

1 **Equilibrium temperature distribution and Hadley circulation** 2 **in an axisymmetric model.**

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10

11 **Abstract**

12

13 The impact of the equilibrium temperature distribution, θ_E , on the Hadley circulation
14 simulated by an axisymmetric model is studied. The θ_E distributions that drive the model are
15 modulated here by two parameters, n and k , the former controlling the horizontal broadness
16 and the latter controlling the vertical stratification of θ_E . In the present study, variations of the
17 θ_E distribution mimic changes of the energy input of the atmospheric system leaving as an
18 almost invariant the equator-poles θ_E difference. Both equinoctial and time-dependent Hadley
19 circulations are simulated and results compared. The results give evidence that concentrated
20 θ_E distributions enhance the meridional circulation and jet wind speed intensities even with a
21 lower energy input. The meridional circulation and the subtropical jet stream widths are
22 controlled by the broadness of horizontal θ_E rather than the vertical stratification, which is
23 important only when θ_E distribution is concentrated at the equator. The jet stream position
24 does not show any dependence with n and k , except when the θ_E distribution is very wide

1 ($n=3$) and in such a case the jet is located at the mid-latitudes and the model temperature
2 clamps to forcing θ_E . Using $n=2$ and $k=1$ we have the formulation of the potential temperature
3 adopted in classical literature. A comparison with other works is performed and our results
4 show that the model running in different configurations (equinoctial, solstitial and time-
5 dependent) yields results similar to one another.

1 Introduction

2 The earth's atmosphere is driven by differential heating of the earth's surface. At the
3 equator, where the heating is larger than that at other latitudes, air rises and diverges poleward
4 in the upper troposphere, descending more or less at 30° latitude. This meridional circulation
5 is known as Hadley cell. Two subtropical jets at the poleward edges of the Hadley form
6 because of earth rotation and the conservation of the angular momentum. A poleward shift
7 (Fu and Lin, 2011) and an enhanced wind speed of these jets (Strong and Davis, 2007) are
8 associated with a possible Hadley cell widening and strengthening, which has been observed
9 in the last decades (Fu et al., 2006; Hu and Fu, 2007; Seidel et al., 2008; Johanson and Fu,
10 2009; Nguyen et al., 2013).

11 There are a few studies suggesting possible causes of these phenomena. One of the
12 theories postulates global warming as a possible cause of Hadley cell widening (Lu et al.,
13 2009). However, the atmosphere is a complex system containing many subsystems interacting
14 with one another and the global warming might not be the only cause that is suggested to
15 explain the widening. Ozone depletion (Lu et al., 2009; Polvani et al., 2011), SST warming
16 (Chen et al., 2013; Staten et al., 2011) and aerosol (Allen et al., 2012) have also been invoked
17 to explain the Hadley cell widening.

18 Climate models vary to some extent in their response and the relationship between global
19 warming and Hadley cell is not straightforward. For instance, Lu et al. (2007) found a smaller
20 widening than the observed one. Gitelman et al. (1997) showed that the meridional
21 temperature gradient decreases with increasing global mean temperature and the same result
22 can be found in recent modeling studies (Schaller et al., 2013).

1 Much of our understanding on the Hadley cell comes from theories using simple models
2 (Schneider 1977, Schneider and Lindzen 1977 and Held and Hou 1980, hereafter HH80) and
3 such a simple model will be adopted here in order to understand how temperature
4 distributions can change the Hadley circulation. How much temperature change impacts the
5 real Hadley circulation is not clear yet, perhaps because of discrepancies between
6 observations, reanalysis (Waliser et al., 1999) and climate model outputs, although these
7 differences are becoming less marked because of newer observational datasets or correction
8 of the older ones (Sherwood 2008, Titchner et al., 2008, Santer et al., 2008). Hence, it is
9 critical to understand the possible mechanisms behind the cell expansion starting from a
10 simple model.

11 The objective of this study is to analyze the sensitivity of a model of the symmetric
12 circulation to the radiative-convective equilibrium temperature distribution. Our point of
13 departure is the symmetric model used by Cessi (1998), which is a bidimensional model
14 considering atmosphere as a thin spherical shell. This model will be briefly described in Sect.
15 2. The model describes mainly a tropical atmosphere, hence it does not allow for eddies.
16 Although eddies may play a central role in controlling the strength and width of the Hadley
17 cell (e.g. Kim and Lee, 2001; Walker and Schneider, 2006), a symmetric circulation, driven
18 by latitudinal differential heating, can exist even without eddies and it is a robust feature of
19 the atmospheric system (Dima and Wallace, 2003). The temperature distributions used in this
20 study represent some paradigms of tropical atmospheres. Among the possible causes that can
21 change temperature distributions there are El Niño, global warming and change of solar
22 activity. We will show, in Sect. 3, that the energy input is not as important as the forcing
23 distribution. Our results are consistent with those obtained both by Hou and Lindzen (1992)

1 (hereafter HL92), and recently by Tandon et al. (2013) who performed experiments similar to
2 those described here. The conclusions will be drawn in Sect. 4.

3 **2 The model**

4 The model used in this study is a bidimensional model of the axis-symmetric atmospheric
5 circulation described in Cessi (1998). The horizontal coordinate is defined as $y = a \sin \phi$ from
6 which we have

$$7 \quad c(y) = \cos \phi = \sqrt{(1 - y^2/a^2)} \quad (1)$$

8 where a is the radius of a planet having a rotation rate Ω , the height of atmosphere is
9 prescribed to be H .

10 The model is similar to the Held and Hou model (HH80), but it prescribes a horizontal
11 diffusion ν_H other than the vertical diffusion ν_v . The prognostic variables are the angular
12 momentum M , defined as $M = \Omega a c^2 + u c$ where u represents the zonal velocity; the zonal
13 vorticity ψ_{zz} with the meridional stream function ψ defined by

$$14 \quad \begin{aligned} \partial_y \psi &\equiv w; \\ \partial_z \psi &\equiv -c v \end{aligned} \quad (2)$$

15 and the potential temperature θ that is forced towards a radiative-convective equilibrium
16 temperature θ_E . Starting from the dimensional equations of the angular momentum, zonal
17 vorticity and potential temperature, we will obtain a set of dimensionless equations. The new
18 equations are non-dimensionalized using a scaling that follows Schneider and Lindzen (1977),
19 but the zonal velocity u is scaled with Ωa . A detailed description can be found in Cessi
20 (1998).

21 The non-dimensional model equations are:

$$1 \quad M_t = \frac{1}{R} \left\{ M_{zz} + \mu [c^4 (c^{-2} M)_y]_y \right\} - J(\psi, M) \quad (3a)$$

$$2 \quad \psi_{zzt} = \frac{1}{(R^2 E^2)} y c^{-2} (M^2)_z - \frac{1}{c^{-2}} J(\psi, c^{-2} \psi_{zz}) + \frac{1}{(R E^2 c^{-2})} \theta_y + \frac{1}{(R c^{-2})} [c^{-2} \psi_{zzzz} + \mu \psi_{zzyy}] \quad (3b)$$

$$3 \quad \theta_t = \frac{1}{R} \left\{ \theta_{zz} + \mu [c^2 \theta_y]_y + \alpha [\theta_E(y, z) - \theta] \right\} - J(\psi, \theta) \quad (3c)$$

4 The term $J(A, B) = A_y B_z - A_z B_y$ is the Jacobian.

5 The thermal Rossby number R ; the Ekman number E , the ratio of the horizontal to the vertical

6 viscosity μ and the parameter α are defined as

$$7 \quad R \equiv gH \Delta_H / (\Omega^2 a^2); \quad E \equiv \nu_V / (\Omega H^2); \quad \mu \equiv (H^2 / a^2) \nu_H / \nu_V; \quad \alpha \equiv H^2 / (\tau \nu_V) \quad (4)$$

8 The term α is the ratio of the viscous timescale across the depth of the model atmosphere to

9 the relaxation time τ toward the radiative-convective equilibrium.

10 The boundary conditions for the set of Eq. 3 are:

$$11 \quad \begin{aligned} M_z &= \gamma(M - c^2), \quad \psi_{zz} = \gamma\psi_z; \\ \psi &= \theta_z = 0 \text{ at } z = 0; \\ M_z &= \psi_{zz} = \psi = \theta_z = 0 \text{ at } z = 1. \end{aligned} \quad (5)$$

12 Where $\gamma = \frac{cH}{\nu_V}$ is the ratio of the spin-down time due to the drag to the viscous timescale, the

13 bottom drag relaxes the angular momentum M to the local planetary value $\Omega a c^2$ through a

14 drag coefficient C .

15 The model flow started from an isothermal state at rest and is maintained by a Newton

16 heating function where the heating rate is proportional to the difference between the model

17 potential temperature and a specified radiative-convective equilibrium temperature

18 distribution, which follows the HH80 one:

$$19 \quad \theta_E = \frac{4}{3} - y^2 + \frac{\Delta_V}{\Delta_H} \left(z - \frac{1}{2} \right). \quad (6)$$

1 Equation 6 is used extensively in dry axisymmetric models (e.g. HH80, Farrell, 1990, Cessi
2 1998) and it is related to the thermal forcing term of the equation system. A statically stable
3 state as a vertical profile of θ_E is also assumed by Eq. 6. HH80 suggested that the impact of
4 latent heat released by water vapor condensation can be incorporated in dry axisymmetric
5 models by modifying the meridional distribution of θ_E . HL92 followed the HH80 argument
6 and altered the concentration of θ_E under the constraint of equal energy input. The resulting θ_E
7 distributions used by HL92 were peaked distributions on and off the equator resulting in a
8 stronger Hadley circulation with respect the circulation obtained applying Eq. 6. Tandon et al.
9 (2013) used narrow and wide thermal forcing to mimic El Niño or global warming effect on a
10 tropical circulation in a Global Circulation Model. On the opposite side, in fact, we can
11 suppose that if a warmer climate happens, especially in the tropical regions, a very weak
12 gradient of the equilibrium temperature θ_E will be more extent in latitude, expanding
13 consequently the tropical region. This is already occurred in the past, especially in the mid
14 Cretaceous and Eocene when the tropics extended up to 60° . This is the so called equable
15 climate (e.g. Greenwood and Wing, 1995) where roughly equal temperatures are present
16 throughout the world. During those geological ages the temperature was generally higher
17 everywhere, but adding a constant to the temperature does not change the response of this
18 kind of models. The equator-pole temperature gradient was smaller than the present situation,
19 whereas we prescribe constant surface equator-pole θ_E gradient. As we shall show afterwards
20 this is necessary to demonstrate that it is the tropical temperature gradient drives the Hadley
21 circulation. Thus, in order to study systematically these different conditions we adopt the
22 strategy to build forcing functions dependent on a parameter that controls the θ_E gradient in
23 the tropical regions. Since, with different horizontal distributions of θ_E we can figure out that
24 even the vertical distribution could be affected by some physical mechanisms that make the

1 atmosphere more or less stable than the stratification described by the z component of Eq. 6.
2 The meridional and vertical changes of equilibrium temperature can be obtained by changing
3 the exponents of y and z in Eq. 6 transforming Eq. 6 in the following equation:

$$4 \quad \theta_E = \frac{4}{3} - |y|^n + \frac{\Delta_V}{\Delta_H} \left(z^k - \frac{1}{2} \right). \quad (7)$$

5 The values n and k control the horizontal distribution of θ_E and its stratification respectively.
6 Small values of n are associated with concentrated θ_E distributions. Increasing n means
7 increasing broadness of the θ_E distribution. Values of k larger than 1 mean more stable upper
8 levels, vice-versa smaller k values means lower levels are more stable than upper levels.
9 Thus, it comes quite natural to explore the response of Hadley circulation by changing the
10 parameters n and k , which control the distribution of θ_E , in closest ranges of 2 and 1
11 respectively. Thus n and k will change from 0.5 to 3 with a 0.5 step, in such a way we have a
12 set of 36 simulations. When $n=2$ and $k=1$ Eq. (7) becomes the reference equilibrium
13 temperature given in Eq. 6 and the experiments performed with $n=2$ and $k=1$ will be
14 considered as the reference experiments.

