiv \nu_V / (\Omega H^2); \quad \mu \equiv (H^2 / a^2) \frac{\nu_H / \nu_V}{\nu_H / \nu_V}; \quad \alpha \equiv H^2 / (\tau \nu_V) \end{array} \right. \quad (4)$$

10 The term α is the ratio of the viscous timescale across the depth of the model atmosphere to

11 the relaxation time τ toward the radiative-convective equilibrium.

12 The boundary conditions for the set of Eq. 3 are:

$$13 \quad \begin{array}{l} M_z = \gamma(M - c^2), \quad \psi_{zz} = \gamma \psi_z; \\ \psi = \theta_z = 0 \text{ at } z = 0; \\ M_z = \psi_{zz} = \psi = \theta_z = 0 \text{ at } z = 1. \end{array} \quad (5)$$

14 Where $\gamma = \frac{CH}{\nu_V}$ is the ratio of the spin-down time due to the drag to the viscous timescale, the

15 bottom drag relaxes the angular momentum M to the local planetary value $\Omega a c^2$ through a

16 drag coefficient C .

17 The model flow started from an isothermal state at rest and is maintained by a Newton

18 heating function where the heating rate is proportional to the difference between the model

1 potential temperature and a specified radiative-convective equilibrium temperature
2 distribution, which follows the HH80 one:

$$3 \quad \theta_E = \frac{4}{3} - y^2 + \frac{\Delta V}{\Delta H} \left(z - \frac{1}{2} \right). \quad (6)$$

4 Equation 6 is used extensively in dry axisymmetric models (e.g. HH80, Farrell, 1990, Cessi
5 1998) and it is related to the thermal forcing term of the equation system. A statically stable
6 state as a vertical profile of θ_E is also assumed by Eq. 6. HH80 suggested that the impact of
7 latent heat released by water vapor condensation can be incorporated in dry axisymmetric
8 models by modifying the meridional distribution of θ_E . HL92 followed the HH80 argument
9 and altered the concentration of θ_E under the constraint of equal energy input. The resulting θ_E
10 distributions used by HL92 were peaked distributions on and off the equator resulting in a
11 stronger Hadley circulation with respect the circulation obtained applying Eq. 6. Tandon et al.
12 (2013) used narrow and wide thermal forcing to mimic El Niño or global warming effect on a
13 tropical circulation in a Global Circulation Model. On the opposite side, in fact, we can
14 suppose that if a warmer climate happens, especially in the tropical regions, a very weak
15 gradient of the equilibrium temperature θ_E will be more extent in latitude, expanding
16 consequently the tropical region. This is already occurred in the past, especially in the mid
17 Cretaceous and Eocene when the tropics extended up to 60° . This is the so called equable
18 climate (e.g. Greenwood and Wing, 1995) where roughly equal temperatures are present
19 throughout the world. During those geological ages the temperature was generally higher
20 everywhere, but adding a constant to the temperature does not change the response of this
21 kind of models. The equator-pole temperature gradient was smaller than the present situation,
22 whereas we prescribe constant surface equator-pole θ_E gradient. As we shall show afterwards
23 this is necessary to demonstrate that it is the tropical temperature gradient drives the Hadley

1 circulation. Thus, in order to study systematically these different conditions we adopt the
2 strategy to build forcing functions dependent on a parameter that controls the θ_E gradient in
3 the tropical regions. Since, with different horizontal distributions of θ_E we can figure out that
4 even the vertical distribution could be affected by some physical mechanisms that make the
5 atmosphere more or less stable than the stratification described by the z component of Eq. 6.
6 The meridional and vertical changes of equilibrium temperature can be obtained by changing
7 the exponents of y and z in Eq. 6 transforming Eq. 6 in the following equation:

$$8 \quad \theta_E = \frac{4}{3} - |y|^n + \frac{4V}{4H} \left(z^k - \frac{1}{2} \right). \quad (7)$$

9 The ~~valuesterms~~ n and k control the horizontal distribution of θ_E and its stratification
10 respectively. Small values of n are associated with concentrated θ_E distributions. Increasing n
11 means increasing broadness of the θ_E distribution. Values of k larger than ~~one~~ mean more
12 stable ~~stratification at~~ upper levels, vice-versa smaller k values means lower levels are more
13 stable than upper levels. Thus, it comes quite natural to explore the response of Hadley
14 circulation by changing the parameters n and k , which control the distribution of θ_E , in closest
15 ranges of 2 and 1 respectively. Thus, n and k will change from 0.5 to 3 with a 0.5 step, in such
16 a way ~~weto~~ have a set of 36 simulations. When $n=2$ and $k=1$ Eq. (7) becomes ~~the reference~~
17 ~~equilibrium temperature given in~~ Eq. 6 and the ~~experimentexperiment~~ performed with ~~such~~
18 ~~values of $n=2$ and $k=1$~~ will be considered as the reference ~~experimentexperiment~~.

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19 -The ~~average θ_E along the latitudes~~ meridional and ~~heights~~ vertical averages of θ_E are
20 shown in Fig. 1. Heating functions with n value equal to 0.5 should not be regarded as unreal,
21 but merely as a simple way to represent a specific state of the atmosphere. The same assertion
22 is valid for all other parameters. As n increases the average temperature increases as well, but
23 the meridional gradient decreases in the tropical regions- (Fig. 1a).

1 | With the prescribed θ_E as specified by Eq. 7, ~~the~~ θ_E values at the boundaries and its
2 | equator-pole difference temperature remain invariant with respect to n , for a given k value.
3 | The energy input is not constant here, which differs from HL92, which analyzed the influence
4 | of concentration heating perturbing the forcing function $\theta_E(y,z)$ in such a way that θ_E
5 | averaged over the domain remained constant. It is easily visible in Fig. 1b. Higher n values,
6 | keeping k invariant, have higher averaged θ_E at all levels. The same is true for k , with higher k
7 | values, for n constant; θ_E at each level is always higher than that with lower ~~k~~ values. The
8 | pole-equator θ_E difference at upper and lower vertical boundaries are the same for all the
9 | experiments having the same k , the vertical averaged θ_E changes as a function of k , for n
10 | constant.

11 | Whether global warming makes the equilibrium temperature distribution narrower or
12 | wider is beyond the aim of the paper. One can expect that global warming broadens the
13 | temperature distribution, but at the same time it could have an impact above all on the sea
14 | surface temperature (SST) bringing more water in the upper atmosphere which changes the
15 | vertical distribution especially of the temperature in the inter-tropical convergence zone
16 | (ITCZ). It is supposed that, in first approximation, oceans force the atmosphere, so we have to
17 | allow for the possibility that increasing SST can change the forcing distribution. Increasing
18 | uniformly SST might could a poleward expansion as showed by Chen et al. (2013) with an
19 | ~~aqua-planet~~aqua-planet model, but in that case the mechanism was supposed to be related
20 | mainly to mid-latitude eddies rather than a tropical forcing. Since other causes can change the
21 | temperature distribution of a planet such as changes in the solar activity for instance, we will
22 | focus on the temperature distribution regardless of its cause.

23 | In this model the atmosphere is dry as in many other studies (e.g. Schneider 1977, HH80,
24 | Caballero et al. 2008), changing the θ_E distribution allows for a change in the static stability.

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1 Looking at the average θ_E along the vertical direction, low values of k are related to low
 2 values of static stability, especially in higher level of the model atmosphere.

3 The Brunt–Väisälä frequency, when the atmosphere reaches the equilibrium will be

$$4 \quad N^2 = \frac{(gk\Delta_V/\Delta_H z^{(k-1)})}{[4/3 - \gamma^n + \Delta_V/\Delta_H(z^k - 1/2)]}. \quad (8)$$

5 It is clear from Eq. 8 that the Brunt–Väisälä frequency does not depend on n at the poles and
 6 equator. On the contrary, it depends on k ; large values of k imply a more stable atmosphere in
 7 the upper levels, especially at poles, making the model atmosphere more similar to the real
 8 one, simulating in some respects a sort of tropopause. Moreover, this is equivalent to creating
 9 a “physical” sponge layer in the upper levels of the model that will have some effects on the
 10 vertical ~~position~~location of the stream function maximum.

11 Starting from Eq. 7 a set of experiments were performed changing n and k in such a way
 12 to have a set of numerical results. In order to isolate the contribution of the θ_E distribution on
 13 the solution of Eq. 3, a set of parameters will be used:

$$14 \quad a = 6.4 \times 10^6 \text{ m} \quad \Omega = 2\pi/(8.64 \times 10^4) \text{ s}^{-1}$$

$$\Delta_H = 1/3 \quad \Delta_V = 1/8$$

$$g = 9.8 \text{ ms}^{-2} \quad C = 0.005 \text{ ms}^{-1}$$

$$H = 8 \times 10^3 \text{ m} \quad \tau = 20 \text{ days}$$

$$15 \quad \nu_V = 5 \text{ m}^2 \text{ s}^{-1} \quad \nu_H = 1.86 \text{ m}^2 \text{ s}^{-1} \quad (9)$$

16 The parameters in Eq. 9 are the same as those used by Cessi (1998).
 17

1 3 Numerical Results

2 This section is divided into three subsections, the first showing the results of the model
3 applying the equinoctial condition, when the sun is assumed to be over the equator. The
4 solution is steady as already shown for instance in Cessi (1998). The second subsection will
5 show the results of the model having a θ_E distribution described by Eq. 7 but moving
6 following a seasonal cycle. ~~The~~ Finally, the case $n=2$ and $k=1$ ~~is~~ will be discussed in the third
7 subsection in comparison with previous studies.

8 3.1 Equinoctial simulations

9 The axially symmetric circulation is forced by axially symmetric heating as in HH80 and
10 many others and as prescribed by Eq. (7). The model started from an isothermal state and it
11 was run for 300 days, even though it reached its equilibrium approximately after 100 days, in
12 order to be sure that the model does not have instabilities in the long run. The stream function
13 values obtained when $n=2$ and $k=1$, i.e. the reference experiment, are about the same of that
14 obtained by HH80. We will show the non-dimensional value, but to have the dimensional
15 values we need to multiply by $\nu_V R \varepsilon^{-1} = 484 \text{ m}^2 \text{ s}^{-1}$.

