Nonlinear Processes in Geophysics (2000) 7: 167–172

Nonlinear Processes in Geophysics

Three-dimensional simulation of the electromagnetic ion/ion beam instability: cross field diffusion

H. Kucharek[1], M. Scholder[1,2], and A. P. Matthews[3]

[1]Max-Planck-Institut für Extraterrestrische Physik, 85741 Garching, Germany
[2]STEL, Nagoya University, Toyokawa, 442, Japan
[3]Department of Physics, University of Natal, Rep. of South Africa

Received: 5 November 1999 – Revised: 28 February 2000 – Accepted: 22 March 2000

Abstract. In a system with at least one ignorable spatial dimension charged particles moving in fluctuating fields are tied to the magnetic field lines. Thus, in one- and two-dimensional simulations cross-field diffusion is inhibited and important physics may be lost. We have investigated cross-field diffusion in self-consistent 3-D magnetic turbulence by fully 3-dimensional hybrid simulation (macro-particle ions, massless electron fluid). The turbulence is generated by the electromagnetic ion/ion beam instability. A cold, low density, ion beam with a high velocity stream relative to the background plasma excites the right-hand resonant instability. Such ion beams may be important in the region of the Earth's foreshock. The field turbulence scatters the beam ions parallel as well as perpendicular to the magnetic field. We have determined the parallel and perpendicular diffusion coefficient for the beam ions in the turbulent wave field. The result compares favorably well (within a factor 2) with hard-sphere scattering theory for the cross-field diffusion coefficient. The cross-field diffusion coefficient is larger than that obtained in a static field with a Kolmogorov type spectrum and similar total fluctuation power. This is attributed to the resonant behavior of the particles in the fluctuating field.

1 Introduction

The problem of particle motion in electromagnetic fields is fundamental for space and astrophysical plasmas. In the mid 1960s the spatial transport of charged particles was explicitly related to magnetic field turbulence (Jokipii, 1966). Subsequent work, mainly based on quasi-linear theory, was concerned with the transport of cosmic rays in interstellar and interplanetary space (e.g, Hasselmann and Wibberenz, 1968; Goldstein et al., 1975; Luhmann, 1976). While pitch angle diffusion leading to a finite mean free path parallel to the magnetic field is fairly well understood, this is not so in the case of transport normal to the average field. Such cross-field diffusion is expected to be particularly important at quasi-perpendicular shocks, where both, the parallel diffusion coefficient and the diffusion coefficient normal to the shock, determine the rate with which particles are accelerated.

Correspondence to: H. Kucharek

Recently, Jokipii et al. (1993) have presented a general theorem according to which charged particles in fields with at least one ignorable spatial coordinate are effectively forever tied to the same magnetic field line, except for motion along the ignorable coordinate. This theorem was derived by Jokipii et al. (1993) in a heuristic manner and has recently been derived rigorously by Jones et al. (1998). Thus, in the previous analyses in which only one- and two-dimensional fluctuations were considered, i.e., where the magnetic field depends on only one or two spatial coordinates, the resulting cross-field diffusion coefficient was only due to field-line mixing (e.g., Jokipii, 1966; Forman et al., 1974). The analysis by Jokipii et al. (1993) did not show that cross-field diffusion actually does occur when a three-dimensional realization of the magnetic field is considered. Giacalone and Jokipii (1994) have therefore investigated test particle motion in a magnetic field which fluctuates in all three spatial directions. The fluctuations were assumed to be static and to have a power spectrum which is Kolmogorov. These authors found that, with such a fully three-dimensional realization of turbulence, cross-field diffusion is indeed possible, and that the simple hard-sphere scattering picture is in reasonable agreement with the simulation.