15 The average θ_E along the latitudes and heights are shown in Fig. 1. Heating functions
16 with n value equal to 0.5 should not be regarded as unreal, but merely as a simple way to
17 represent a specific state of the atmosphere. The same assertion is valid for all other
18 parameters. As n increases the average temperature increases as well, but the meridional
19 gradient decreases in the tropical regions.

20 With the prescribed θ_E as specified by Eq. 7, the θ_E values at the boundaries and its
21 equator-pole difference temperature remain invariant with respect to n , for a given k value.
22 The energy input is not constant here, which differs from HL92, which analyzed the influence
23 of concentration heating perturbing the forcing function $\theta_E(y,z)$ in such a way that θ_E

1 averaged over the domain remained constant. It is easily visible in Fig. 1b. Higher n values,
 2 keeping k invariant, have higher averaged θ_E at all levels. The same is true for k , with higher k
 3 values, for n constant; θ_E at each level is always higher than that with lower k values. The
 4 pole-equator θ_E difference at upper and lower vertical boundaries are the same for all the
 5 experiments having the same k , the vertical averaged θ_E changes as a function of k , for n
 6 constant. Whether global warming makes the equilibrium temperature distribution narrower
 7 or wider is beyond the aim of the paper. One can expect that global warming broadens the
 8 temperature distribution, but at the same time it could have an impact above all on the sea
 9 surface temperature (SST) bringing more water in the upper atmosphere which changes the
 10 vertical distribution of the temperature in the inter-tropical convergence zone (ITCZ). It is
 11 supposed that, in first approximation, oceans force the atmosphere, so we have to allow for
 12 the possibility that increasing SST can change the forcing distribution. Increasing uniformly
 13 SST might could a poleward expansion as showed by Chen et al. (2013) with an aquaplanet
 14 model, but in that case the mechanism was supposed to be related mainly to mid-latitude
 15 eddies rather than a tropical forcing. Since other causes can change the temperature
 16 distribution of a planet such as changes in the solar activity for instance, we will focus on the
 17 temperature distribution regardless of its cause.

18 In this model the atmosphere is dry as in many other studies (e.g. Schneider 1977, HH80,
 19 Caballero et al. 2008), changing the θ_E distribution allows for a change in the static stability.
 20 Looking at the average θ_E along the vertical direction, low values of k are related to low
 21 values of static stability, especially in higher level of the model atmosphere.

22 The Brunt–Väisälä frequency, when the atmosphere reaches the equilibrium will be

$$23 \quad N^2 = \frac{(gk\Delta_V/\Delta_H z^{(k-1)})}{[4/3 - y^n + \Delta_V/\Delta_H(z^k - 1/2)]}. \quad (8)$$

1 It is clear from Eq. 8 that the Brunt–Väisälä frequency does not depend on n at the poles and
 2 equator. On the contrary, it depends on k ; large values of k imply a more stable atmosphere in
 3 the upper levels, especially at poles, making the model atmosphere more similar to the real
 4 one, simulating in some respects a sort of tropopause. Moreover, this is equivalent to creating
 5 a physical sponge layer in the upper levels of the model that will have some effects on the
 6 vertical position of stream function maximum.

7 Starting from Eq. 7 a set of experiments were performed changing n and k in such a way
 8 to have a set of numerical results. In order to isolate the contribution of the θ_E distribution on
 9 the solution of Eq. 3, a set of parameters will be used:

$$\begin{aligned}
 10 \quad a &= 6.4 \times 10^6 \text{ m} & \Omega &= 2 \pi / (8.64 \times 10^4) \text{ s}^{-1} \\
 & \Delta_H = 1/3 & & \Delta_V = 1/8 \\
 & g = 9.8 \text{ ms}^{-2} & & C = 0.005 \text{ ms}^{-1} \\
 & H = 8 \times 10^3 \text{ m} & & \tau = 20 \text{ days} \\
 11 \quad \nu_V &= 5 \text{ m}^2 \text{ s}^{-1} & & \nu_H = 1.86 \text{ m}^2 \text{ s}^{-1} \tag{9}
 \end{aligned}$$

12 The parameters in Eq. 9 are the same as those used by Cessi (1998).

13

14 **3 Numerical Results**

15 This section is divided into three subsections, the first showing the results of the model
 16 applying the equinoctial condition, when the sun is assumed to be over the equator. The
 17 solution is steady as already shown for instance in Cessi (1998). The second subsection will
 18 show the results of the model having a θ_E distribution described by Eq. 7 but moving

1 following a seasonal cycle. The case $n=2$ and $k=1$ is discussed in the third subsection in
2 comparison with previous studies.

3 **3.1 Equinoctial simulations**

4 The axially symmetric circulation is forced by axially symmetric heating as in HH80 and
5 many others and as prescribed by Eq. (7). The model started from an isothermal state and it
6 was run for 300 days, even though it reached its equilibrium approximately after 100 days, in
7 order to be sure that the model does not have instabilities in the long run. The stream function
8 values obtained when $n=2$ and $k=1$, the reference experiment are about the same of that
9 obtained by HH80. We will show the non-dimensional value, but to have the dimensional
10 values we need to multiply by $\nu_V R \varepsilon^{-1} = 484 \text{ m}^2 \text{ s}^{-1}$.

11 The absolute value of the maximum stream function intensity at the equilibrium
12 conditions for the 36 experiments is shown in Fig. 2. When $n=0.5$, with k constant, the
13 circulation is always the strongest. The stream function intensity is inversely proportional to n
14 (Fig. 2a). With $n=0.5$ the experiment resembles the one described in HL92 where they
15 concentrated the latitudinal extent of heating and this led to a more intense circulation.
16 However, they imposed the forcing function $\theta_E(x, y)$ in such a way that its average over the
17 domain remained the same as in the control experiment, i.e. without changing the energy
18 input. They found that concentration of the heating through a redistribution of heat within the
19 Hadley cell led to a more intense circulation without altering its meridional extent. Instead,
20 here, it is evident from Fig. 1 that the experiment with $n=0.5$ has an energy input lower than
21 the other cases. Nevertheless, the Hadley circulation is always more intense than the other
22 cases and contrary to higher n value experiments, the circulation is confined close to the
23 equator. Thus the results of HL92 are extended to a more general case with a lower energy

1 input. It is worth noticing the constraint of an equal pole-equator gradient of mean θ_E is
2 assumed here differently from HL92 (Fig. 1a).

3 The dependence on k is not as straightforward as the one on n , instead. The stream
4 function reaches the highest value for $n=0.5$ and $k=3$. With a high n values the Hadley cell
5 stream function intensity is lower and the dependence on k loses its importance. In other
6 words, in our model, the symmetric circulation strength is modulated by k only when the
7 equilibrium temperature distribution is concentrated to the equator.

8 Figure 2b shows the maximum zonal wind speed as function of n and k , it is inversely
9 proportional to n , the dependence on k is not as clear as the one on n and when $n=3$ it almost
10 vanishes in accordance with the behavior of the maximum stream function. These results are
11 in agreement with HL92, who found a stronger zonal wind when the forcing was concentrated
12 at the equator.

13 Some studies define the border of a Hadley cell as that by the zero line of the 500 hPa
14 stream function (e.g. Frierson et al., 2007). Since in this kind of model the zero stream
15 function is at the poles, it is problematic to define an edge of the Hadley cell based on the zero
16 stream function. Moreover, the circulation intensity changes greatly in our experiments, so it
17 is problematic to define an edge of the Hadley cell based on an absolute value of the
18 circulation itself. Hence, we will define the position of the cell equal to the position of the
19 maximum value of stream function, in this way we will study a possible poleward shift of the
20 cell as a function of the two parameters n and k . The edge of the cell might be defined by
21 values of isolines that are relative with respect to the maximum value, for example 1/4 of the
22 stream function. For the sake of clarity this definition is an operational one and does not
23 follow the definition used for example by Dima and Wallace (2003) or Frierson et al. (2007).

1 The latitude of the maximum stream function value shows a general dependence on n
2 and k . It increases with n and decreases with k . However, as shown in Fig. 3a, this dependence
3 is not straightforward or linear, although we have a few exceptions, for instance when
4 $k=n=0.5$. Hence in general when n increases, and the total energy input is larger, the stream
5 function is weaker and the Hadley cell moves poleward. This result is in agreement with other
6 model outcomes (Frierson et al, 2007, Lu et al., 2008; Gastineau et al., 2008; and Tandon et
7 al., 2013). The model predicts a weakening of circulation, in contrast with the strengthening,
8 together with widening, of the Hadley circulation for the past three decades observed by Liu
9 et al. (2012) and Hu and Fu (2007). However, Liu et al. (2012) showed that if the
10 observations start from 1870, the Hadley cell has become more narrow and stronger.

11 The height of the maximum stream function value is confined for almost all the
12 simulations under 2200 m and the general rule is that when n increases, the height of
13 maximum lowers, however a few experiments, those with $k=0.5$ and $n=0.5, 1$ and 1.5 , have
14 the maximum value between 4300 and 5600 m exhibiting an increase in the height with n
15 (Fig. 3b).

16 In general, the location of the maximum zonal wind speed does not show any evident
17 relationship with the parameters n and k . It is always confined between 26° and 29° off the
18 equator; however when $n=3$, there is an abrupt transition to about 48° , independently from the
19 k value. In Table 1, we show the latitude of the maximum wind speed when $k=1$ for different
20 n values.

21 The difference between θ_E and θ , once the model reaches the equilibrium, is quite
22 interesting. Figure 4 shows meridional distributions of θ_E and θ for $n=3$ and $k=0.5, 1$ and 3 . In
23 Fig. 4a, θ_E is under θ , when $k=1$ we find θ_E is over θ in a region around the equator (Fig. 4b),

1 with θ_E crossing θ at about 47° , finding again the equal area condition suggested by HH80
2 and that explains approximately the jet location, whereas in Fig.4c, with $k=3$, we can see how
3 θ_E is over θ . Despite these differences in the distributions of θ_E and θ the model produces
4 with these different k values almost the same solution, in terms of circulation strength and jet
5 location. For other values of n the results are similar, but the differences between θ_E and θ are
6 not so visible.