16 The absolute value of the maximum stream function intensity at the equilibrium
17 conditions for the 36 experiments is shown in Fig. 2. When $n=0.5$, with k constant, the
18 circulation is always the strongest. The stream function intensity is inversely proportional to n
19 (Fig. 2a). With $n=0.5$ the experiment resembles the one described in HL92 where they
20 concentrated the latitudinal extent of heating and this led to a more intense circulation.
21 However, they imposed the forcing function $\theta_E(x, y)$ in such a way that its average over the
22 domain remained the same as in the control experiment, i.e. without changing the energy
23 input. They found that concentration of the heating through a redistribution of heat within the

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1 Hadley cell led to a more intense circulation without altering its meridional extent. Instead,
2 here, it is evident from Fig. 1 that the experiment with $n=0.5$ has an energy input lower than
3 the other cases. Nevertheless, the Hadley circulation is always more intense than the other
4 cases and ~~contrary opposite~~ to higher n value ~~experiments~~ experiment results, the circulation is
5 confined ~~close~~ closer to the equator. Thus, the results of HL92 are extended to a more general
6 case with a lower energy input. It is worth noticing the constraint of an equal pole-equator
7 gradient of mean θ_E is assumed here differently from HL92 (Fig. 1a).

8 The dependence on k is not as straightforward as the one on n , instead. The stream
9 function reaches the highest value for $n=0.5$ and $k=3$. With ~~a~~ high n values the Hadley cell
10 ~~stream function intensity~~ strength is lower and the dependence on k loses its importance. In
11 other words, in our model, the symmetric circulation strength is modulated by k only when
12 the equilibrium temperature distribution is concentrated to the equator.

13 Figure 2b shows the maximum zonal wind speed as function of n and k , it is inversely
14 proportional to n , the dependence on k is not as clear as the one on n and when $n=3$ it almost
15 vanishes in accordance with the behavior of the maximum stream function. These results are
16 in agreement with HL92, who found a stronger zonal wind when the forcing was concentrated
17 at the equator.

18 Some observative studies define the border of ~~the~~ Hadley cell ~~as that by the zero line~~
19 ~~of where~~ the ~~500 hPa~~-stream function goes to zero at 500 hPa (e.g. Frierson et al., 2007). Since
20 in ~~this kind of our~~ model the zero stream function is at the poles, it is problematic to define an
21 edge of the Hadley cell based on the zero stream function. Moreover, ~~the~~-circulation intensity
22 changes greatly in our experiments, so it is ~~problematic~~ puzzling to define an edge of the
23 Hadley cell based on an absolute value of the circulation itself. Hence, we will ~~define~~ look at

1 | the ~~position of the cell equal to the position~~location of the maximum ~~value of~~ stream function,
2 | ~~in this way and~~ we will ~~study a possible~~analyze its poleward shift ~~of the cell~~ as a function of
3 | the two parameters n and k . The edge of the cell might be also defined by values of isolines
4 | that are relative with respect to the maximum value, for example 1/4 of the stream function.
5 | For the sake of clarity this definition ~~is~~would be an operational one and does not follow the
6 | definition used in observative studies, for example by Dima and Wallace (2003) or Frierson et
7 | al. (2007).

8 | The latitude of the maximum stream function value shows a general dependence on n
9 | and k . It increases with n and decreases with k . However, as shown in Fig. 3a, this dependence
10 | is not straightforward or linear, although we have a few exceptions, for instance when
11 | $k=n=0.5$. Hence, in general, when n increases, and the temperature gradient at tropics
12 | decreases, even though the total energy input is larger, the stream function is weaker and the
13 | Hadley cell moves poleward. This result is in agreement with other model outcomes (Frierson
14 | et al, 2007, Lu et al., 2008; Gastineau et al., 2008; and Tandon et al., 2013). The model
15 | predicts a weakening of circulation, in contrast with the strengthening, together with
16 | widening, of the Hadley circulation for the past three decades observed by Liu et al. (2012)
17 | and Hu and Fu (2007). However, Liu et al. (2012) showed that if the observations start from
18 | 1870, the Hadley cell has become more narrow and stronger.

19 | The height of the maximum stream function value is confined for almost all the
20 | simulations under 2200 m and the general rule is that when n increases, the height of
21 | maximum lowers, however a few experiments, those with $k=0.5$ and $n=0.5, 1$ and 1.5 , have
22 | the maximum value between 4300 and 5600 m ~~exhibiting an increase in the height with n~~
23 | ~~(Fig. (Fig. 3b).~~

1 In general, the location of the maximum zonal wind speed does not show any evident
2 relationship with the parameters n and k . It is always confined between 26° and 29° off the
3 equator; however, when $n=3$, there is an abrupt transition to about 48° , independently from
4 the k value. In Table 1, we show the latitude of the maximum wind speed when $k=1$ for
5 different n values.

6 The difference between θ_E and θ , once the model reaches the equilibrium, is quite
7 interesting- ~~when k is not equal to one.~~ Figure 4 shows meridional distributions of θ_E and θ
8 for $n=3$ and $k=0.5, 1$ and 3 . In Fig. 4a, ~~$k=0.5$, θ_E is under θ~~ , when $k=1$ we find θ_E is
9 over θ in a region around the equator (Fig. 4b), with θ_E crossing θ at about 47° , finding again
10 the equal area condition suggested by HH80 and that explains approximately the jet location;
11 whereas in Fig.4c, with $k=3$, we can see how θ_E is over θ . ~~Despite these differences in the~~
12 ~~distributions of θ_E and θ the model produces~~ Nevertheless, all simulations with these different
13 ~~k values $n=3$ give~~ almost the same solution, in terms of circulation strength and jet location-
14 (Figs. 2 and 3). For other values of n the results are similar, but the differences between θ_E
15 and θ are not so visible.

16 We can understand these findings in the light of Cessi (1998) ~~results obtained who~~
17 ~~analyzed the model described~~ by ~~expanding the the set of Eqs. 3 by using an asymptotic~~
18 ~~expansion of the~~ variables M , θ and $-\psi$ in power series of ~~the Rossby number R~~ . The ~~term R^2~~
19 ~~in nonlinear expansion part, the~~ meridional advection, ~~in the nonlinear term of the expansion,~~
20 depends on the differences between θ_E and θ , ~~$\theta_E - \theta$~~ ; on the cube of the meridional
21 temperature gradient; and ~~quasi-~~linearly on the imposed stratification; deducing that for
22 unstable ~~stratifications~~stratification, this term would appear as a negative diffusivity term, ~~a~~
23 ~~condition that can exist even with some stable stratifications~~ (Cessi, 1998), ~~whereas it acts as~~

1 ~~a positive diffusion.~~ This seems to be ~~the case, in our simulation case~~ when $k=0.5$. ~~The~~
2 ~~thermal energy obtained in the model is larger than the imposed temperature (Fig.4a).~~
3 Although the stratification imposed by Eq. 7 is stable, i.e. $\frac{\partial \theta_E}{\partial z} > 0$, the second derivative is
4 negative when $k=0.5$, reducing the stability at upper levels, so this situation can be seen as a
5 way to simulate the effect of the latent heat released by water vapor condensation. ~~When $k=3$~~
6 ~~the air in upper levels is very stable and the flow has to do more work, giving rise to a sort of~~
7 ~~implicit dissipation. Nevertheless, the model acts to bring the vertical temperature gradient in~~
8 ~~a more stable configuration and the Hadley circulation is in any case reproduced~~
9 ~~demonstrating the robustness of the model~~ Running the model with an enhanced vertical
10 viscosity (five time the value defined in Eq. 9) the situation described by Fig. 4a changes to
11 look like that of Fig. 4b. Defining stratification with $k=0.5$ is consequently equivalent to
12 reduce the actual vertical diffusivity.

13 When $k=3$ the air in upper levels is very stable and the upward flow has to do more work
14 to rise at upper levels; most of the thermal energy that drives the model atmosphere is
15 evidently dissipated by this work, reducing the actual energy with respect to that provided by
16 θ_E . We performed some runs with reduced vertical viscosity, the actual value of θ in Fig. 4c
17 slightly increases becoming closer to θ_E , but it remains constantly under the θ_E curve, even
18 for values of vertical viscosity close to 0.1 (with vertical viscosity very close to zero or
19 negative the model blows up). This should not interpreted as an unphysical result, but it has to
20 be seen as the difficulty of flow temperature to relax to θ_E because of very stable imposed
21 stratification In any case, the Hadley circulation is still reproduced demonstrating the
22 robustness of the model.

1 With n getting larger, the θ_E distribution becomes flatter in the tropical region and θ
2 clamps to θ_E . In general, we expect that a vigorous circulation occurs in a fast rotating planet
3 unless the thermal gradient becomes small ~~in the~~ tropics. In such a case the angular
4 momentum homogenization is equivalent to a weakening of the rotation (Cessi, 1998). If the
5 circulation is proportional to the cube of the meridional temperature gradient, it is quite
6 evident that when such a gradient has high values in the tropical region the circulation is
7 vigorously driven by this term, whereas when it approaches to zero it is the term $\theta_E - \theta$
8 dominates. HH80 found that the edge of the Hadley cell was at the mid-latitudes when the
9 planetary rotation was lower than that of the earth. Since this phenomenon is here observed
10 for a wider forcing distribution, this common result may be attributed to ~~a low efficiency in at~~
11 the process of homogenization of momentum and temperature in the equatorial region.