The limitation of the Giacalone and Jokipii (1994) simulation is that the assumed turbulence, although being three-dimensional, is static and not self-consistently generated. Furthermore, the energy density in the fluctuations was arbitrarily set to be equal to the energy density of the background field. We wish in the present paper to take the simulation by Giacalone and Jokipii (1994) one step further and to study cross-field diffusion in magnetic field turbulence which is self-consistent. We have performed fully three-dimensional simulations of the resonant electromagnetic ion/ion beam instability, i.e. of the turbulence produced by a low density, high speed ion beam penetrating into a background plasma, and we have analyzed the cross-field diffusion properties of the beam ions. The linear dispersion properties of the ion beam instability are well known and the nonlinear properties have been studied by 1- and 2-D simulations in considerable detail. But apart from a purely theoretical interest in this instability, strongly nonthermal distribution functions, including field-aligned beams, have been found in the region of the Earth's foreshock, and it has been hypothesized that the ion beam instability plays an important part in the structure of the foreshock region. First, we will summarize briefly the physics of the electromagnetic ion/ion beam instability and we will then report on the 3-D simulations with particular emphasis on the occurrence of cross-field diffusion.

2 The Electromagnetic Ion/Ion Right-Hand Instability

In the case of two ion components represented by drifting Maxwellian distributions several electromagnetic instabilities may arise (e.g., Gary, 1991). For an isotropic, low density, cool ion beam (beam velocity >> thermal velocity) the ion/ion right-hand resonant instability is the growing mode of lowest threshold. For this instability, both the electrons and the core ions are nonresonant, while this mode is beam resonant. The maximum growth rate is found for propagation parallel to the beam. For parallel propagation the mode is right-hand circularly polarized, has positive helicity, and propagates in the direction of the beam. Nonlinear properties of the electromagnetic instabilities have been investigated by one-dimensional hybrid simulations by Akimoto et al. (1993). Winske and Quest (1986) have compared results of 1-D and 2-D hybrid simulations for a plasma with a high drift velocity and a cool beam and found no important differences between the 1-D and 2-D simulations, with parallel propagating modes dominating also the 2-D simulation in the nonlinear stage. Hellinger and Mangeney (1999) have recently reported 2-D simulations of the right-hand resonant ion beam instability. They found that obliquely propagating right-hand resonant mode occur in the nonlinear state and that the beam ions are resonant with the parallel and the oblique mode. Trapping of beam particles in the oblique modes results in first order density fluctuations, which gives rise to beam filamentation. The oblique modes are also expected to lead to cross-field diffusion of the beam particles. As outlined above, 1-D and 2-D simulations are principally not able to give information of cross-field diffusion. This is only possible when there is no ignorable spatial dimension, i.e. when the simulations are fully three-dimensional.

3 Simulation Model

The simulation is done with a three-dimensional hybrid code (macro-particle ions, inertialess electron fluid) described by Matthews (1994). In this code the current advanced method is used with cyclic leapfrog for the magnetic field. This scheme requires the fields to be know at half time steps ahead. This is obtained by advancing the current density to this time step. All variables are functions of time t and three spatial variables x, y, z. The time is expressed in units of the inverse of the proton gyro-frequency, $\Omega_i = eB_o/mc$, where c is the speed of light, e is the magnitude of the electron charge, B_o is the initial magnetic field strength, and m is the proton mass. The unit velocity is the Alfvén velocity v_A. Distances are in units of the proton inertial length, $\lambda = c/\omega_{pi}$, where ω_{pi} is the proton plasma frequency. The simulations have been performed in a cube of size $L_x \times L_y \times L_z$, where $L_x = 32\lambda$, $L_y = 80\lambda$, and $L_z = 32\lambda$. In this cube we use a grid size of $\Delta x = 0.5$, $\Delta y = 1.0$ and $\Delta z = 0.5$. For the particles and the fields periodic boundary conditions have been assumed in all three directions. For the simulations a low density beam ($n_b/n_c = 0.04$, where n_b denotes the beam density and n_c denotes the core density) has been used, which flows with a drift velocity of $u_b = 6v_A$ relative to a background plasma. We note that Hellinger and Mangeney (1999) have used in their 2-D simulation the same beam density but a somewhat higher beam velocity of $10v_A$. Both the core plasma and the beam are Maxwellians. The simulation is performed in the electron rest frame. For each species we used 24 macro-ions per cell. The initial magnetic field as well as the bulk velocity of the beam ions was oriented in the y-direction. During the simulation we have followed the trajectories of 1.3×10^6 beam ions and have determined their mean square displacement parallel and perpendicular to the average magnetic field. The diffusion tensor is defined in terms of the Fokker-Planck coefficients given by