7 We can understand these findings in the light of Cessi (1998) results obtained by
8 expanding the variables M , θ and ψ in power series of R . The term R^2 in nonlinear expansion
9 part, the meridional advection depends on the differences between θ_E and θ , on the cube of the
10 meridional temperature gradient, and linearly on the imposed stratification deducing that for
11 unstable stratifications, this term would appear as a negative diffusivity term (Cessi, 1998),
12 whereas it acts as a positive diffusion. This seems to be the case, in our simulation when
13 $k=0.5$. The thermal energy obtained in the model is larger than the imposed temperature
14 (Fig.4a). Although the stratification imposed by Eq. 7 is stable, i.e. $\frac{\partial\theta_E}{\partial z} > 0$, the second
15 derivative is negative when $k=0.5$ reducing the stability at upper levels, so this situation can
16 be seen as a way to simulate the effect of the latent heat released by water vapor
17 condensation. When $k=3$ the air in upper levels is very stable and the flow has to do more
18 work, giving rise to a sort of implicit dissipation. Nevertheless, the model acts to bring the
19 vertical temperature gradient in a more stable configuration and the Hadley circulation is in
20 any case reproduced demonstrating the robustness of the model.

21 With n getting larger, the θ_E distribution becomes flatter in the tropical region and θ
22 clamps to θ_E . In general, we expect that a vigorous circulation occurs in a fast rotating planet
23 unless the thermal gradient becomes small in the tropics. In such a case the angular

1 momentum homogenization is equivalent to a weakening of the rotation (Cessi, 1998). If the
2 circulation is proportional to the cube of the meridional temperature gradient, it is quite
3 evident that when such a gradient has high values the circulation is vigorously driven by this
4 term, whereas when it approaches to zero it is the term $\theta_E - \theta$ dominates. HH80 found that
5 the edge of the Hadley cell was at the mid-latitudes when the planetary rotation was lower
6 than that of the earth. Since this phenomenon is here observed for a wider forcing distribution,
7 this common result may be attributed to a low efficiency in the process of homogenization of
8 momentum and temperature.

9 In order to explain equable climates like those supposed to be occurred in Cretaceous
10 and Eocene, Farrell (1990) formulated an axisymmetric model starting from the Held and Hou
11 model and a forcing with $n=2$ and $k=1$ where the temperature gradients became flat because
12 of a dissipation term. For high values of n the θ distributions are similar to those obtained by
13 our forcing conditions. In some respects, flattening of forcing distributions is equivalent to
14 have the same dissipation term in the Farrell (1990) model. The poleward shift of the
15 subtropical jets was also observed by HH80 when increasing the vertical diffusion.

16 Figure 5 shows the stream function and the zonal wind speed for the experiments
17 $n=k=0.5$ (Fig. 5a) and $n=k=3$ (Fig. 5b). The parameter n controls the Hadley cell and jet
18 stream widths. The results show that such with $n=k=0.5$ the Hadley cell and jet streams are
19 quite narrow. As far as the vertical position of the maximum value of the stream function is
20 concerned, the experiments with $k=0.5, 1$ and 1.5 exhibit particular behavior with respect to
21 the other experiments. The stream function has its maximum at upper levels. This is related to
22 the different stratification imposed by the parameter k . Stratification with low values of k
23 favor air to move to higher levels with respect to experiments with higher k values.

1

2 **3.2 Time-dependent simulations**

3 Since heating depends on solar irradiation, it is of interest to analyze the solutions
4 obtained by the annually periodic thermal forcing and to compare it with the steady solutions
5 described previously in this paper. Starting from Eq. (7), we can formulate an equilibrium
6 temperature distribution having the maximum heating off the equator at latitude y_0 :

$$7 \quad \theta_E = \frac{4}{3} - |y - y_0|^n + \frac{\Delta_V}{\Delta_H} \left(z^k - \frac{1}{2} \right). \quad (10)$$

8 where y_0 in Eq. (10) is dependent on time according to

$$9 \quad y_0(t) = \sin\left(\frac{\varphi_0\pi}{180}\right) \cdot \sin\left(\frac{2\pi t}{360\text{days}}\right) \quad (11)$$

10 where φ_0 is the maximum latitude off the equator where heating is maximum. Equations 11
11 and 12 are the same used by Fang and Tung (1999) with the choice of maximum extension of
12 φ_0 consistent with the choice of Lindzen and Hou (1988), i.e. $\varphi_0 = 6^\circ$. A prescribed
13 equilibrium temperature varying seasonally makes the simulations more realistic. As
14 described previously, here we will focus on the average and maximum values, in absolute
15 terms, of the stream function and zonal speed obtained during 360 days of simulations. The
16 averaged values are obtained in these cases by averaging the outputs obtained every 30 days,
17 starting from the minimum corresponding to the summer Hadley cell in the boreal
18 hemisphere.

19 The annual averages of the time-dependent and equinoctial circulations shows that
20 maximum stream functions and zonal wind speeds behave quite similarly (Fig. 6),

1 nevertheless the instantaneous Hadley circulation almost never resembles the modeled
2 circulation (Fang and Tung, 1999) as well as the real one (Dima and Wallace, 2003).

3 The maximum stream function is obtained here when $k=n=0.5$ (Fig. 6a). In general, for
4 $n=0.5$, we have stronger circulations and winds. These simulations confirm the inverse
5 relationship between stream function strength and n . The circulation strength expressed as
6 annually averaged value is weaker when compared with that obtained in the equinoctial
7 experiments, when n is low and k is high, otherwise it is only slightly stronger, but it is never
8 twice as strong as that of the equinoctial solution as found by Fang and Tung (1999). When
9 $n=2$ and $k=1$ our results are consistent with those obtained by Walker and Schneider (2005) as
10 discussed in the Subsect. 3.3. For example, there is not an analog maximum when $n=0.5$ and
11 $k=3$ found in the steady solution. The annually averaged maximum wind speed shows only a
12 slight dependence on k when k is low.

13 The meridional position and the height of the maximum stream function show that there
14 is no clear dependency on n and k (Fig. 7). The difference between the time-dependent
15 simulations and the average of the steady solutions is quite interesting. It is to be noticed that
16 the latitude of the stream function maximum in the time-dependent solution is in the range of
17 12.5° and 16° (Fig. 7a), whereas in the equinoctial solutions the correspondent latitude is
18 within a larger range. The maximum stream function is located at higher levels, between 4500
19 and 6000, when k is equal or less to one when and n is less than 2.5. Otherwise the maximum
20 is positioned under 3000 m except (Fig. 7b). Although the averaged results seem interesting,
21 they are impressively similar to those obtained by the steady experiments, they are obtained
22 by averaging snapshots of the time-dependent simulations and thus a cautionary note should
23 be made about these results.

1 More than the steady solution, it is evident that the height of the maximum stream
2 function is lower when $k=3$. In the steady solution this phenomenon is not that evident. When
3 $k=3$, the vertical gradient of θ_E is higher in upper levels making those levels more stable and it
4 prevents, evidently more than the equinoctial solution, air from moving higher leaving
5 circulation occurring at lower levels. The case $k=3$ is equivalent to imposing a “natural”
6 sponge layer at the top of the model. Thus it does not come as a surprise that the maximum
7 stream function is lower than those observed in simulations with other k values. This result is
8 analogous to that of Walker and Schneider (2005) that removed the maximum stream function
9 at higher levels found by Lindzen and Hou (1988) adopting a numerical sponge layer at the
10 top of the model. A comparison with previous works of the simulations with $n=2$ and $k=1$ will
11 be discussed in the Subsect. 3.3. On the contrary, with low k values, the presence of weaker
12 θ_E gradient at upper levels favors air to move higher and the maximum stream function is
13 observed at upper levels. There are more time-dependent simulations with respect to the
14 steady solutions that exhibit this upper level maximum stream function.

15 The position of the jet stream is almost similar to the one observed in the steady solution.
16 It is confined between 28° and 30° , with latitude of averaged jet remaining almost at the same
17 place, except when $n=3$ the jets are located at about 44° confirming the abrupt transition of
18 the jet stream position when $n=3$ already found for the equinoctial experiment. Fu and Lin
19 (2011) suggest that the jets moved poleward of about 1° per decade in the last several years
20 but Strong and Davis (2007) observed that Northern hemisphere subtropical jet shifted
21 poleward over the east Pacific, while an equatorward shift of the subtropical jet was found
22 over the Atlantic basin. Excluding the case $n=3$, all the other subtropical jets in the different
23 experiments have the position of the maximum very close to one another and the shifting
24 range is very limited. Thus, when a vigorous circulation occurs the jet location must be

1 located at about 30° , whereas reducing too much the tropical gradient the process of
2 homogenization becomes weaker like in a slow rotating planet and this is confirmed in the
3 time-dependent solution. Both Tandon et al. (2013) and Kang and Polvani (2011) found a
4 discrepancy in this area with the jets that do not follow the Hadley cell edge. In an
5 axisymmetric model, defining the Hadley edge as a function of the stream function and
6 connecting it to the jet location is problematic because of lacking of a zero value of the stream
7 function.

8 Figure 8 shows the annually averaged circulation for the same cases as shown in Fig. 5,
9 which is obtained by annually averaged heating. It is impressive how the steady and time-
10 dependent solutions resemble each other. As in Fang and Tung (1999) the annual mean
11 meridional circulation has the same extent, but differently from them the strength of the
12 annual mean circulation of the time-dependent solution is almost the same of the steady
13 solution.

14 When the heating center is off the equator the intensity of the winter cell is stronger,
15 whereas the cell of the summer hemisphere is weak and sometimes almost absent. Figures 9
16 shows the maxima of the stream function and zonal wind speed at the winter solstitial as a
17 function of n and k . The maximum stream function as a function of n and k has the same
18 configuration of the steady solution. Here, as expected the maximum intensity of the
19 meridional circulation (Fig. 9a) reached during the simulation is twice as strong as that of the
20 steady solution or the annually averaged time dependent solution and it has about the same
21 strength of the observed circulation. However the winds are much stronger too, in contrast
22 with observations. The zonal wind has a different configuration instead, the maximum zonal
23 wind speed is obtained when $n=1$ (Fig.9b).

1 We can inspect a couple of simulations when the stream function reaches its maximum
2 in the boreal hemisphere. Figure 10 shows the stream function and the zonal wind speed when
3 $n=2$ and $k=0.5$ (Fig. 10a, b) and $n=2$ and $k=3$ (Fig. 10 c, d). When $k=0.5$ (upper panels) the
4 boreal (winter) circulation is much stronger when $k=0.5$, with the austral (summer) circulation
5 almost absent. The vertical extent is larger and the maximum is located at higher levels. The
6 summer and winter jets are both more intense than their counterparts for $k=3$. The tropical
7 easterly winds are in this case stronger than those for $k=3$ (13.8 ms^{-1} vs 11.4 ms^{-1}) and the
8 easterly region is also wider. When $k=3$, it is noted that the winter cell is located closer to the
9 equator than the summer cell.

10 **3.3 A discussion on the case $n=2$ $k=1$**

11 When $n=2$ and $k=1$, corresponding to the classic case discussed in many studies, we
12 found that the time-dependent solution is only slightly stronger than the steady solution.
13 Lindzen and Hou (1988) proposed a study of the Hadley circulation in which the maximum
14 heating was 6° off the equator. In their non-time-dependent model, the solution showed an
15 average circulation much stronger with respect to the equinoctial solution. Lindzen and Hou
16 (1988) suggested that this exceptional strength was due to a nonlinear amplification of the
17 annually averaged response to seasonally varying heating, although Dima and Wallace (2003)
18 in a study on the seasonality of the Hadley circulation did not observe any nonlinear
19 amplification.