12 In order to explain equable climates like those supposed to be occurred in Cretaceous
13 and Eocene, Farrell (1990) formulated an axisymmetric model starting from ~~the that of~~ Held
14 and Hou ~~model~~ and used a forcing with $n=2$ and $k=1$ ~~where the~~, introducing a radiative-
15 diffusive term to make flatter the model temperature gradients ~~became flat because of a~~
16 dissipation term at tropics. For high values of n the θ distributions are similar to those
17 obtained by ~~our forcing conditions~~ Farrell with high values of its diffusive parameter. In some
18 respects, flattening of the forcing distributions ~~distribution~~ is equivalent to have ~~the same~~
19 dissipation a diffusive term in the, and this also explains Fig. 4c. The Farrell model (1990)
20 ~~model. The~~ showed a poleward shift of the zonal jet and it has to be noticed that a poleward
21 shift of the subtropical jets was also observed by HH80 when increasing the vertical
22 ~~diffusion~~ viscosity.

1 Figure 5 shows the stream function and the zonal wind speed for the experiments
 2 $n=k=0.5$ (Fig. 5a) and $n=k=3$ (Fig. 5b). The parameter n controls the Hadley cell and jet
 3 stream widths. The results show that such with $n=k=0.5$ the Hadley cell and jet streams are
 4 quite narrow. As far as the vertical position of the maximum value of the stream function is
 5 concerned, ~~the~~ experiments with $k=0.5, 1$ and 1.5 exhibit particular behavior with respect to
 6 the other experiments. The stream function has its maximum at upper levels. This is related to
 7 the different stratification imposed by the parameter ~~k~~ . ~~Stratification with low values of k~~
 8 favor air to move to higher levels with respect to experiments with higher k values.

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10 3.2 Time-dependent simulations

11 Since heating depends on solar irradiation, it is of interest to analyze the solutions
 12 obtained by the annually periodic thermal forcing and to compare it with the steady solutions
 13 described previously in this paper. Starting from Eq. ~~(7)~~, we can formulate an equilibrium
 14 temperature distribution having the maximum heating off the equator at latitude y_0 :

$$15 \theta_E = \frac{4}{3} - |y - y_0|^n + \frac{\Delta_V}{\Delta_H} \left(z^k - \frac{1}{2} \right). \quad (10)$$

16 where y_0 in Eq. ~~(10)~~ is dependent on time according to

$$17 y_0(t) = \sin\left(\frac{\varphi_0 \pi}{180}\right) \cdot \sin\left(\frac{2\pi t}{360 \text{ days}}\right) \quad (11)$$

18 where φ_0 is the maximum latitude off the equator where heating is maximum. Equations
 19 ~~10~~, with $n=2$ and ~~11~~, with $k=1$, and ~~11~~ are the same used by Fang and Tung (1999) with the
 20 choice of maximum extension of φ_0 consistent with the choice of Lindzen and Hou (1988),
 21 i.e. $\varphi_0 = 6^\circ$. A prescribed equilibrium temperature varying seasonally makes the simulations

1 | more realistic. ~~As described previously, here we~~We will focus on the average and maximum
2 | values, in absolute terms, of the stream function and zonal speed obtained during 360 days of
3 | simulations. The averaged values are obtained in these cases by averaging the outputs
4 | obtained every 30 days, starting from the minimum corresponding to the summer Hadley cell
5 | in the boreal hemisphere.

6 | The annual averages of the time-dependent and equinoctial circulations shows that
7 | maximum stream functions and zonal wind speeds behave quite similarly (Fig. 6),
8 | nevertheless the instantaneous Hadley circulation almost never resembles the modeled
9 | circulation (Fang and Tung, 1999) as well as the real one (Dima and Wallace, 2003).

10 | The maximum stream function is obtained here when $k=n=0.5$ (Fig. 6a). In general, for
11 | $n=0.5$, we have stronger circulations and winds. These simulations confirm the inverse
12 | relationship between stream function strength and n . The circulation strength expressed as
13 | annually averaged value is weaker when compared with that obtained in the equinoctial
14 | experiments, when n is low and k is high, otherwise it is only slightly stronger, but it is never
15 | twice as strong as that of the equinoctial solution as found by Fang and Tung (1999). When
16 | $n=2$ and $k=1$ our results are consistent with those obtained by Walker and Schneider (2005) as
17 | discussed in the Subsect. 3.3. For example, there is not an analog maximum when $n=0.5$ and
18 | $k=3$ found in the steady solution. The maximum of the annually averaged ~~maximum~~ wind
19 | speed shows only a slight dependence on k when k is low.

20 | The meridional position and the height of the maximum stream function show that there
21 | is no clear dependency on n and k (Fig. 7). The difference between the time-dependent
22 | simulations and the average of the steady solutions is quite interesting. It is to be noticed that
23 | the latitude of the stream function maximum in the time-dependent solution is in the range of

1 12.5° and 16° (Fig. 7a), whereas in the equinoctial solutions the correspondent latitude is
2 within a larger range. The maximum stream function is located at higher levels, between 4500
3 and 6000, when k is equal or less to one when and n is less than 2.5. Otherwise the maximum
4 is positioned under 3000 m ~~except~~ (Fig. 7b). ~~Although the~~ The location and strength of
5 averaged results ~~seem interesting, they~~ are impressively similar to those obtained by ~~the~~
6 ~~steady~~ experiments, ~~they are obtained by averaging snapshots of the time dependent~~
7 ~~simulations and thus a cautionary note should be made about these results with steady~~
8 ~~solution.~~

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9 More than the steady solution, it is evident that the height of the maximum stream
10 function is lower when $k=3$. In the steady solution this phenomenon is not that evident. When
11 $k=3$, the vertical gradient of θ_E is higher in upper levels making those levels more stable and it
12 prevents, evidently more than the equinoctial solution, air from moving higher leaving
13 circulation occurring at lower levels. The case $k=3$ is equivalent to imposing a “natural”
14 sponge layer at the top of the model. Thus it does not come as a surprise that the maximum
15 stream function is lower than those observed in simulations with other k values. This result is
16 analogous to that of Walker and Schneider (2005) that removed the maximum stream function
17 at higher levels found by Lindzen and Hou (1988) adopting a numerical sponge layer at the
18 top of the model. A comparison with previous works of the simulations with $n=2$ and $k=1$ will
19 be discussed in the Subsect. 3.3. On the contrary, with low k values, the presence of weaker
20 θ_E gradient at upper levels favors air to move higher and the maximum stream function is
21 observed at upper levels. ~~There are more time dependent simulations with respect to the~~
22 ~~steady solutions that exhibit this upper level maximum stream function.~~

1 The position of the jet stream is almost similar to the one observed in the steady solution.
2 It is confined between 28° and 30° , with latitude of averaged jet remaining almost at the same
3 place, except when $n=3$ the jets are located at about 44° confirming the abrupt transition of
4 the jet stream position when $n=3$ already found for the equinoctial experiment. Fu and Lin
5 (2011) suggest that the jets moved poleward of about 1° per decade in the last several years
6 but Strong and Davis (2007) observed that Northern hemisphere subtropical jet shifted
7 poleward over the east Pacific, while an equatorward shift of the subtropical jet was found
8 over the Atlantic basin. Excluding the case $n=3$, all the other subtropical jets in the different
9 experiments have the position of the maximum very close to one another and the shifting
10 range is very limited. Thus, when a vigorous circulation occurs the jet location must be
11 located at about 30° , whereas reducing too much the tropical gradient the ~~process of~~
12 homogenization ~~becomes weaker~~ looks like ~~in that of~~ a slow rotating planet and this is
13 confirmed in the time-dependent solution. Both Tandon et al. (2013) and Kang and Polvani
14 (2011) found a discrepancy in this area with the jets that do not follow the Hadley cell edge.
15 In an axisymmetric model, defining the Hadley edge as a function of the stream function and
16 connecting it to the jet location is problematic because of lacking of a zero value of the stream
17 function.

18 Figure 8 shows the annually averaged circulation for the same cases as shown in Fig. 5,
19 which is obtained by annually averaged heating. It is impressive how the steady and time-
20 dependent solutions resemble each other. As in Fang and Tung (1999) the annual mean
21 meridional circulation has the same extent, but differently from them the strength of the
22 annual mean circulation of the time-dependent solution is almost the same of the steady
23 solution.

1 When the heating center is off the equator the intensity of the winter cell is stronger,
2 whereas the cell of the summer hemisphere is weak and sometimes almost absent. Figures 9
3 shows the maxima of the stream function and zonal wind speed at the winter solstitial as a
4 function of n and k . The maximum stream function as a function of n and k has the same
5 configuration of the steady solution. Here, as expected the maximum intensity of the
6 meridional circulation (Fig. 9a) reached during the simulation is twice as strong as that of the
7 steady solution or the annually averaged time dependent solution and it has about the same
8 strength of the observed circulation. However, the winds are much stronger too, in contrast
9 with observations. The zonal wind has a different configuration instead, the maximum zonal
10 wind speed is obtained when $n=1$ (Fig.9b).

11 We can inspect a couple of simulations when the stream function reaches its maximum
12 in the boreal hemisphere. Figure 10 shows the stream function and the zonal wind speed when
13 $n=2$ and $k=0.5$ (Fig. 10a, b) and $n=2$ and $k=3$ (Fig. 10 c, d). When $k=0.5$ (upper panels) the
14 boreal (winter) circulation is much stronger when $k=0.5$, with the austral (summer) circulation
15 almost absent. The vertical extent is larger and the maximum is located at higher levels. The
16 summer and winter jets are both more intense than their counterparts for $k=3$. The tropical
17 easterly winds are in this case stronger than those for $k=3$ (13.8 ms^{-1} vs 11.4 ms^{-1}) and the
18 easterly region is also wider. When $k=3$, it is noted that the winter cell is located closer to the
19 equator than the summer cell.