$$\kappa_{ij} = \frac{< \Delta r_i \Delta r_j >}{2\Delta t}$$

where $\Delta \mathbf{r}$ is the displacement of a particle from its original position after a time Δt. The angle brackets denote ensemble averages, which can also be viewed as averages over many particle trajectories during one run:

$$< \Delta r_i \Delta r_j > = \frac{1}{N} \sum_{k=1}^{N} \Delta r_i \Delta r_j ,$$

where N is the number of particles used in the average. The diffusion coefficient κ_{xx} across the ambient mag-

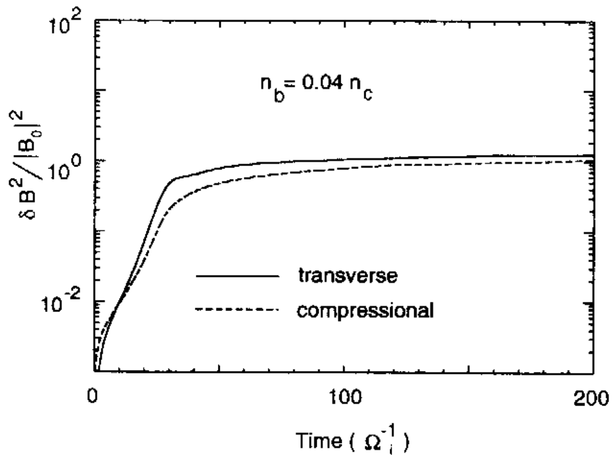

Fig. 1. Energy density $(\delta B/B_0)^2$ of the magnetic field fluctuations as a function of time.

netic field should be directly comparable with $\kappa_\perp$, while κ_{yy} is the parallel diffusion coefficient.

4 Simulation Results

Figure 1 shows the time history of the average fluctuating magnetic field energy densities $W_y = (\delta B_y/B_0)^2$ (dashed line) and $W_x = (\delta B_x/B_0)^2$ (full line) in the compressional and transverse magnetic field component in a logarithmic versus linear representation. There is a phase of exponential growth around $t = 20\Omega_i^{-1}$ with a growth rate of $\gamma = 0.23\Omega_i$. This compares favorably well with the linear growth rate for parallel propagation. Both modes, a compressional and a transverse wave mode are excited. The power in both modes is of the order of one relative to the background magnetic field energy density. The transverse mode has a slightly higher maximum power at the end of the simulation than the compressional mode.

Figure 2 shows the magnetic field fluctuations at the end of the simulation at $t = 200\Omega_i^{-1}$. On the left-hand side we present the fluctuation $\delta B_x(x, y, z_0) = B_x(x, y, z_0) - B_{x0}(x, y, z_0)$ of the transverse magnetic field component in a gray scale format in the x-y plane for a fixed value of spatial component $z = z_0$. Here $B_{x0}(x, y, z_0)$ indicates the initial magnetic field component. Black contours corresponds to the minimum value and white corresponds to the maximum value. The range between the minimum and the maximum value is linearly divided into ten steps with different gray scale. The transverse component exhibits long wavelength waves propagating parallel to the magnetic field. The left-hand side shows a similar gray scale plot of the fluctuations of the parallel magnetic field component, $\delta B_y = B_y(x, y, z_0) - B_{y0}(x, y, z_0)$. These waves seem to be of smaller wavelengths and no uniform propagation direction is immediately discernible.