20 With the parameters used for equinoctial and time-dependent simulations we performed
21 an experiment like that of Lindzen and Hou (1988), with $\phi_0 = 6^\circ$ that will be referred to as
22 solstitial experiment. We found that the winter circulation is stronger by a factor three with
23 respect to the steady solution obtained with the equinoctial heating consistent with the result

1 of the axisymmetric model in Walker and Schneider (2005). However, the average circulation
2 obtained by averaging two solstitial experiments, with $\phi_0 = 6^\circ$ and $\phi_0 = -6^\circ$ respectively is
3 only 1.5 times stronger than the steady solution with $\phi_0 = 0^\circ$, and it has a maximum in the
4 upper levels of the model domain as in Lindzen and Hou (1988). We suggest that this
5 maximum is due to a numerical effect caused by averaging the single solstitial experiments
6 rather than a spurious effect caused by the rigid lid as suggested in Walker and Schneider
7 (2005), even though a sponge layer actually lowers the maximum stream function height and
8 we can see the effects of a stronger vertical gradient in the upper levels especially in the time-
9 dependent solution (cf Fig. 3 and Fig. 7). Single solstitial experiments did not show a
10 maximum in upper levels and so the equinoctial and time-dependent experiments (Figs. 11a
11 and 11b). Consequently the only operation performed to produce Fig. 11c, which exhibits the
12 upper levels maxima, was to average the two solstitial experiments, which causes the
13 maximum at upper levels.

14 Finally, we notice that comparing a time-dependent solution with $\varphi_0 = 6^\circ$ with the
15 equivalent steady solution having the heating off the equator is not properly correct, since for
16 the time-dependent model φ_0 represents only the maximum extension of heating, hence a
17 more correct comparison between time and no time-dependent solutions should be performed
18 with the time-dependent solution having $\varphi_0 = 3^\circ$. In such a case, the average solution is only
19 slightly weaker than the Hadley circulation driven by annually averaged heating or by a time-
20 dependent heating which does not show any maximum in the upper levels. Thus, the results of
21 equinoctial, time-dependent and solstitial ($\varphi_0 = 3^\circ$) experiments are mutually consistent.

1 **4 Conclusions**

2 The forcing of an Earth-like planet can change for several reasons. For instance, a
3 change of forcing distribution can be caused by different factors such as global warming or
4 long-term variation of solar activity.

5 Under the assumption of an equal equator-pole difference at the surface we used an
6 axisymmetric model to study the sensitivity of the tropical atmosphere to different θ_E
7 distributions modulated by two parameters, n that controls the broadness of the distribution
8 and k that modulates how the θ_E is distributed vertically. Equinoctial and time-dependent
9 solutions were simulated and compared. Moreover for the case $n=2$ and $k=1$, corresponding to
10 the classical distribution used in literature, a few solstitial experiments were also run. When
11 $n=2$ and $k=1$, the annually averaged circulation of equinoctial, time-dependent and solstitial
12 experiments are quite close to one another, consistent with the results of Walker and
13 Schneider (2005). However, the results differ from those of Lindzen and Hou (1988) and
14 Fang and Tung (1999). As in all those works the maximum of the stream function of the
15 solstitial experiment is at upper levels, but it seems to be related to a spurious effect of the
16 averaging operation rather than a spurious effect due to the rigid lid.

17 The results provide evidence that concentrated equilibrium temperature distributions
18 enhance the meridional circulation and jet wind speed intensities, confirming findings of
19 Lindzen and Hou (1988) even though these authors imposed the same energy input. However,
20 in the present study the concentrated distribution at the equator has lower energy input.

21 The width of the Hadley cell is proportional to n , but when the cell width increases its
22 intensity decreases. Poleward shift of the Hadley circulation with warming is very robust as it
23 has been observed in many models and over large range of climates (Frierson et al., 2007).

1 Since the equator-pole gradient is the same for all the experiments with the same k ; it evident
2 that the gradient in the tropical region controls the circulation strength. The term k controlling
3 the imposed stratification has influence on the actual temperature distribution that can differ
4 remarkably from θ_E distribution.

5 Vertical stratification is important in determining the position and intensity of the Hadley
6 cell and jet when n is low, i.e. when for whereas k loses its importance when the θ_E
7 distribution is wider. This latter result is consistent with results of Tandon et al. (2013) who
8 found that the Hadley cell expansion and jet shift had relatively little sensitivity to the change
9 of the lapse rate. Consequently, the subtropical jet stream intensities are controlled by the
10 broadness of horizontal equilibrium temperature rather than the stratification, with higher
11 values of the jet when the thermal forcing is concentrated to the equator. In the case of time
12 dependent solution with $n=0.5$ (concentrated heating) and k takes the extreme values (0.5 and
13 3) the simulated maximum stream function has the same magnitude order of the observed
14 stream function, ten times larger than that obtained in HH80 and with the reference
15 simulation, even though with stronger winds too.

16 The jet stream position does not show any dependence with n and k , except when the
17 θ_E distribution is the widest ($n=3$); in such a case an abrupt change occurs and the maximum
18 of the zonal wind jet is located at mid-latitudes (47° in steady solution and 44° in annually
19 averaged time-dependent solution). This behavior can explained by using the analytic study of
20 this model performed by Cessi (1998) claiming that when the meridional gradient becomes
21 too small the process of homogenization of temperature and momentum occurs slowly and the
22 circulation behaves as that of a slow rotating planet that exhibits poleward shift of the
23 subtropical jets.

1

2

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1

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1 Table 1. Latitudes (in degrees) of the maximum wind speed for the equinoctial and time-
2 dependent solutions when $k=1$ as a function of the parameter n .

3

n	0.5	1	1.5	2	2.5	3
Equinoctial	27.4	28.7	27.4	26.1	28.7	47.7
Time dependent	28.7	28.7	28.7	27.4	27.4	44.4

4

5

1 **Figure Captions**

2 Figure 1. Meridional (a) and vertical (b) average of non-dimensional equilibrium temperature
3 as a function of n with $k=1$ (a) and k with $n=0.5, 1$ and 1.5 (b). Dimensional values are
4 obtained multiplying by $\theta_0=300$ K.

5 Figure 2. Maximum non-dimensional stream function (a) and zonal wind speed [ms^{-1}] (b) as
6 function of parameters n and k for the steady solution. Dimensional values of the stream
7 function are obtained multiplying by $\nu_V R \varepsilon^{-1} = 484 \text{ m}^2 \text{s}^{-1}$.

8 Figure 3. Latitude [degree] (a) and Height [m] (b) of maximum non-dimensional stream
9 function.

10 Figure 4. Vertically averaged the θ (blue line) and θ_E (red line) for the simulations with $n=3$
11 and $k=0.5$ (a), $k=1$ (b) and $k=3$ (c). Dimensional values are obtained multiplying by $\theta_0=300$
12 K.

13 Figure 5. Non-dimensional stream function (contours) and zonal wind speed [ms^{-1}] (colors)
14 for the steady cases $n=0.5, k=0.5$ (a) and $n=3, k=3$ (b). Dimensional values of the stream
15 function are obtained multiplying by $\nu_V R \varepsilon^{-1} = 484 \text{ m}^2 \text{s}^{-1}$.

16 Figure 6. Maximum of annually averaged non dimensional stream function (a) and zonal wind
17 speed [ms^{-1}] (b) as function of parameters n and k for the time-dependent simulations.
18 Dimensional values of the stream function are obtained multiplying by $\nu_V R \varepsilon^{-1} = 484 \text{ m}^2 \text{s}^{-1}$.

19 Figure 7. Latitude [degree] (a) and Height [m] (b) of maximum annually averaged non-
20 dimensional stream function for the time-dependent solution.

21 Figure 8. Annually averaged non-dimensional stream function (contours) and zonal wind
22 speed [ms^{-1}] (colors) for the steady cases $n=0.5, k=0.5$ (a) and $n=3, k=3$ (b). Dimensional
23 values of the stream function are obtained multiplying by $\nu_V R \varepsilon^{-1} = 484 \text{ m}^2 \text{s}^{-1}$.

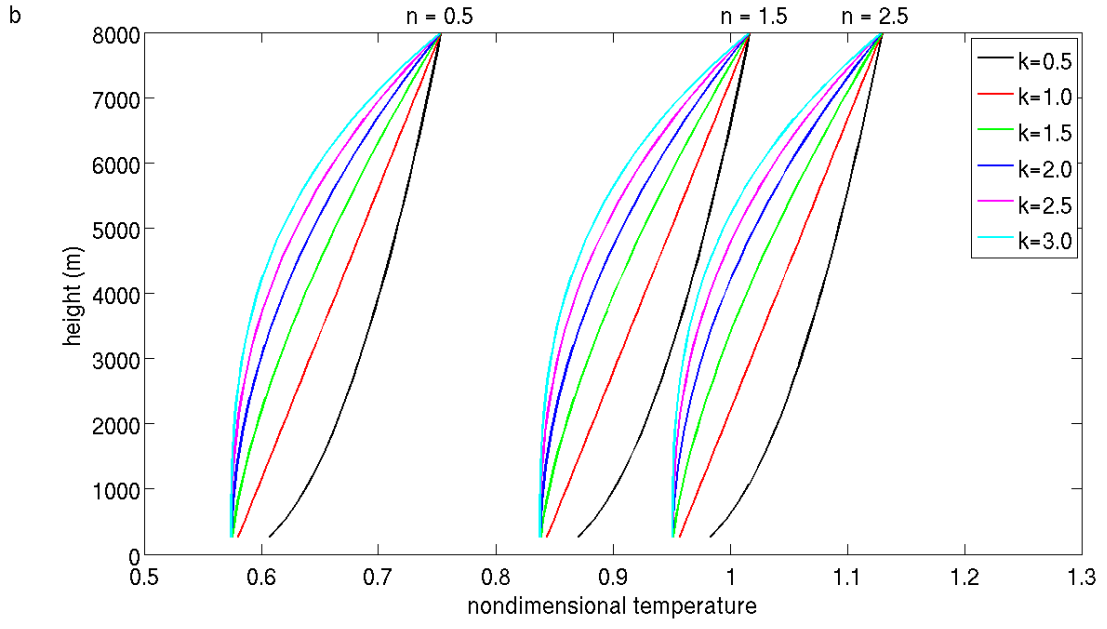
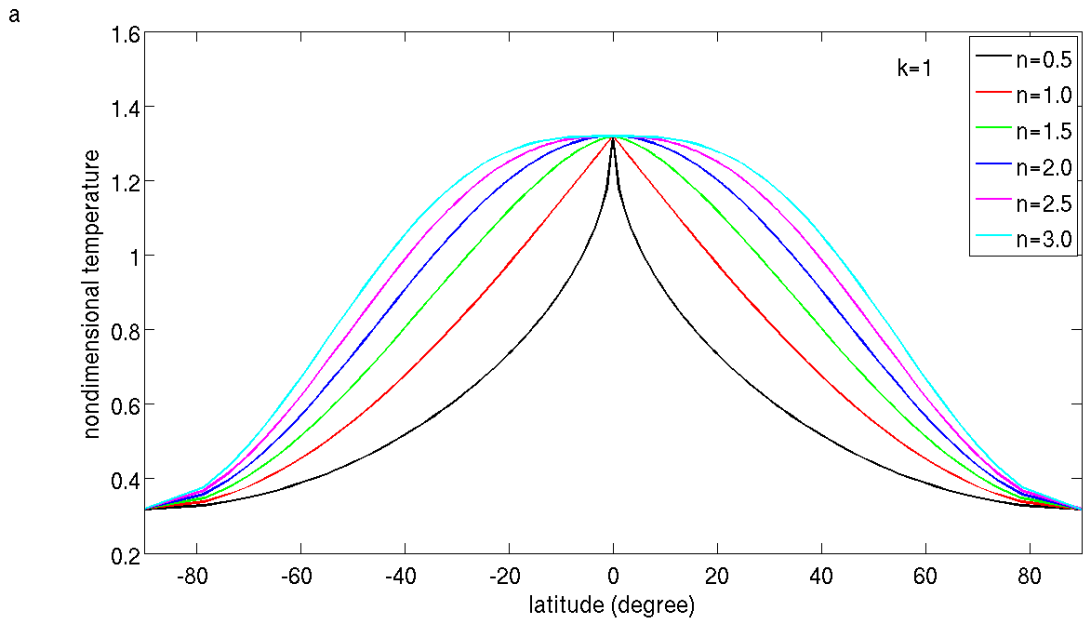
24 Figure 9. Maximum of non-dimensional stream function (a) and zonal wind speed [ms^{-1}] (b)
25 as function of parameters n and k for the time-dependent simulations. Dimensional values of
26 the stream function are obtained multiplying by $\nu_V R \varepsilon^{-1} = 484 \text{ m}^2 \text{s}^{-1}$.