20 **3.3 A discussion on the case $n=2$ $k=1$**

21 When $n=2$ and $k=1$, corresponding to the classic case discussed in many studies, we
22 found that the time-dependent solution is only slightly stronger than the steady solution.
23 Lindzen and Hou (1988) proposed a study of the Hadley circulation in which the maximum

1 heating was 6° off the equator. In their non-time-dependent model, the solution showed an
2 average circulation much stronger with respect to the equinoctial solution. Lindzen and Hou
3 (1988) suggested that this exceptional strength was due to a nonlinear amplification of the
4 annually averaged response to seasonally varying heating, although Dima and Wallace (2003)
5 in a study on the seasonality of the Hadley circulation did not observe any nonlinear
6 amplification.

7 With the parameters used for equinoctial and time-dependent simulations we performed
8 an experiment like that of Lindzen and Hou (1988), with $\phi_0 = 6^\circ$ -that will be referred to as
9 solstitial experiment. We found that the winter circulation is stronger by a factor three with
10 respect to the steady solution obtained with the equinoctial heating consistent with the result
11 of ~~the axisymmetric model in~~ Walker and Schneider (2005). However, the average circulation
12 obtained by averaging two solstitial experiments, with $\phi_0 = 6^\circ$ and $\phi_0 = -6^\circ$ respectively is
13 only 1.5 times stronger than the steady solution with $\phi_0 = 0^\circ$, and it has a maximum in the
14 upper levels of the model domain as in Lindzen and Hou (1988). We suggest that this
15 maximum is due to a numerical effect caused by averaging the single solstitial experiments
16 rather than a spurious effect caused by the rigid lid as suggested in Walker and Schneider
17 (2005), even though a sponge layer actually lowers the maximum stream function height and
18 we can see the effects of a ~~stronger~~strong vertical gradient in the upper levels especially in the
19 time-dependent solution (cf. Fig. 3 and Fig. 7). Single solstitial experiments did not show a
20 maximum in upper levels and so the equinoctial and time-dependent experiments (Figs. 11a
21 and 11b). Consequently the only operation performed to produce Fig. 11c, which exhibits the
22 upper levels maxima, was to average the two solstitial experiments, which causes the
23 maximum at upper levels.

1 | Finally, we notice that comparing ~~the~~ time-dependent solution with $\varphi_0 = 6^\circ$ with the
2 | equivalent steady solution having the heating off the equator is not properly correct, since for
3 | the time-dependent model φ_0 represents only the maximum extension of heating, hence a
4 | more correct comparison between ~~time~~steady and ~~no~~ time-dependent solutions should be
5 | performed with the time-dependent solution having $\varphi_0 = 3^\circ$. In such a case, the average
6 | solution is only slightly weaker than the Hadley circulation driven by annually averaged
7 | heating or by a time-dependent heating which does not show any maximum in the upper
8 | levels. Thus, the results of equinoctial, time-dependent and solstitial ($\varphi_0 = 3^\circ$) experiments
9 | are mutually consistent.

10 | 4 Conclusions

11 | The forcing of an Earth-like planet can change for several reasons. For instance, a
12 | change of forcing distribution can be caused by different factors such as global warming or
13 | long-term variation of solar activity.

14 | Under the assumption of an equal equator-pole difference at the surface we used an
15 | axisymmetric model to study the sensitivity of the tropical atmosphere to different θ_E
16 | distributions modulated by two parameters, n that controls the broadness of the distribution
17 | and k that modulates how the θ_E is distributed vertically. Equinoctial and time-dependent
18 | solutions were simulated and compared. Moreover for the case $n=2$ and $k=1$, corresponding to
19 | the classical distribution used in literature, a few solstitial experiments were also run. When
20 | $n=2$ and $k=1$, the annually averaged circulation of equinoctial, time-dependent and solstitial
21 | experiments are quite close to one another, consistent with the results of Walker and
22 | Schneider (2005). However, the results differ from those of Lindzen and Hou (1988) and
23 | Fang and Tung (1999). As in all those works the maximum of the stream function of the

1 solstitial experiment is at upper levels, but it seems to be related to a spurious effect of the
2 averaging operation rather than a spurious effect due to the rigid lid.

3 The results provide evidence that concentrated equilibrium temperature distributions
4 enhance the meridional circulation and jet wind speed intensities, confirming findings of
5 Lindzen and Hou (1988) even though these authors imposed the same energy input. However,
6 in the present study the concentrated distribution at the equator has lower energy input.

7 The width of the Hadley cell is proportional to n , but when the cell width increases its
8 intensity decreases. Poleward shift of the Hadley circulation with warming is very robust as it
9 has been observed in many models and over large range of climates (Frierson et al., 2007).
10 Since the equator-pole gradient is the same for all the experiments with the same k ; it evident
11 that the gradient in the tropical region controls the circulation strength. The term k controlling
12 the imposed stratification has influence on the actual temperature distribution that can differ
13 remarkably from θ_E distribution.

14 Vertical stratification is important in determining the position and intensity of the Hadley
15 cell and jet when n is low, i.e. when for whereas k loses its importance when the θ_E
16 distribution is wider. This latter result is consistent with results of Tandon et al. (2013) who
17 found that the Hadley cell expansion and jet shift had relatively little sensitivity to the change
18 of the lapse rate. Consequently, the subtropical jet stream intensities are controlled by the
19 broadness of horizontal equilibrium temperature rather than the stratification, with higher
20 values of the jet when the thermal forcing is concentrated to the equator. In the case of time
21 dependent solution with $n=0.5$ (concentrated heating) and k takes the extreme values (0.5 and
22 3) the simulated maximum stream function has the same magnitude order of the observed

1 stream function, ten times larger than that obtained in HH80 and with the reference
2 simulation, even though with stronger winds too.

3 The jet stream position does not show any dependence with n and k , except when the
4 θ_E distribution is the widest ($n=3$); in such a case an abrupt change occurs and the maximum
5 of the zonal wind jet is located at mid-latitudes (47° in steady solution and 44° in annually
6 averaged time-dependent solution). This behavior can be explained by using the analytic study of
7 this model performed by Cessi (1998) claiming that when the meridional gradient becomes
8 too small ~~the process of homogenization of temperature and momentum occurs slowly and~~ the
9 circulation behaves as that of a slow rotating planet that exhibits poleward shift of the
10 subtropical jets.

11
12

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2 **References**

3 Allen, R. J., Sherwood, S. C., Norris, J. R., and Zender, C. S.: Recent Northern Hemisphere
4 tropical expansion primarily driven by black carbon and tropospheric ozone. *Nature*, 485,
5 350–354, 2012.

6 Caballero, R., Pierrehumbert, R. T., and Mitchell, J. L.: Axisymmetric, nearly inviscid
7 circulations in non-condensing radiative-convective atmospheres. *Q. J. Roy. Met. Soc.*, 134,
8 1269-1285, 2008.

9 Cessi, P.: angular momentum and temperature homogenization in the symmetric circulation
10 of the atmosphere. *J. Atmos. Sci.*, 55, 1997-2015, 1998.

11 Chen, G., Lu J., and Sun L.: Delineating the eddy–zonal flow interaction in the atmospheric
12 circulation response to climate forcing: Uniform SST warming in an idealized aquaplanet
13 Model. *J. Atmos. Sci.*, 70, 2214–2233, doi:<http://dx.doi.org/10.1175/JAS-D-12-0248.1>,
14 2013.

15 Dima, I. M., and Wallace, J. M.: On the seasonality of the Hadley cell. *J. Atmos. Sci.*, 60,
16 1522-1527, 2003.

17 Fang, M., and Tung, K. K.: Time-dependent nonlinear Hadley circulation. *J. Atmos. Sci.*, 56,
18 1797-1807, 1999.

19 Farrell, B. F.: 1990: Equable Climate Dynamics. *J. Atmos. Sci.*, 47, 2986–2995, doi:
20 [http://dx.doi.org/10.1175/1520-0469\(1990\)047<2986:ECD>2.0.CO;2](http://dx.doi.org/10.1175/1520-0469(1990)047<2986:ECD>2.0.CO;2), 1990.

21 Frierson, D. M. W., Lu, J., Chen, G.: Width of the Hadley cell in simple and comprehensive

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- 1 general circulation models. *Geophys. Res. Lett.*, L18804, 2007.
- 2 Fu, Q., Johanson, C. M., Wallace, J. M., and Reichler, T.: Enhanced mid-latitude tropospheric
3 warming in satellite measurements. *Science*, 312, 1179, 2006.
- 4 Fu, Q., and Lin, P.: Poleward shift of subtropical jets inferred from satellite-observed lower
5 stratospheric temperatures. *J. Climate*, 24, 5597-5603, doi:10.1175/JCLI-D-11-00027.1, 2011.
- 6 Gastineau, G., Le Treut, H. Li, L.: Hadley circulation changes under global warming
7 conditions indicated by coupled climate models *Tellus*, 60, 863-884, doi:10.1111/j.1600-
8 0870.2008.00344.x, 2008.
- 9 Gitelman, A. I., Risbey, J. S., Kass, R. E., and Rosen, R. D.: Trends in the surface meridional
10 temperature gradient. *Geophys. Res. Lett.*, 24, 1243-1246, 1997.
- 11 Greenwood, D. R. and Wing, S. L.: Eocene continental climates and latitudinal temperature
12 gradients, *Geology*, 23, 1044-1048, 1995.
- 13 Held, I. M., and Hou, A. Y.: Nonlinear axially symmetric circulation in a nearly inviscid
14 atmosphere. *J. Atmos. Sci.*, 37, 515-533, 1980.
- 15 Hou, A.Y., and Lindzen, R. S.: The influence of concentrated heating on the Hadley
16 circulation. *J. Atmos. Sci.*, 49, 1233-1241, 1992.
- 17 Hu, Y., and Fu, Q.: Observed poleward expansion of the Hadley circulation since 1979.
18 *Atmos. Chem. Phys.*, 7, 5229-5236, 2007.
- 19 Johanson, C. M., and Fu, Q.: Hadley cell widening: Model simulations versus observations. *J.*
20 *Climate*, 22, 2713-2725, 2009.