More detailed information of the wave propagation

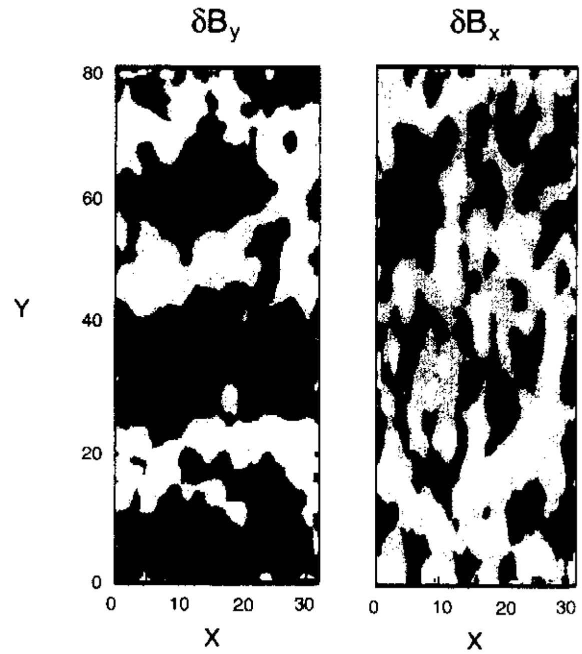

Fig. 2. Gray scale plots of the magnetic field fluctuations parallel (δB_y) (left-hand panel) and perpendicular (δB_x) (right-hand panel) to the average magnetic field direction at the end of the simulation.

direction can be obtained by a two-dimensional Fourier analysis of the transverse and parallel component of the magnetic field, respectively. The result is summarized in Figure 3, where we show the power in gray-scaled representation in the $k_x - k_y$ plane.

From the distribution of the power of the B_x component on the left-hand side it can be seen that the transverse waves propagate dominantly parallel to the magnetic field with a wavenumber of $\sim 0.2\lambda^{-1}$. This correspond to a parallel wavelength of about 30 ion inertial lengths. Inspection of the distribution of the power in the B_y component (right-hand side) exhibits smaller wavelength waves propagating at oblique angles. The results of the wave properties of the magnetic field fluctuations are similar to those reported by Hellinger and Mangeney (1999).

We now turn to the behavior of the beam ions in the 3-D fluctuating field. In Figure 4 we show the distributions of a randomly chosen subset of 10000 particles at different times during the simulation. The dashed histogram shows the initial distribution with its maximum value at the initial beam bulk velocity of $v = 6v_A$. Also shown are the distribution of the same subset of particles at $t\Omega_i = 100$ (indicated by diamonds). At this time the system is beyond exponential growth phase of the instability. The maximum of the distribution is shifted to a somewhat lower bulk speed of $|v| \sim 5v_A$, which is due to the conversion of kinetic energy of the beam ions to wave energy. The histogram (full line) shows the dis-

Fir

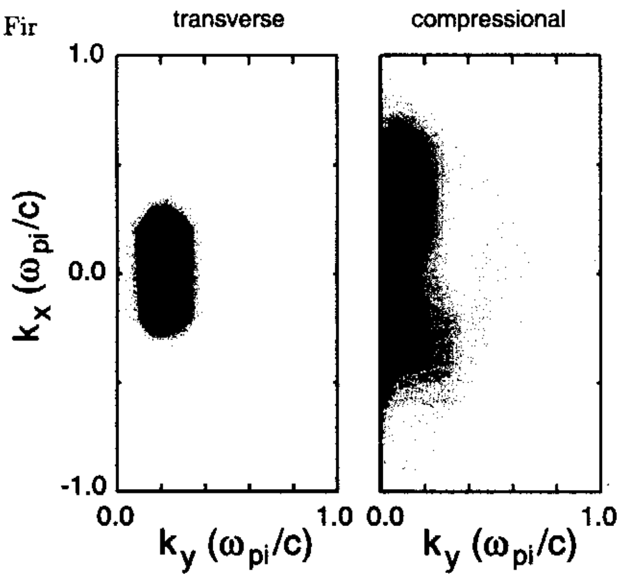

Fig. 3. Gray scale plot of the power of the transverse and parallel component of the magnetic field in the k_x-k_y plane at $t = 200\Omega_i^{-1}$. The heavy black lines indicate the half width of the distribution.