27 Figure 10. Boreal winter circulation, non-dimensional stream function (a and c) and zonal
28 wind speed [ms^{-1}] (b and d) for the time-dependent simulation with $n=2, k=0.5$ (upper panels)

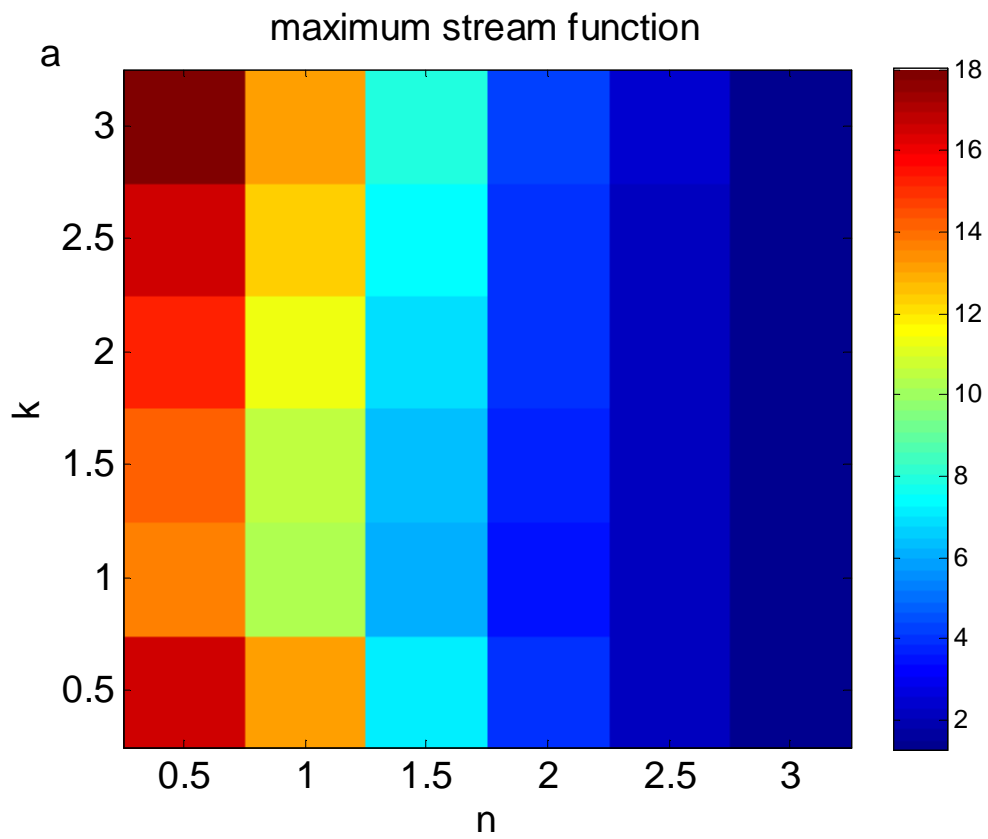
1 and $n=2, k=3$ (lower panels). Dashed lines indicate negative values. Dimensional values of the
2 stream function are obtained multiplying by $\nu_V R \varepsilon^{-1} = 484 \text{ m}^2 \text{s}^{-1}$.

3 Figure 11. Non-dimensional stream function (contours) and zonal wind speed ms^{-1}] (colors)
4 when $n=2$ and $k=1$ for the steady solution (a), annually averaged for the time-dependent
5 solution (b) and averaged for maximum heating 6° off the equator (c). Dimensional values of
6 the stream function are obtained multiplying by $\nu_V R \varepsilon^{-1} = 484 \text{ m}^2 \text{s}^{-1}$.

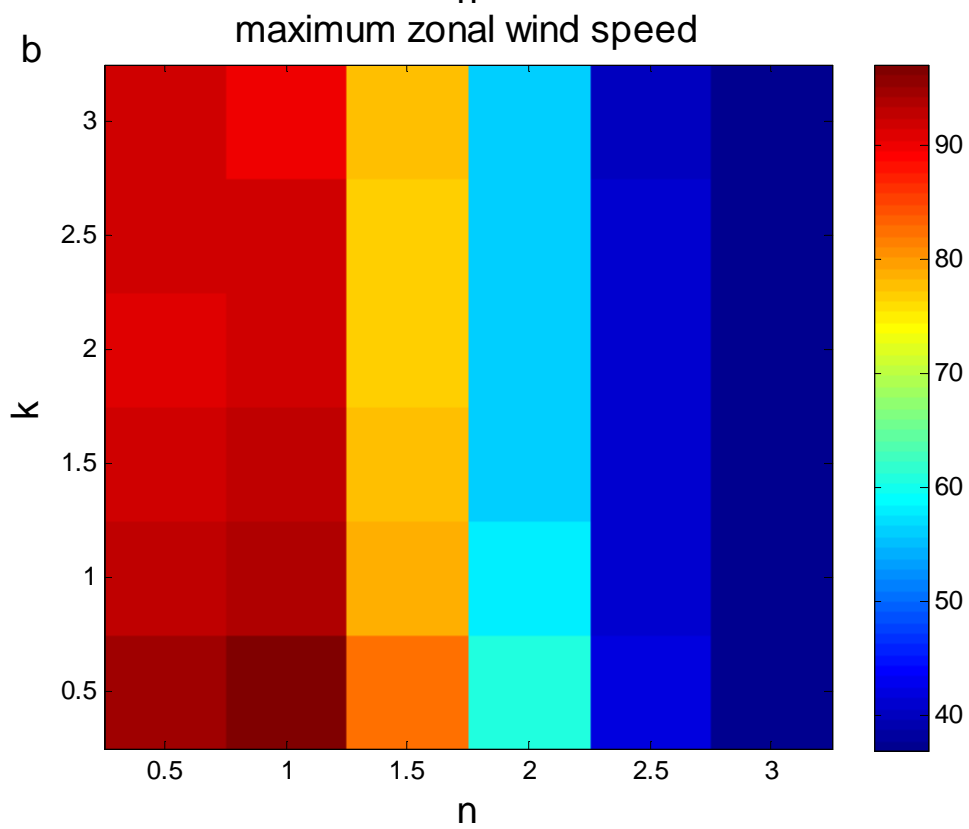
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3 Figure 1.

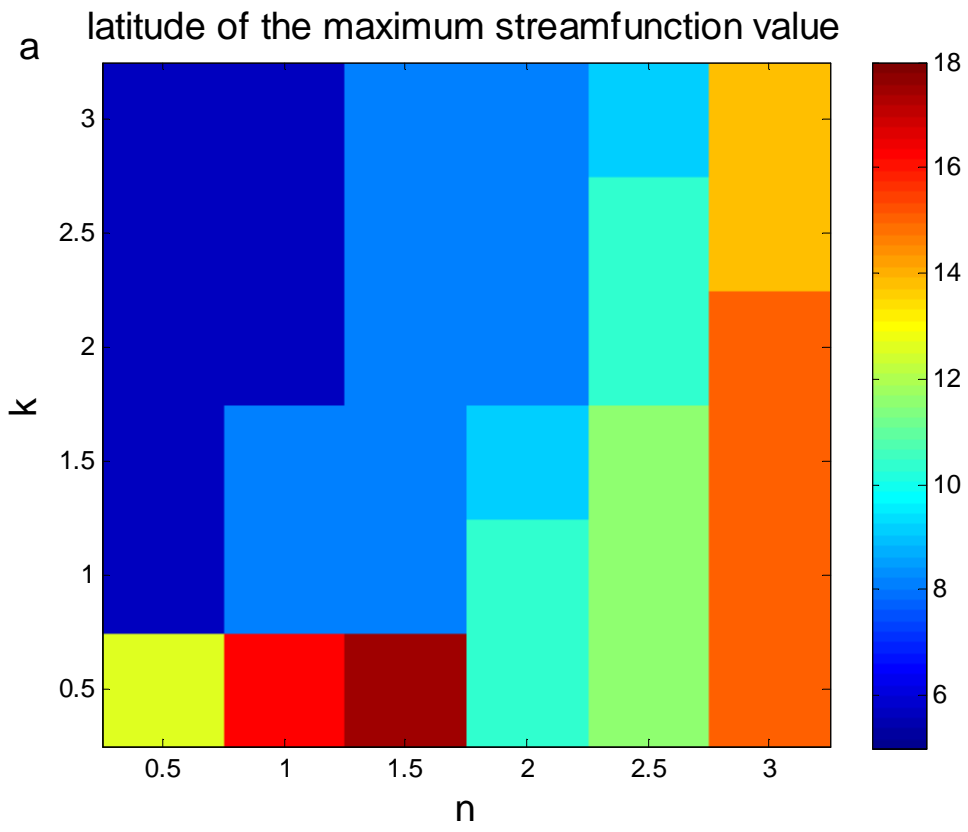


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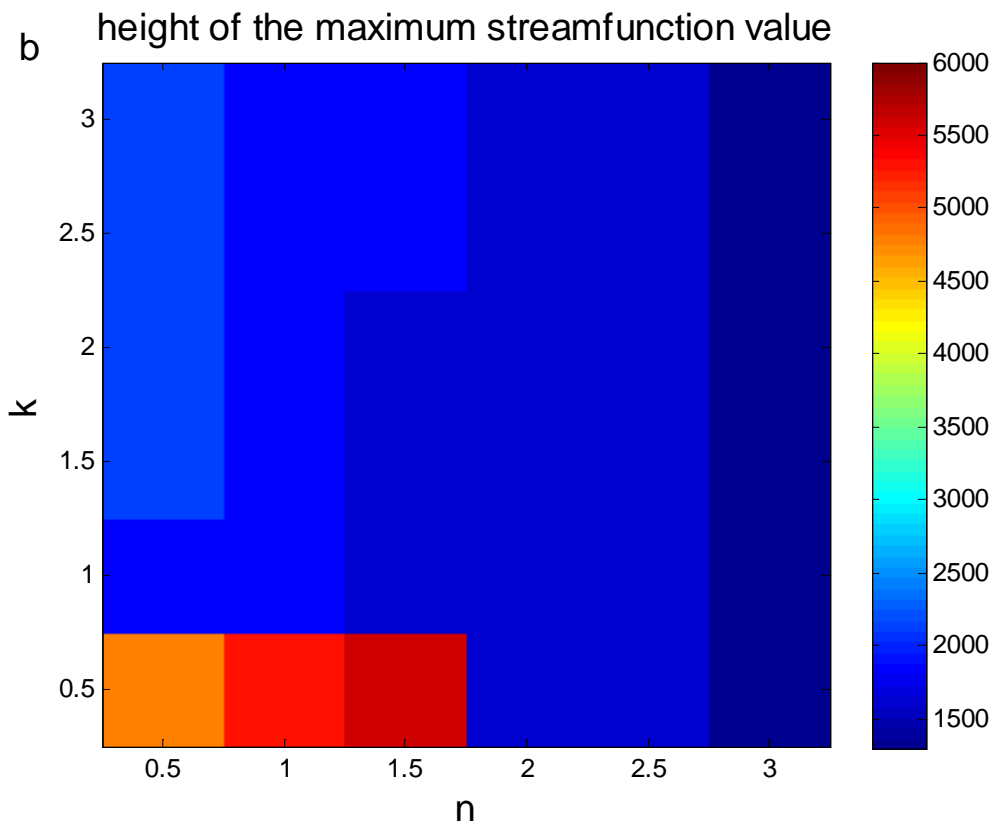


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3 Figure 2.