- 1 Kang, S. M., and Polvani, L. M.: The interannual relationship between the latitude of the
2 eddy-driven jet and the edge of the Hadley Cell. *J. Climate* 24(2), 563-68, [568](#),
3 doi:10.1175/2010JCLI4077.1, 2011.
- 4 Kim, H.-K., and Lee, S.: Hadley cell dynamics in a primitive equation model. Part II:
5 Nonaxisymmetric flow. *J. Atmos. Sci.*, 58, 2859-2871, 2001.
- 6 Lindzen, R. S., and Hou, A. Y.: Hadley circulations for zonally averaged heating centered off
7 the Equator, *J. Atmos. Sci.*, 2416-2427, 1988.
- 8 Liu, J., Song, M., Hu, Y., and Ren, X.: Changes in the strength and width of the Hadley
9 Circulation since 1871. *Clim. Past*, 8, 1169-1175, doi:10.5194/cp-8-1169-2012, 2012.
- 10 Lu, J., Chen, G., and Frierson, D. M. W.: Response of the zonal mean atmospheric circulation
11 to El Niño versus global warming. *J. Climate*, 21, 5835-5851, doi:
12 <http://dx.doi.org/10.1175/2008JCLI2200.1>, 2008.
- 13 Lu, J., Deser, C., and Reichler, T.: Cause of the widening of the tropical belt since 1958.
14 *Geophys. Res. Lett.*, 36, L03803, doi:10.1029/2008GL036076, 2009.
- 15 Lu, J., Vecchi, G. A., and Reichler, T.: Expansion of the Hadley cell under global warming.
16 *Geophys. Res. Lett.* 34, L06805, 2007.
- 17 Nguyen, H., Evans, A., Lucas, C., Smith, I., and Timbal, B.: The Hadley circulation in
18 reanalyses: Climatology, variability, and change. *J. Climate*, 26, 3357-3376, 2013.
- 19 Polvani, L. M., Waugh, D. W., Correa, G. J. P., and Son, S.-W.: Stratospheric ozone
20 depletion: the main driver of 20th ~~Century~~ century atmospheric circulation changes in the
21 Southern Hemisphere. *J. Climate*, 24, 795-812, doi:10.1175/2010JCLI3772.1, 2011.

- 1 Santer, B. D., Thorne, P. W., Haimberger, L., Taylor, K. E., Wigley, T. M. L., Lanzante, J. R.,
2 Solomon, S., Free, M., Gleckler, P. J., Jones, P. D., Karl, T. R., Klein, S. A., Mears, C.,
3 Nychka, D., Schmidt, G. A., Sherwood, S. C., and Wentz, F. J.: Consistency of modelled and
4 | observed temperature trends in the tropical troposphere. *Int. J. Climatol.*, 28-(13), 1703-1722,
5 2008.
- 6 Schaller, N., Cermak, J., Wild, M., and Knutti, R.: The sensitivity of the modeled energy
7 budget and hydrological cycle to CO₂ and solar forcing. *Earth Syst. Dynam.*, 4, 253-266,
8 doi:10.5194/esd-4-253-2013, 2013.
- 9 | Schneider, E. K.: Axially symmetric steady-state models of the basic state for instability and
10 | climate studies. Part II. Nonlinear calculations. *J. Atmos. Sci.*, 34, 280-296, 1977.
- 11 Schneider, E. K., and Lindzen, R. S.: Axially symmetric steady state models of the basic state
12 of instability and climate studies. Part I: Linearized calculations. *J. Atmos. Sci.*, 34, 253-279,
13 1977.
- 14 Seidel, D. J., Fu, Q., Randel, W. J., and Reichler, T. J.: Widening of the tropical belt in a
15 | changing climate. *Nat. Geosci.*, 1, 21-24, 2008.
- 16 Sherwood, S. C., Meyer, C. L., Allen, R. J., and Titchner, H. A.: Robust tropospheric
17 | warming revealed by iteratively homogenized radiosonde data. *J. Climate*, 21-(20), 5336-
18 5352, 2008.
- 19 Staten, P. W., Rutz, J., Reichler, T. and Lu, J.: Breaking down the tropospheric circulation
20 response by forcing. *Clim. Dynam.*, 39, 2361-2375, doi:10.1007/s00382-011-1267-y, 2011.
- 21 Strong, C., and Davis, R. E.: Winter jet stream trends over the Northern Hemisphere. *Q. J. R.*

1 | Meteorol. Soc. 133, 2109–2115, 2007.

2 | Tandon, N. F., Gerber, E. P., Sobel, A. H. and Polvani, L. M.: Understanding Hadley ~~Cell~~cell
3 | expansion versus contraction: Insights from simplified models and implications for recent
4 | observations. J. Climate, 26-(12), 4304–21, doi:10.1175/JCLI-D-12-00598.1, 2013.

5 | Titchner, H. A., Thorne, P.W., McCarthy, M.P., Tett, S. F. B., Haimberger L., and Parker, D.
6 | E.: Critically reassessing tropospheric temperature trends from radiosondes using realistic
7 | validation experiments. J. Climate, 22-(3), 465-485, 2008.

8 | Walker, C. C., and Schneider, T.: Response of idealized Hadley circulations to seasonally
9 | varying heating. Geophys. Res. Lett., 32, L06813, doi:10.1029/2004GL022304, 2005.

10 | Walker, C. C., and Schneider, T.: Eddy influences on Hadley circulations: Simulations with
11 | an idealized GCM. J. Atmos. Sci., 63, 3333–3350, 2006.

12 | Waliser, D. E., Shi, Z., Lanzante, J. R., and Oort, A. H.: The Hadley circulation:
13 | ~~assessing~~Assessing NCEP/NCAR reanalysis and sparse in situ estimates. Clim. Dyn., 15, 719-
14 | 735, 1999.

1 Table 1. Latitudes (in degrees) of the maximum wind speed for the equinoctial and time-
2 dependent solutions when $k=1$ as a function of the parameter n .

3

n	0.5	1	1.5	2	2.5	3
Equinoctial	27.4	28.7	27.4	26.1	28.7	47.7
Time dependent	28.7	28.7	28.7	27.4	27.4	44.4

4

5

1 **Figure Captions**

2 Figure 1. Meridional (a) and vertical (b) average of non-dimensional equilibrium temperature
3 as a function of n -with $k=1$ (a) and k with $n=0.5, 1$ and 1.5 (b). Dimensional values are
4 obtained multiplying by $\theta_0=300$ K.

5 Figure 2. Maximum non-dimensional stream function (a) and zonal wind speed [ms^{-1}] (b) as
6 function of parameters n and k for the steady solution. Dimensional values of the stream
7 function are obtained multiplying by $\nu_V R \varepsilon^{-1} = 484 \cdot \text{m}^2 \text{s}^{-1}$.

8 Figure 3. Latitude [degree] (a) and Height [m] (b) of maximum non-dimensional stream
9 function.

10 Figure 4. Vertically averaged the θ (blue line) and θ_E (red line) for the simulations with $n=3$
11 and $k=0.5$ (a), $k=1$ (b) and $k=3$ (c). Dimensional values are obtained multiplying by $\theta_0=300$
12 K.

13 Figure 5. Non-dimensional stream function (contours) and zonal wind speed [ms^{-1}] (colors)
14 for the steady cases $n=0.5, k=0.5$ (a) and $n=3, k=3$ (b). Dimensional values of the stream
15 function are obtained multiplying by $\nu_V R \varepsilon^{-1} = 484 \cdot \text{m}^2 \text{s}^{-1}$.

16 Figure 6. Maximum of annually averaged non dimensional stream function (a) and zonal wind
17 speed [ms^{-1}] (b) as function of parameters n and k for the time-dependent simulations.
18 Dimensional values of the stream function are obtained multiplying by $\nu_V R \varepsilon^{-1} = 484 \cdot \text{m}^2 \text{s}^{-1}$.

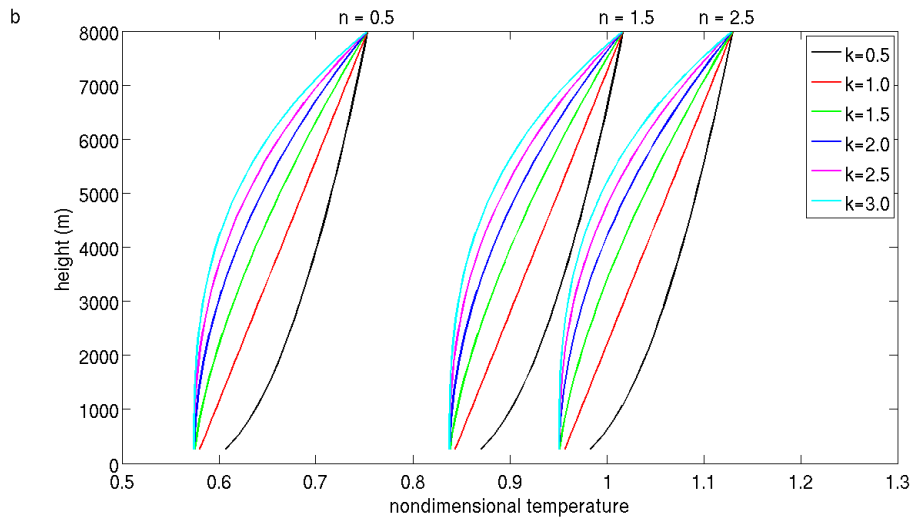
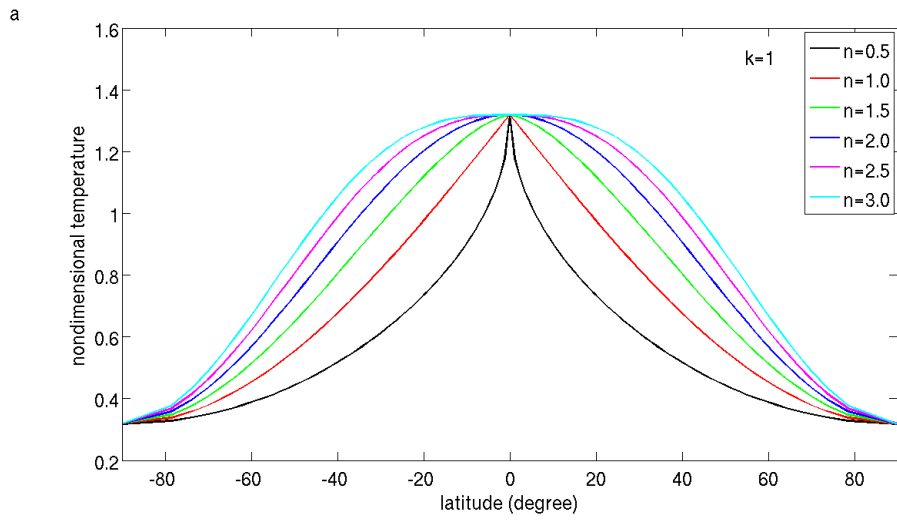
19 Figure 7. Latitude [degree] (a) and Height [m] (b) of maximum annually averaged non-
20 dimensional stream function for the time-dependent solution.