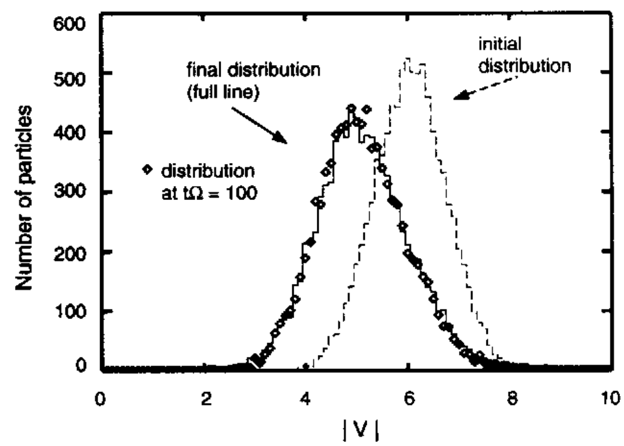

Fig. 4. Distribution of a subset of 10000 randomly chosen beam ions at three different times during the simulation. The dashed line indicates the initial distribution, whereas the full line and diamonds represent the distribution at $t = 200\Omega_i^{-1}$ and $t = 100\Omega_i^{-1}$, respectively.

tribution at the end of the simulation ($t\Omega_i = 200$). A comparison of the distribution at $t\Omega_i = 100$ with the final one shows no significant difference. This demonstrates that when cross-field diffusion sets in and the variances exhibit a linear dependence on time the beam ions are not heated. Thus, on average, an increase of the variance in the perpendicular direction due to increase of the average gyro-radius can be excluded.

Figure 5 shows the orbit of a beam ion between $\Omega_i t = 150$ and $\Omega_i t = 200$ projected onto the plane perpendicular (top) and onto the $x - y$ plane parallel (bottom) to the average magnetic field. The starting and the end point of the trajectory is indicated by an asterisk and a dot, respectively. As can be seen from the projection of the ion trajectory on the plane perpendicular to the magnetic field the ion moves sometimes a considerable distance perpendicular to the average magnetic field without performing a complete gyration.

Figure 6 shows the mean square displacement of a particle ensemble perpendicular and parallel to the average magnetic field versus time. The mean square displacement in the parallel direction, $\langle(\Delta y)^2\rangle$, increases during the growth phase of the instability according to t^2, while the mean square displacement perpendicular to the magnetic field (top panel) only exhibits the gyromotion (rather small on this scale). The t^2 dependence of $\langle(\Delta y)^2\rangle$ simply represents the motion of the beam relative to the core ions. In the nonlinear phase both $\langle(\Delta y)^2\rangle$ and $\langle(\Delta x)^2\rangle$ become a linear function of time.

The linear dependence of the mean square displacements on time can be used as an indication for a diffusive process and the gyro-centers of the particles actually shift field lines. In the case that particles remain strictly tied to the field lines which themselves are randomly walking in space the mean square displacement perpen-