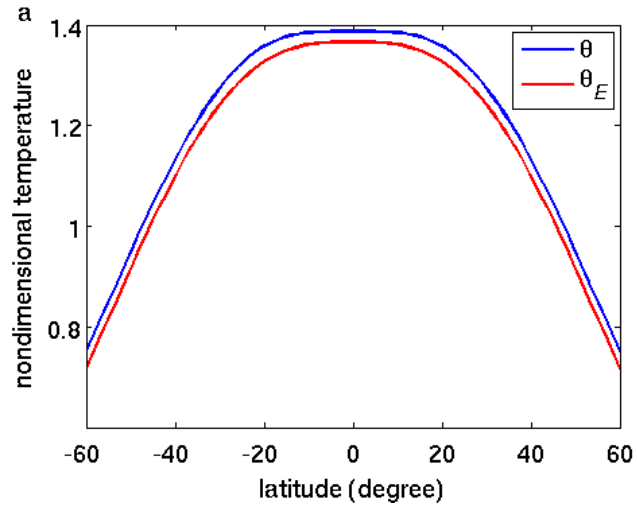


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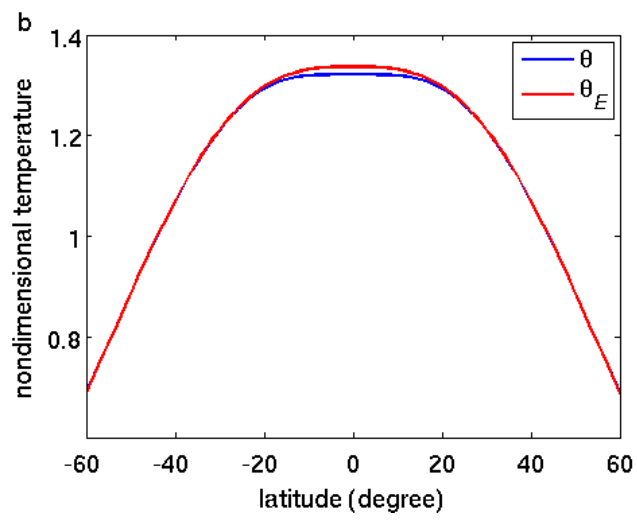


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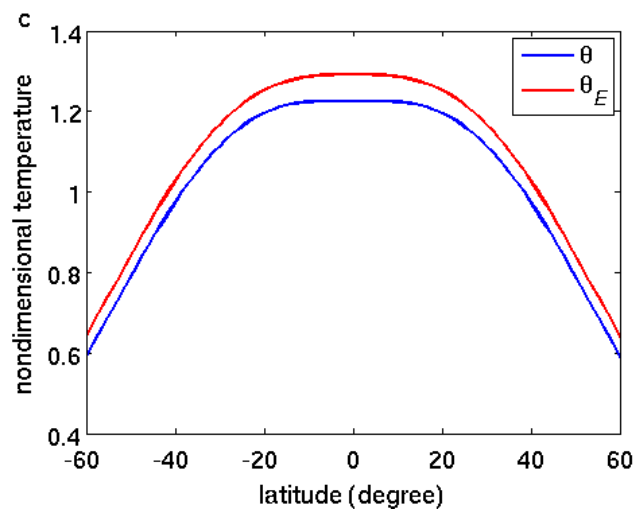
3 Figure 3.



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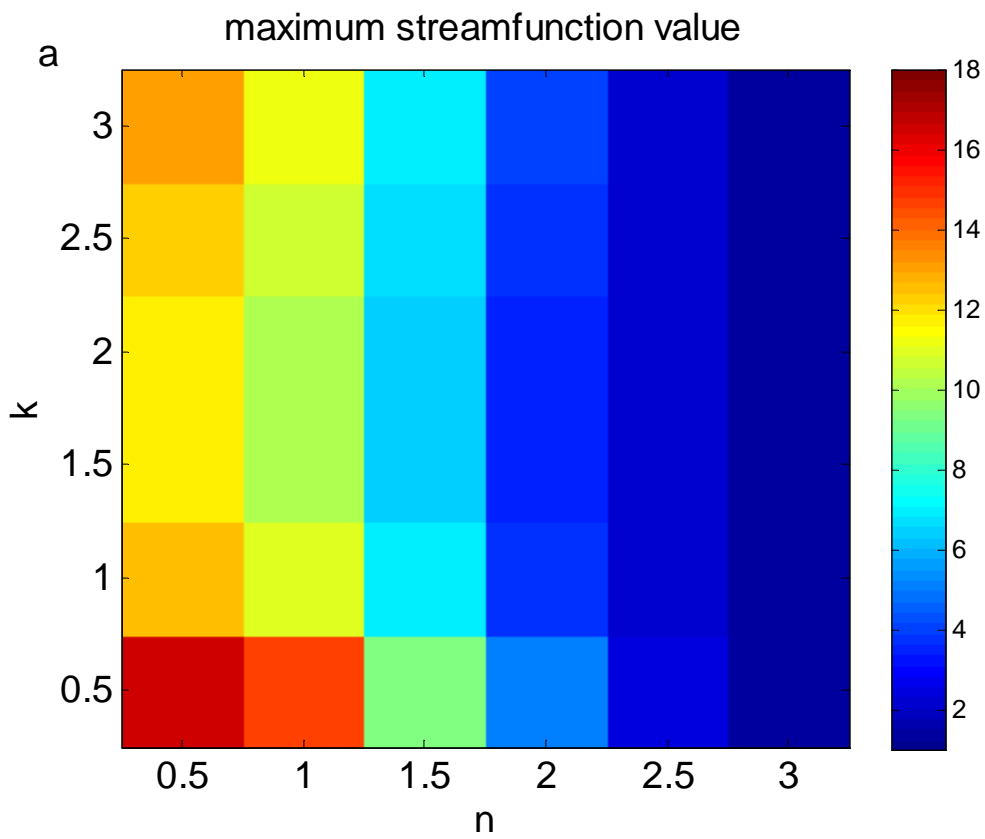


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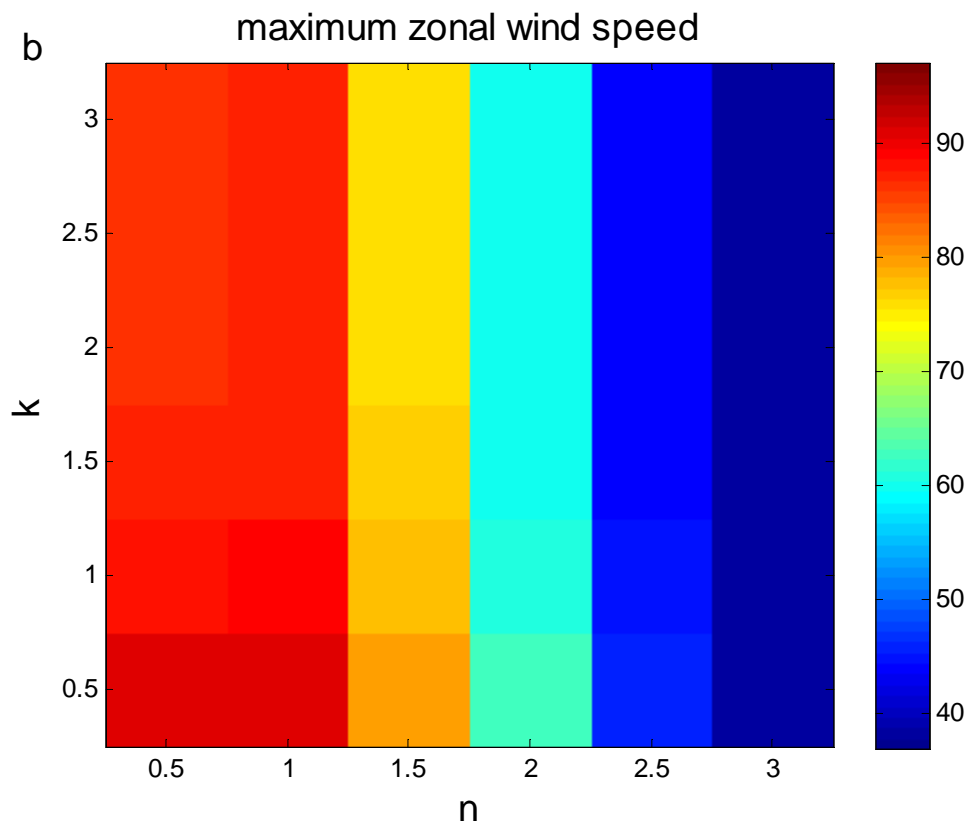


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4 Figure 4.