21 Figure 8. Annually averaged non-dimensional stream function (contours) and zonal wind
22 speed [ms^{-1}] (colors) for the steady cases $n=0.5, k=0.5$ (a) and $n=3, k=3$ (b). Dimensional
23 values of the stream function are obtained multiplying by $\nu_V R \varepsilon^{-1} = 484 \cdot \text{m}^2 \text{s}^{-1}$.

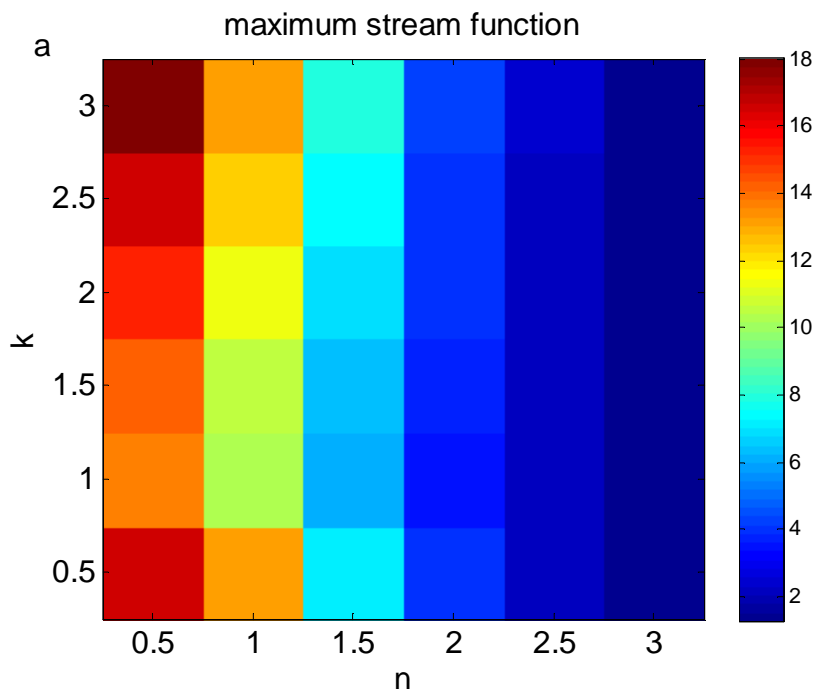
24 Figure 9. Maximum of non-dimensional stream function (a) and zonal wind speed [ms^{-1}] (b)
25 as function of parameters n and k for the time-dependent simulations. Dimensional values of
26 the stream function are obtained multiplying by $\nu_V R \varepsilon^{-1} = 484 \cdot \text{m}^2 \text{s}^{-1}$.

27 Figure 10. Boreal winter circulation, non-dimensional stream function (a and c) and zonal
28 wind speed [ms^{-1}] (b and d) for the time-dependent simulation with $n=2, k=0.5$ (upper panels)

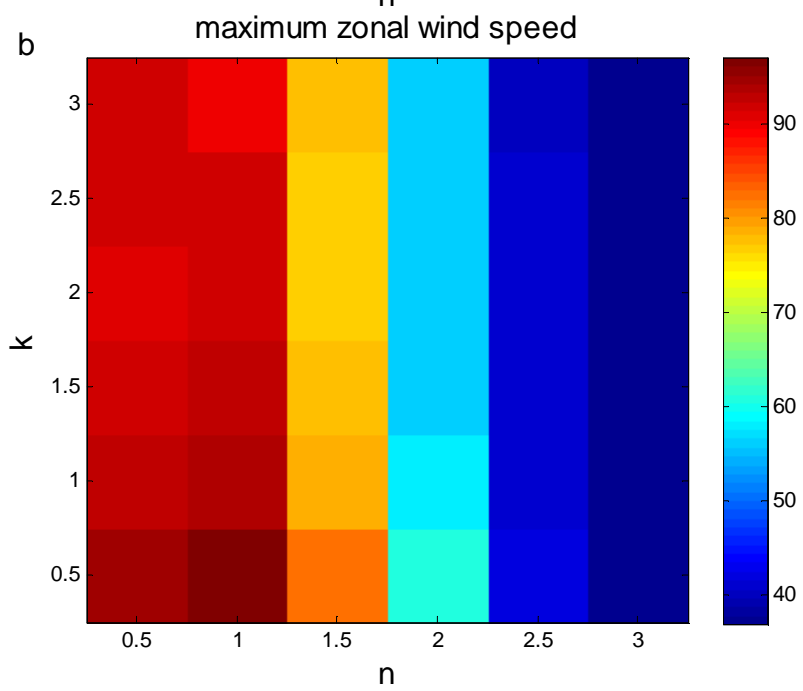
1 and $n=2, k=3$ (lower panels). Dashed lines indicate negative values. Dimensional values of the
2 | stream function are obtained multiplying by $\nu_V R \varepsilon^{-1} = 484 \text{ m}^2 \text{ s}^{-1}$.
3 Figure 11. Non-dimensional stream function (contours) and zonal wind speed ms^{-1}] (colors)
4 when $n=2$ and $k=1$ for the steady solution (a), annually averaged for the time-dependent
5 | solution (b) and averaged for maximum heating 6° off the equator (c). Dimensional values of
6 | the stream function are obtained multiplying by $\nu_V R \varepsilon^{-1} = 484 \text{ m}^2 \text{ s}^{-1}$.
7



3 Figure 1.

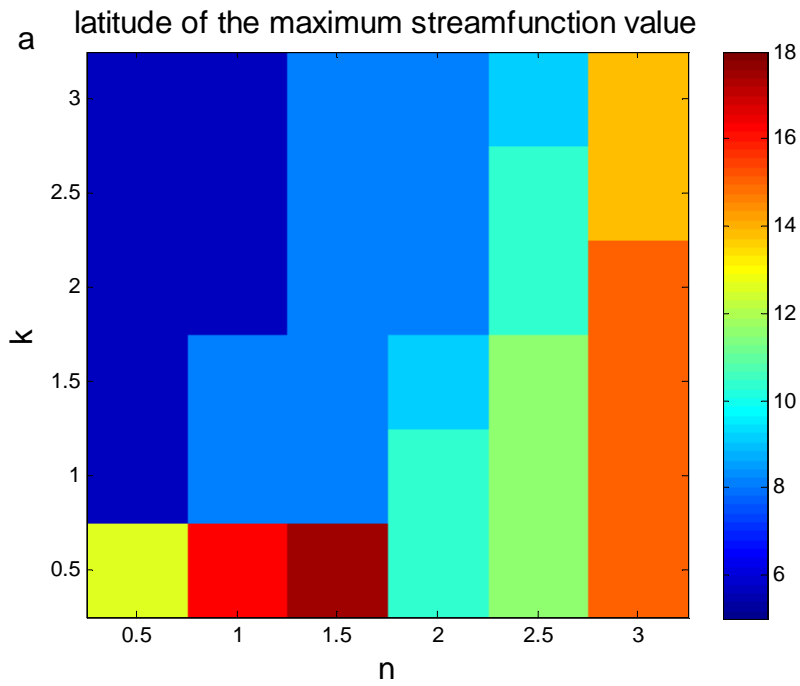


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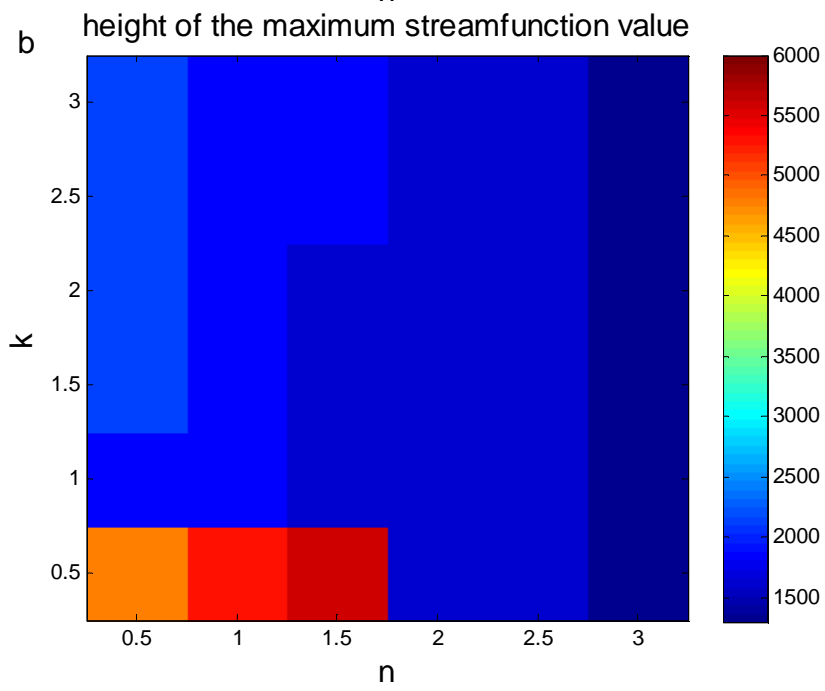


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3 Figure 2.