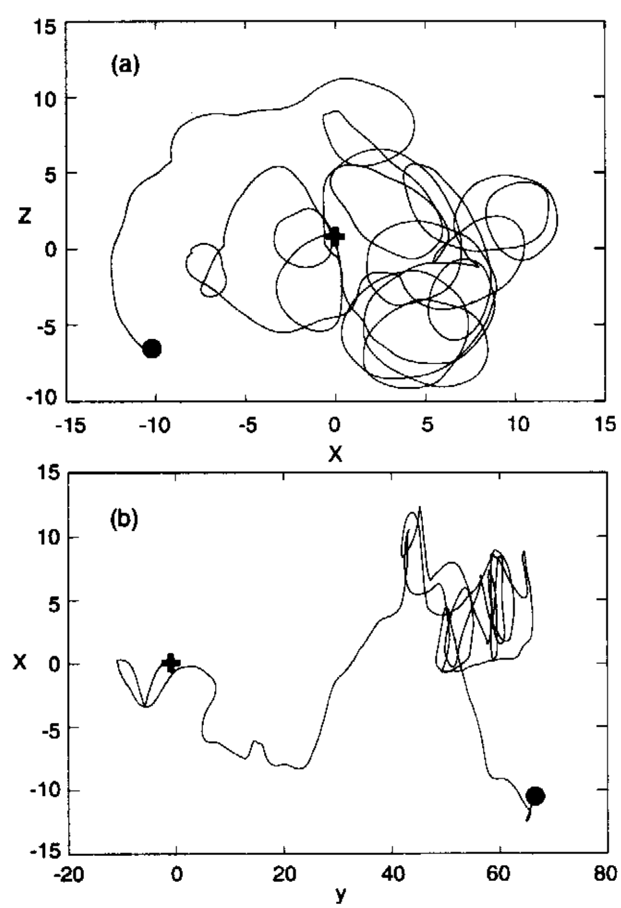

Fig. 5. Projection of the trajectory of a beam ion onto a plane perpendicular (a) and parallel (b) to the average magnetic field direction during a time period of $\Delta t = 50\Omega_i^{-1}$ starting at $t = 150\Omega_i^{-1}$. The asterisks and the dots indicate the starting and the end points, respectively.

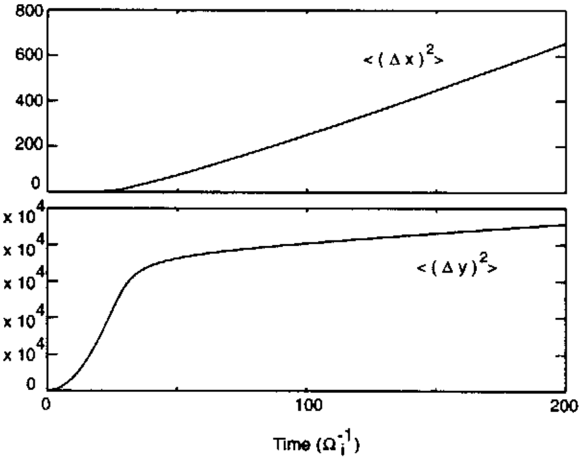

Fig. 6. The mean square distance of the ions relative to their original position at $\Omega_i t = 0$ as a function of time. Top (bottom) panel shows the mean square distance of an ion ensemble perpendicular (parallel) to the average magnetic field.

dicular to the magnetic field increases as $\langle(\Delta x)^2\rangle \propto t^{1/2}$ in contrast to the $\langle(\Delta x)^2\rangle \propto t$ dependence of a diffusive process. From the slopes of the curves in the upper panels one can derive diffusion coefficients $\kappa_{||} = \kappa_{yy} \sim 25\lambda_o^2\Omega_i$ and $\kappa_\perp = \kappa_{xx} \sim 2.38\lambda_o^2\Omega_i$. In terms of the beam velocity u_b the perpendicular diffusion coefficient can be written as $\kappa_\perp = 0.064u_b^2/\Omega_i$. In units of the gyroradius of the beam ions, $r_g = M_A v_A/\Omega_i = M_A\lambda_o$, these coefficients can be expressed in units of gyroradius square times gyro-frequency: $\kappa_{||} \sim 0.71r_g^2\Omega_i$ and $\kappa_\perp \sim 0.064r_g^2\Omega_i$ leading to a ratio of $\kappa_\perp/\kappa_{||} = 0.09$. The parallel mean free path is given by $\lambda_{||} = 3\kappa_{||}/(r_g\Omega_i)$. From the simulation data we obtain a parallel mean free path $\lambda_{||} \sim 2.14r_g$. Using the idealized model for the cross-field diffusion which is based on hard sphere scattering one obtains