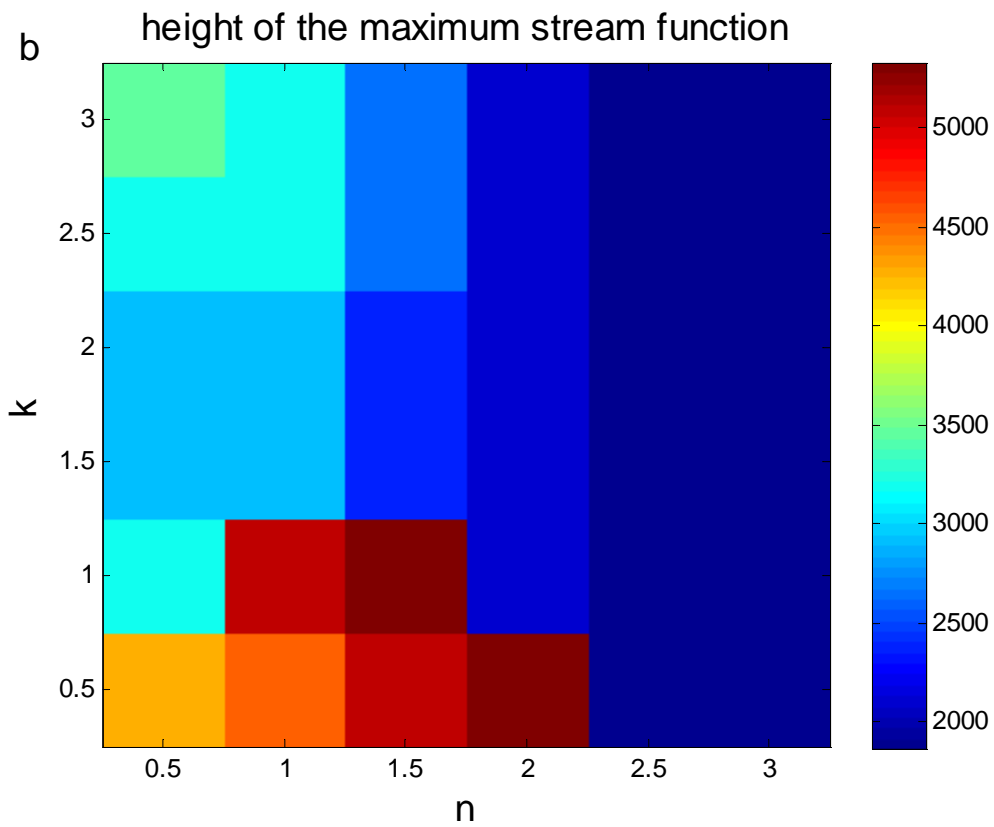
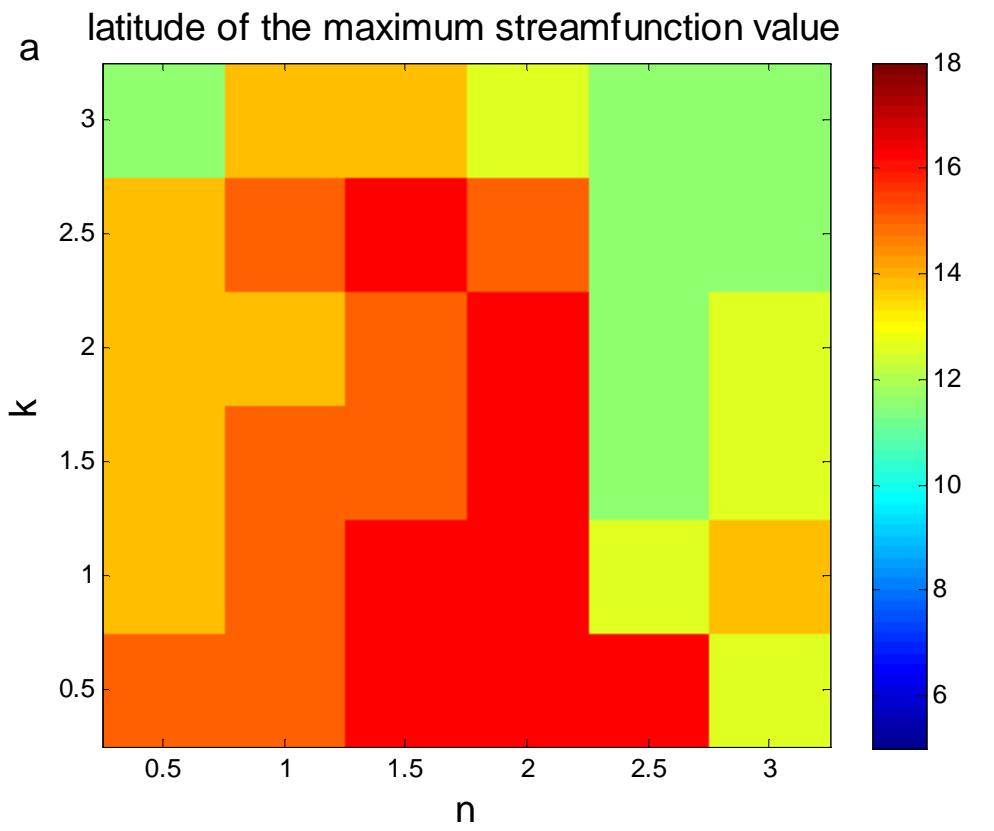


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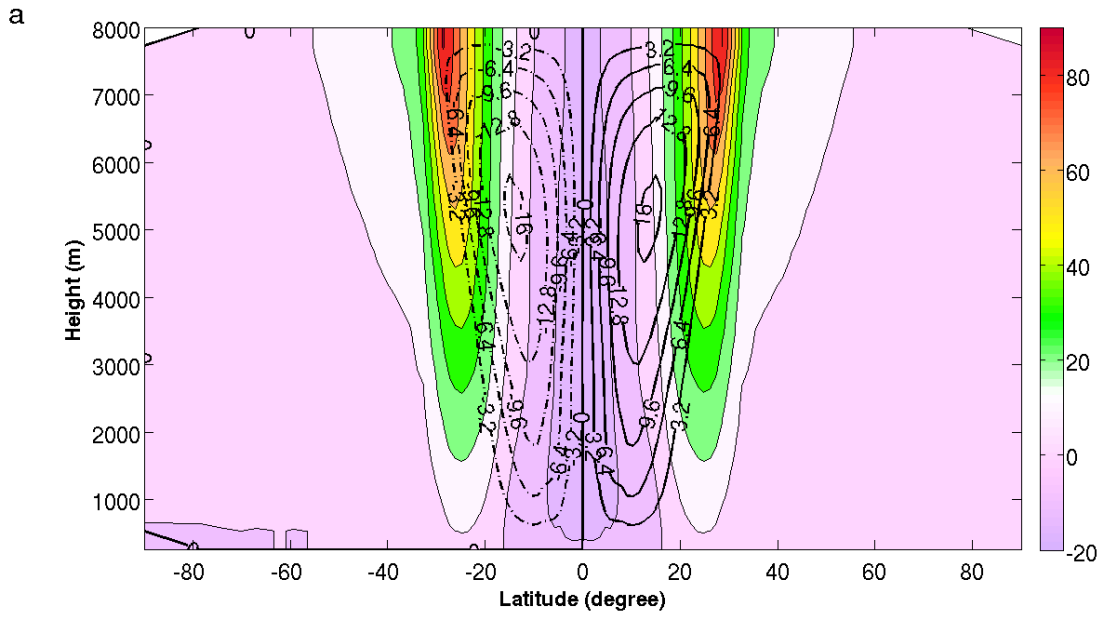
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3 Figure 6.



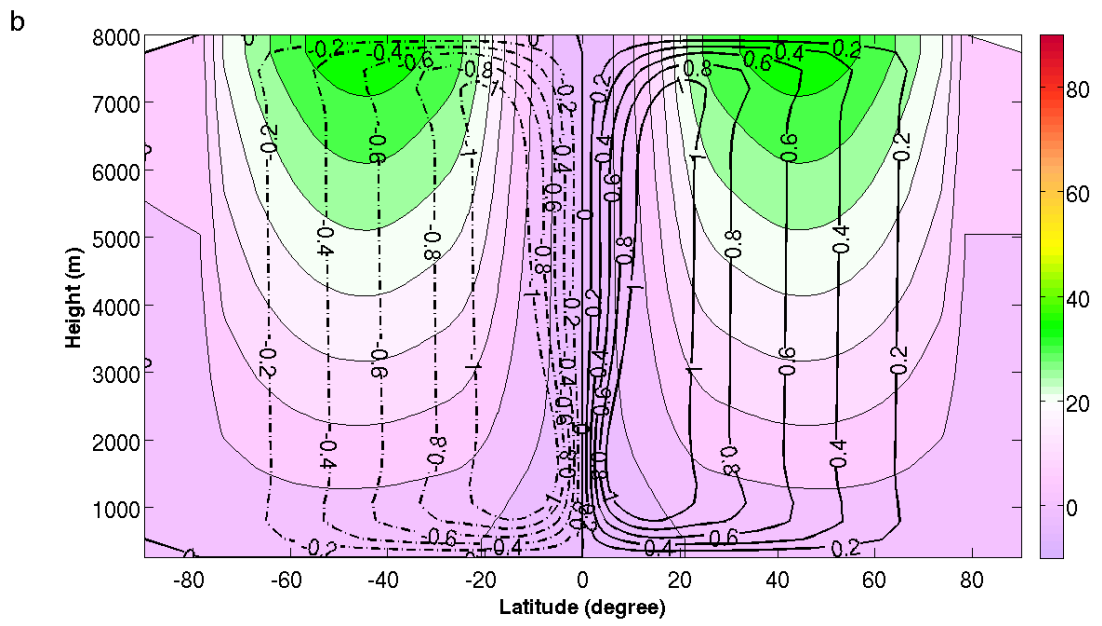
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2 Figure 7.



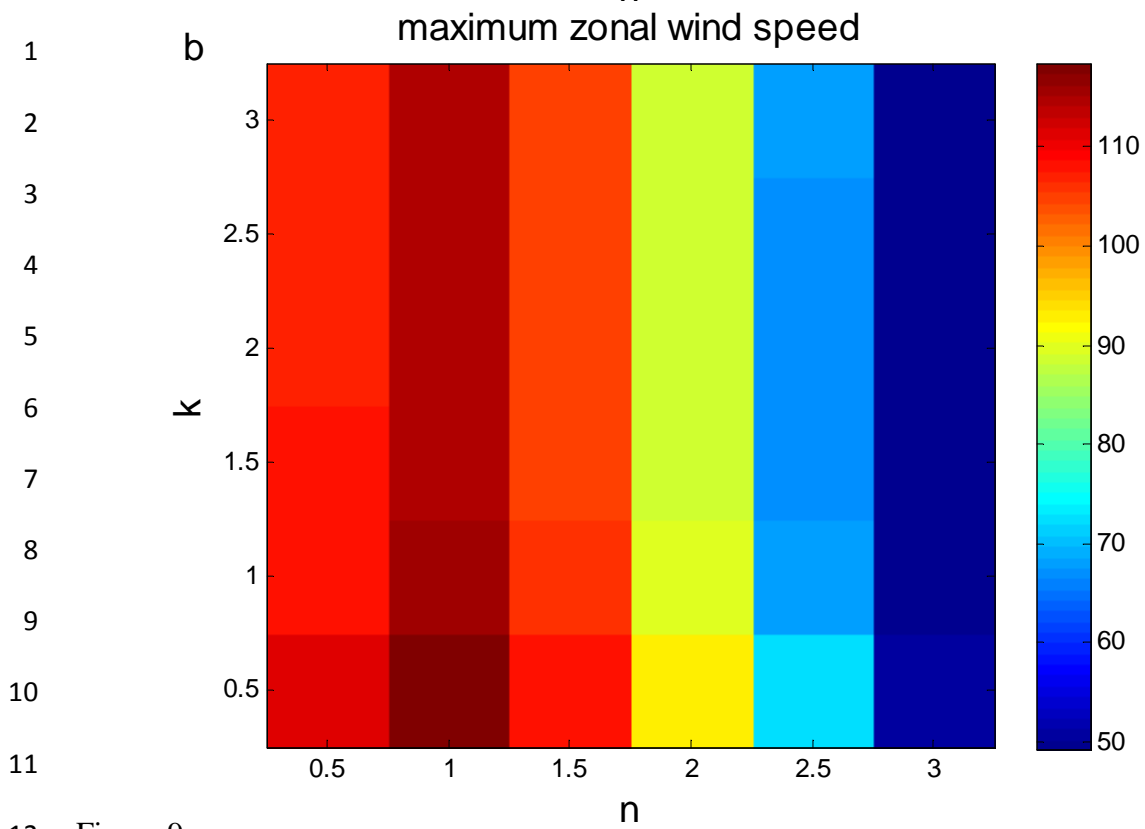
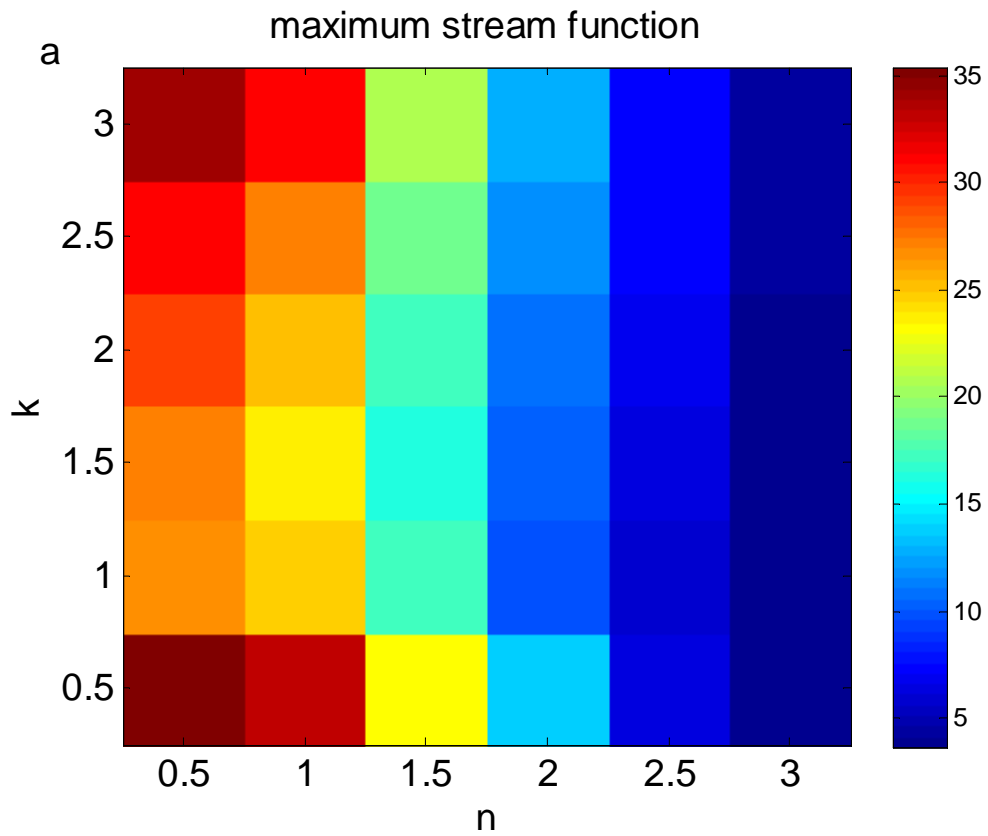
1