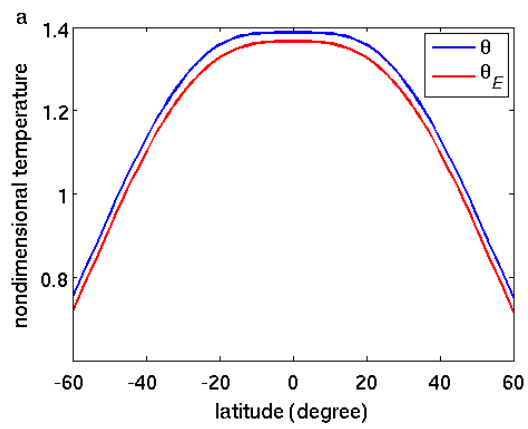


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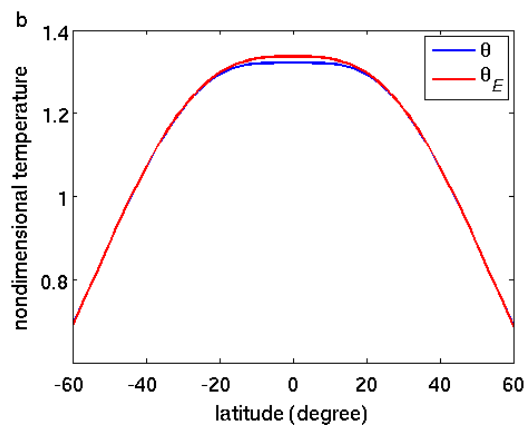


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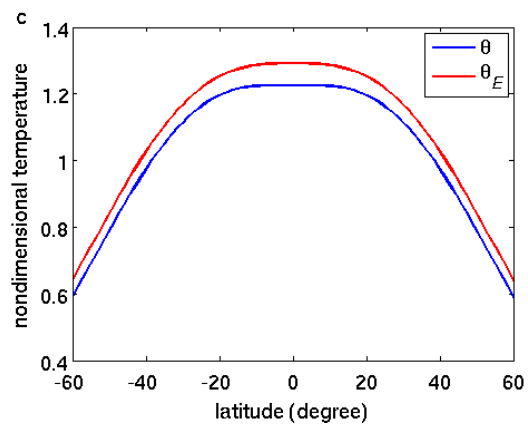
3 Figure 3.



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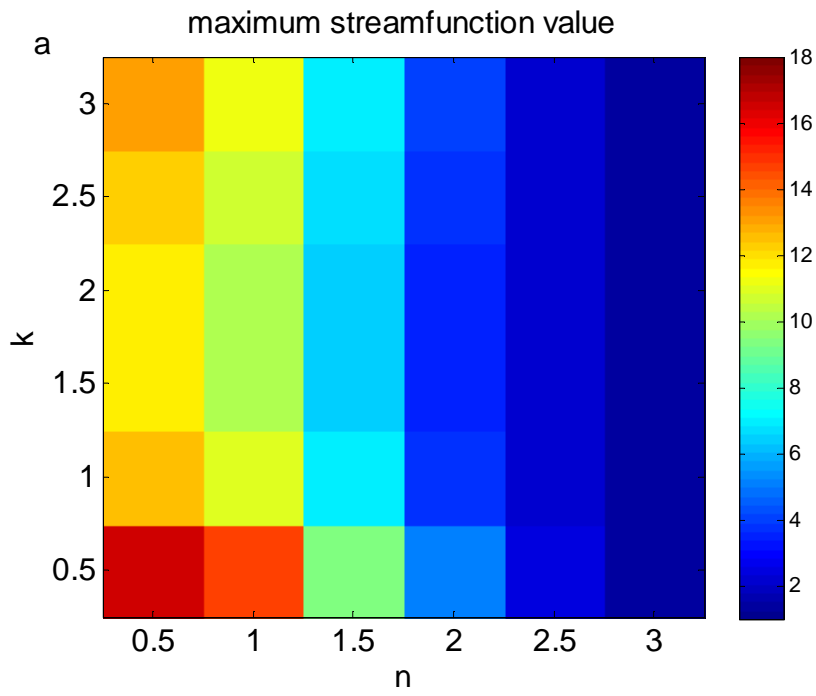


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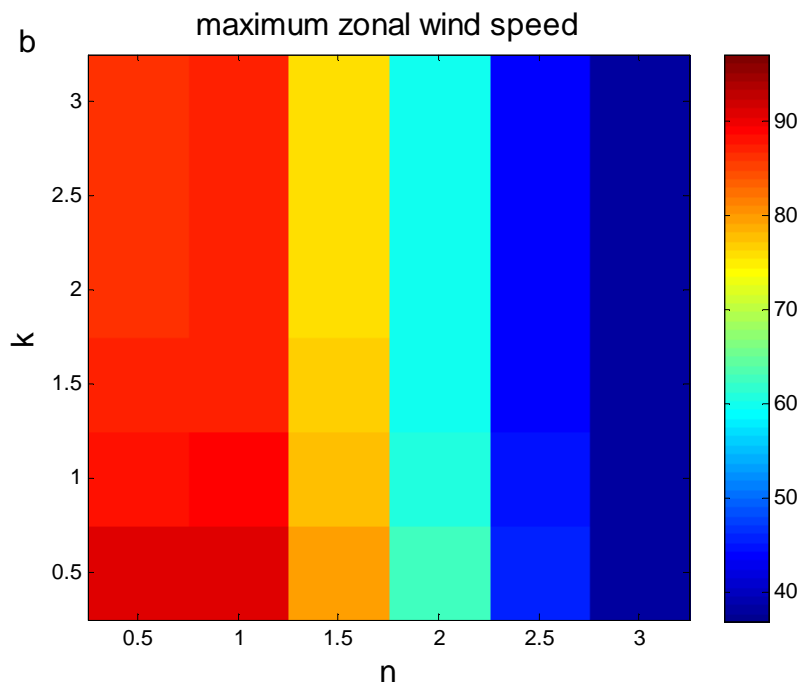


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4 Figure 4.

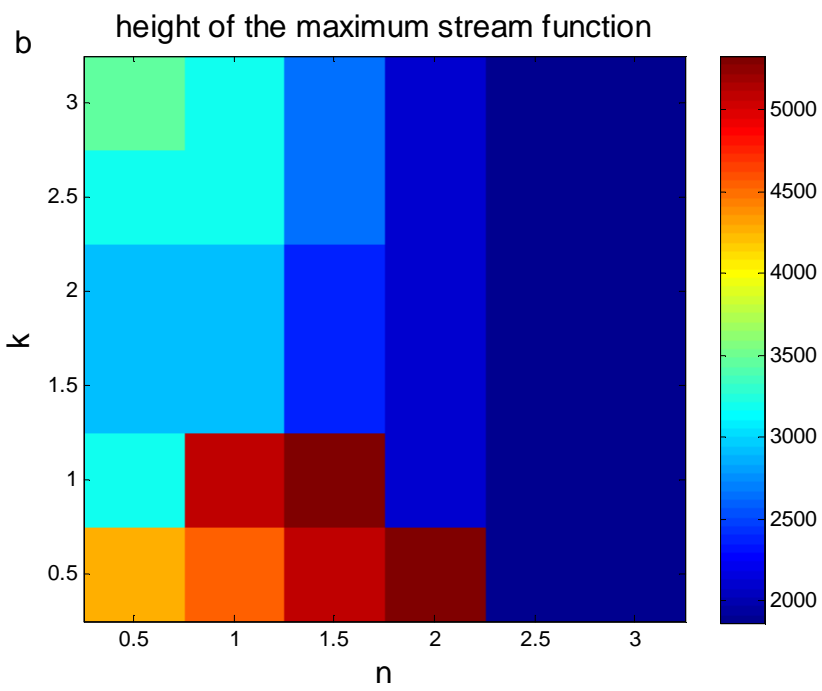
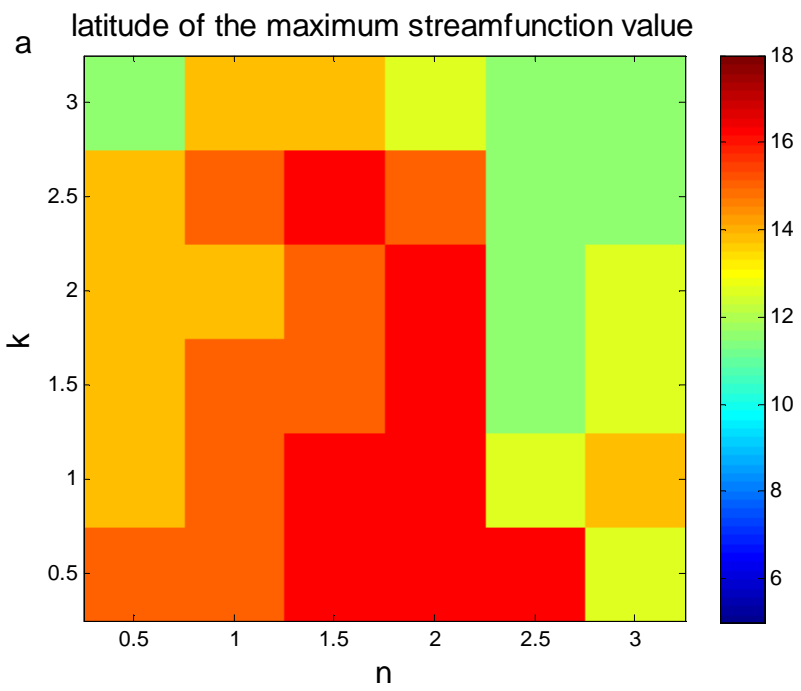