$$\frac{\kappa_\perp}{\kappa_{||}} = \frac{1}{1 + (\lambda_{||}/r_g)^2}$$

for the ratio of the perpendicular and the parallel diffusion coefficient (Parker, 1665; Chapman and Cowling, 1970). Inserting the value for the mean free path obtained from the present simulation into the above equation we obtain $\kappa_\perp/\kappa_{||} \sim 0.18$, which is in good agreement with the ratio of the two diffusion coefficients obtained from the present simulation.

The parallel scattering time is of the order of $\tau = \lambda_{||}/v \sim 3\Omega_i^{-1}$ and is short in comparison with the nonlinear growth time of the beam instability ($\sim 30\Omega_i^{-1}$). Thus, the nonlinear growth time is expected to determine the spatial extent of a beam emitted from the bow shock along the magnetic field. Parallel beams of specularly reflected ions could therefore be expected up to a distance of $\sim 30M_A$ ion inertial length upstream of the shock, where M_A is the shock Alfvenic Mach number. In the case presented here the upstream distance would be about 200 ion inertial lengths. At quasi-parallel bow shocks the reflected ions are free to escape upstream where they can interact directly with the incoming flow. Specularly reflected ion distributions that fit this description have been observed at large distances (i.e. more than several ion gyro-radii) from the Earth's bow shock (Gosling et al., 1982). Lee (1982) has proposed that cross-field diffusion to the flanks of the bow shock is a possible process leading to exponential spectra of upstream diffuse ions. The extend of the lateral spatial scale of diffuse upstream events has been determined by Wibberenz et al. (1985) from radial gradients to $A \sim 8R_E$, consistent with the prediction of the e-folding energy obtained with the theory of Lee (1982). Using the theory of Eichler (1981) this value of A results in a ratio $\kappa_\perp/\kappa_{||} \sim 0.7$ which is considerably larger than the value obtained by the present simulation. From our value of $\kappa_\perp \sim 2.38\lambda_o^2\Omega_i$ we have determined a cross-field diffusion time of several hours. This small perpendicular diffusion is consistent with the large lateral density gradients observed by Wibberenz et al. (1985) but is inconsistent with the theory of Lee.

5 Summary

A self-consistent three-dimensional hybrid simulation of an ion/ion beam instability has been performed in order to determine the diffusion coefficient parallel and perpendicular to the average magnetic field. The results obtained from the simulation can be summarized as follows.

In the self-consistent three-dimensional magnetic field turbulence created by an ion/ion beam instability the beam ions diffuse parallel as well as perpendicular to the magnetic field. The transport of ions across magnetic field lines is a consequence of the interaction of charged particles with obliquely propagating waves. The ratio of the perpendicular to the parallel diffusion coefficient is by a factor of two smaller than the ratio predicted by a hard sphere scattering theory. The parallel scattering time is short in comparison with the time for the instability to reach the nonlinear state. Thus the latter time will determine the extent of the beams in front of the bow shock. Hellinger and Mangeney (1999) obtained a filamentary structure with dimensions of the order of 10 ion inertial lengths. From the cross-field diffusion coefficient we obtain for such structures a characteristic diffusion time of $50\Omega_i^{-1}$.