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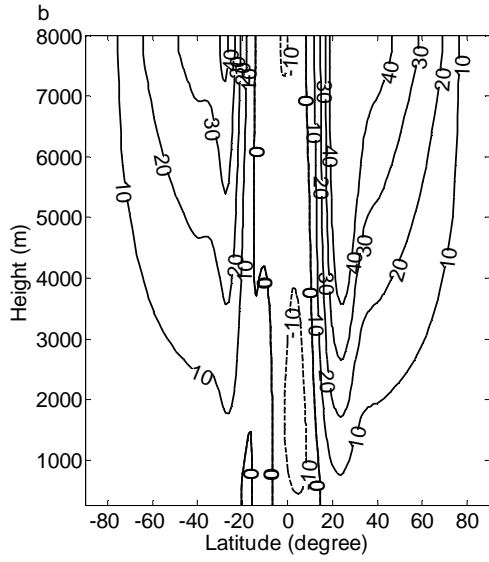
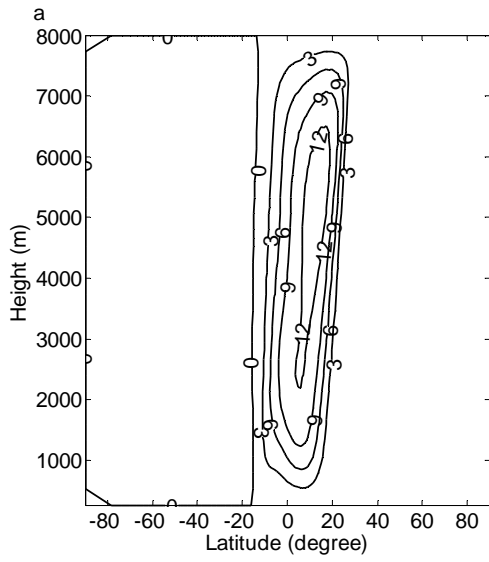


3

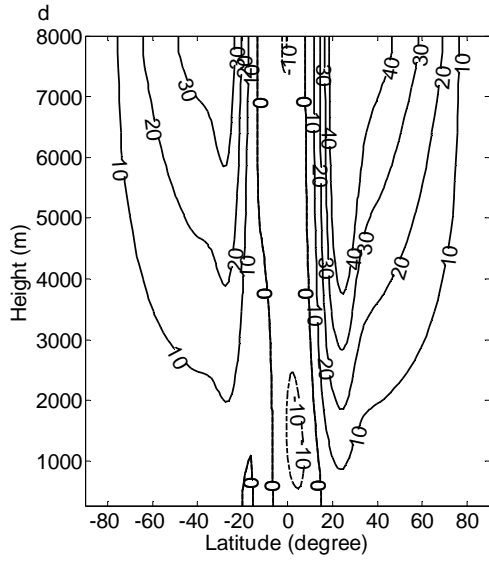
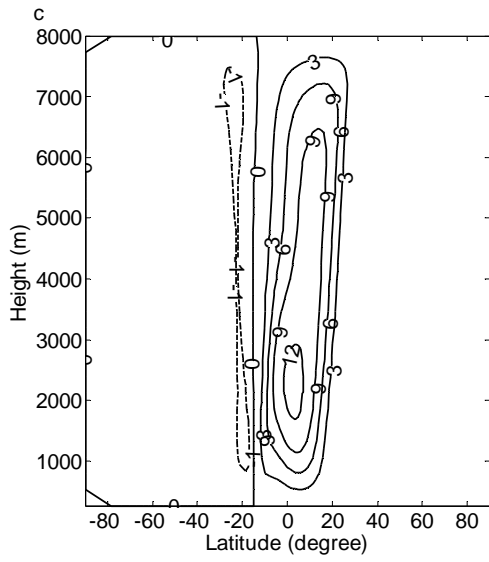
4 Figure 8.



12 Figure 9.

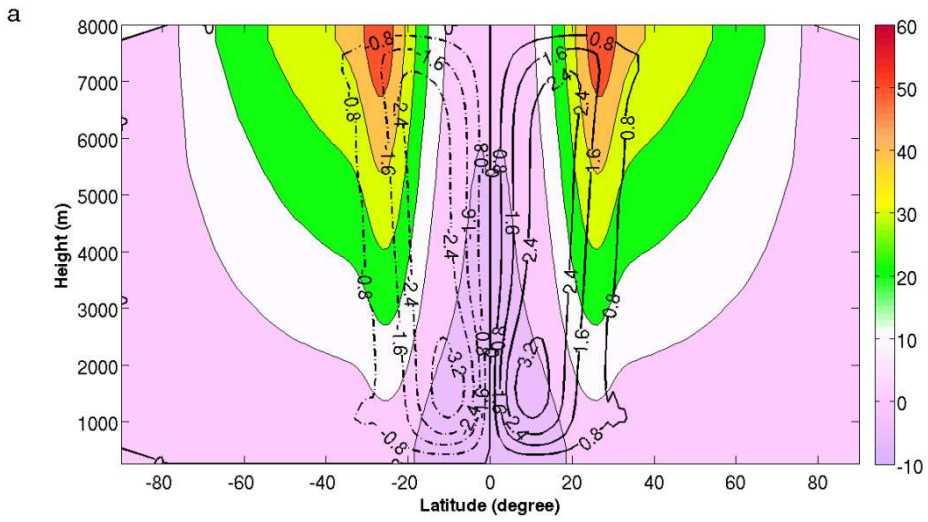


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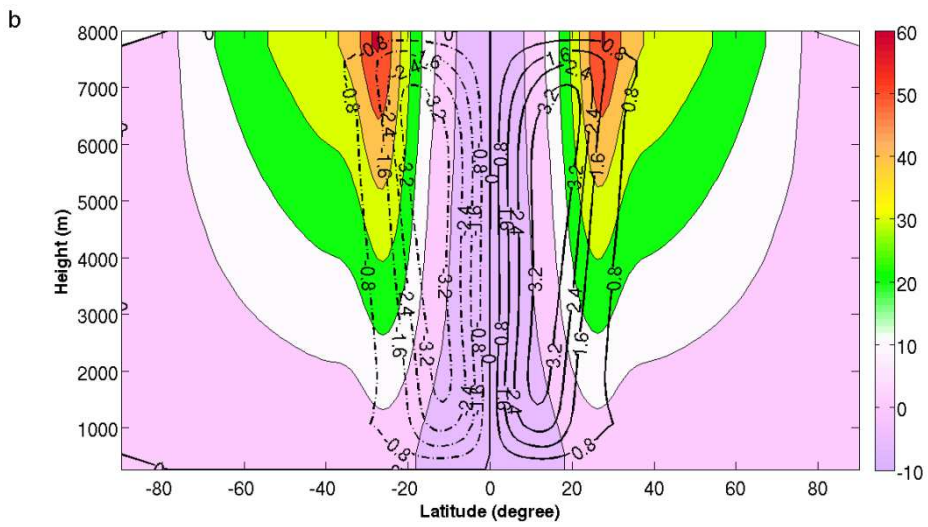


2

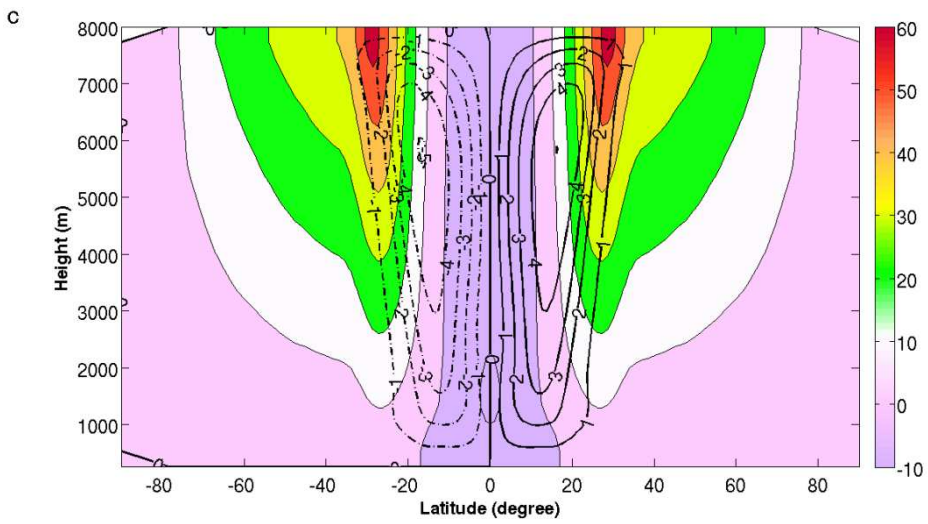
3 Figure 10.



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4 Figure 11.