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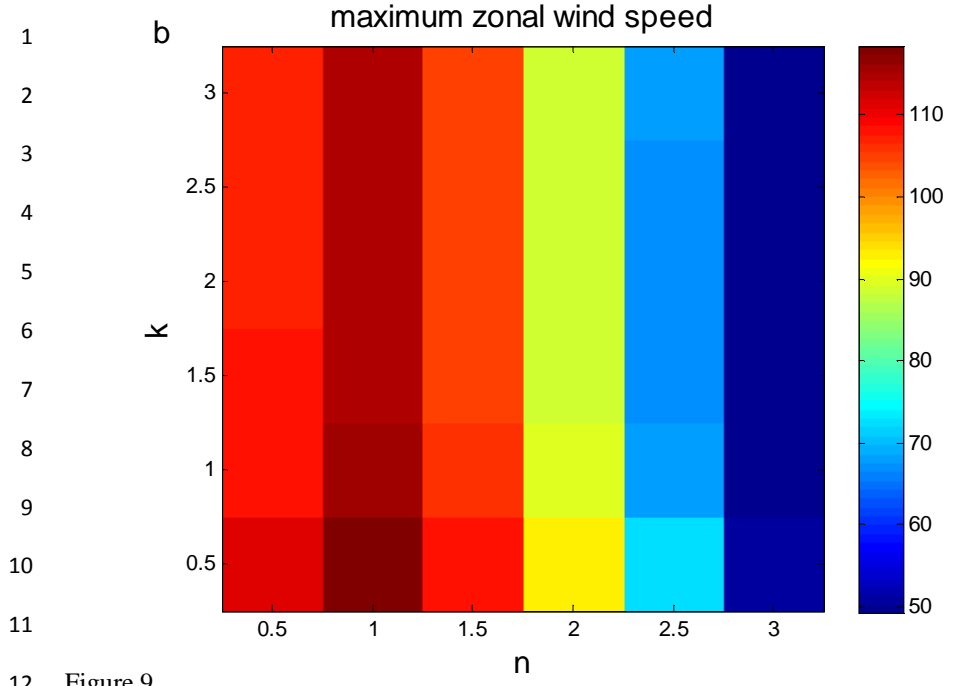
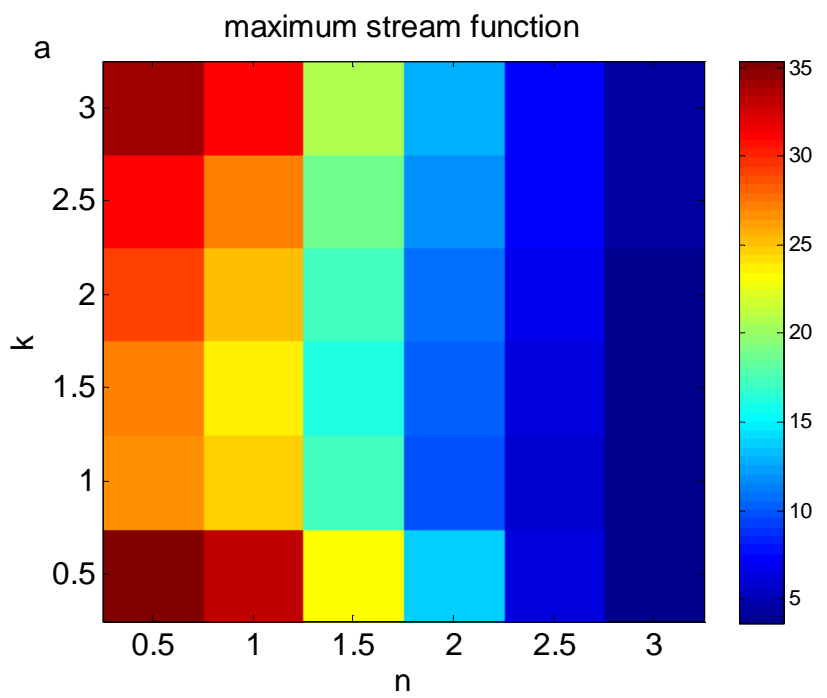
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3 Figure 6.

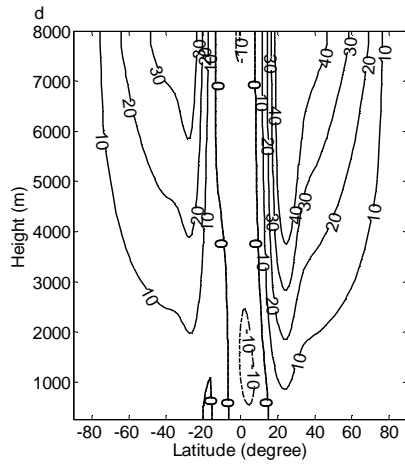
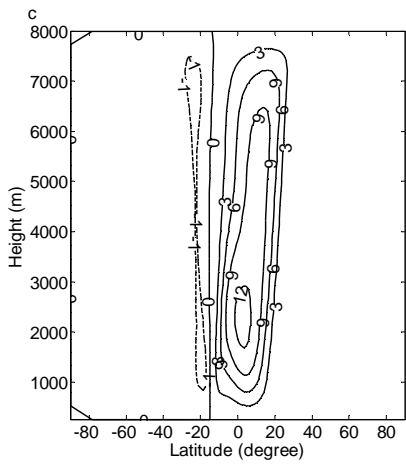
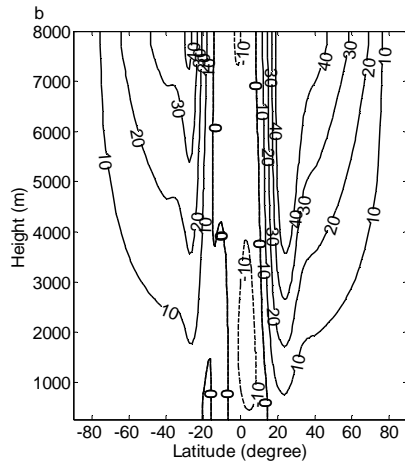
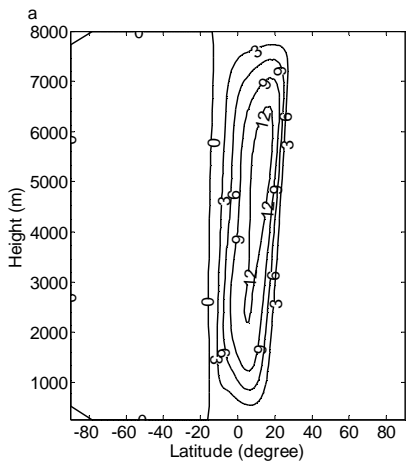


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2 Figure 7.



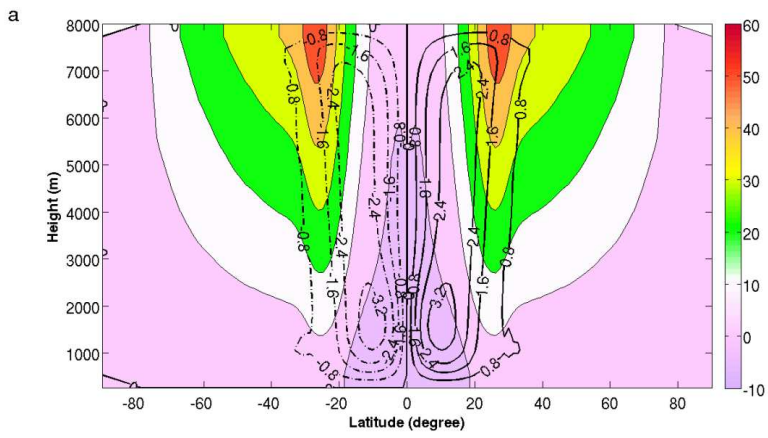
12 Figure 9.



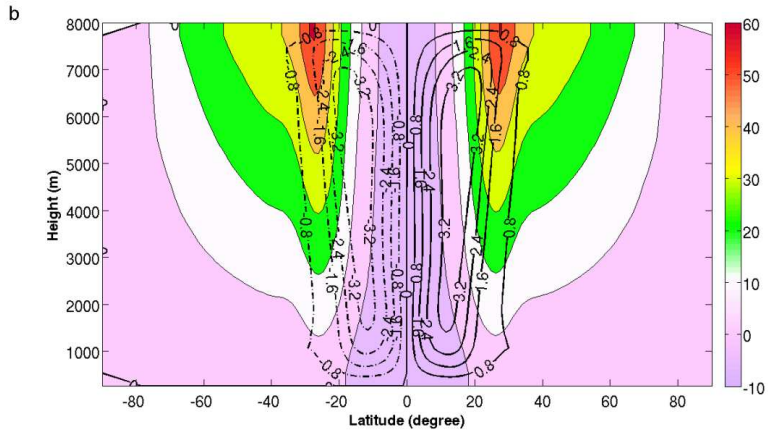
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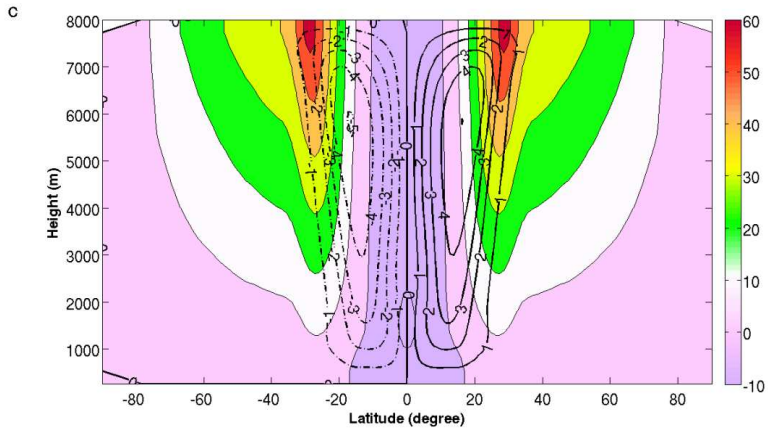
3 Figure 10.



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4 Figure 11.