In terms of the beam velocity u_b the perpendicular diffusion coefficient can be written as $\kappa_\perp = 0.064u_b^2/\Omega_i$. This is almost an order of magnitude larger than the value derived by Giacalone et al. (1994) for scattering in static 3-D magnetic field turbulence with a Kolmogorov type distribution and similar total power in the fluctuations. Helllinger and Mangeney (1999) have shown that the beam ions interact resonantly with the oblique modes. This suggests that resonant interaction can considerably enhance cross-field diffusion.

This small value for perpendicular diffusion is consistent with the large lateral density gradients observed by Wibberenz et al. (1984) but is inconsistent with the theory by Lee (1982) who has proposed cross-field diffusion to the flanks of the bow shock as a possible loss process leading to exponential spectra of upstream events.

Acknowledgements. M. Scholer would like to thank Y. Kamide for his kind hospitality at the Solar Terrestrial Environment Laboratory.

References

Akimoto, K., Winske, D., Gary, S. P., and Thomsen M. F., Nonlinear evolution of electromagnetic ion beam instabilities, *J. Geophys. Res.*, *98*, 1419, 1993.

Eichler, D., Energetic particle spectra in finit shocks: The Earth's bow shock, *Astrophys. J.*, *244*, 711, 1981.

Forman, M. A., Jokipii, J. R. and Owens, A. J., Cosmic-ray streaming perpendicular to the mean magnetic field *Astrophys. J.*, *192*, 535, 1974.

Chapman, S., and Cowling, T. G., *The mathematical theory of non-uniform gasses*, Cambridge University Press, 1970.

Gary, S. P., Electromagnetic ion/ion beam instabilities and their consequences in space plasmas: A review, *Space Science Rev.*, *56*, 373, 1991.

Giacalone, J., and Jokipii, J. R., Charged particle motion in multi-dimensional magnetic-field turbulence, *Astrophys. J.*, *430*, L137, 1994.

Goldstein, M. L., Klimas, A. J., Sandri, G., Mirroring in the Fokker-Planck coefficient for cosmic-ray pitch-angle scattering in homogeneous magnetic turbulence, *Astrophys. J.*, *195*, 787, 1975.

Gosling, J. T., Thomsen, M. F., Bame, S. J., Feldman, W. C. Paschmann, G., Sckopke, N., Evidence for specularly reflected ions upstream from the quasi-parallel bow shock, *J. Geophys. Res.*, *9*, 1333, 1982.

Hasselmann, K. G., and Wibberenz G., Scattering of charged particles by random electromagnetic fields, *Z. Geophys.*, *34*, 353, 1968.

Hellinger, P., and Mangeney A., Electromagnetic ion beam instabilities: oblique pulsations, *J. Geophys. Res.*, *104*, 4669, 1999.

Jokipii, J. R., Cosmic-ray Propagation I. Charged particles in a random magnetic field, *Astrophys. J.*, *146*, 480, 1966.

Jokipii, J. R., Kóta, J. and Giacalone, J., Perpendicular transport in 1- and 2-dimensional simulations, *Geophys. Res. Lett.*,*20*, 1759, 1993.

Jones, F. C., Jokipii, J. R. and Baring M. G., Charged particle motion in electromagnetic fields having at least one ignorable spatial coordinate, *Astrophys. J.*, *509*, 238, 1998.

Lee, M. A., Coupled hydromagnetic wave excitation and ion acceleration upstream of the Earth's bow shock, *J. Geophys. Res.*, *87*, 5063, 1982.

Luhmann, J. G., A quasi-linear kinetic equation for cosmic rays in the interplanetary medium, *J. Geophys. Res.*, *81*, 2089, 1976.

Matthews, A. P., Current advanced method and cyclic leabfrog for 2-D multispecies hybrid plasma simulation, *J. Comp. Phys.*, *112*, 102, 1994.

Parker, E. N., The passage of energetic charged particles through interplanetary space, *Planet. Space Sci.*, *13*, 9, 1965.

Wibberenz, G., Zöllich, F., Fischer, H. M., and Keppler, E., Dynamics of intense upstream ion events, *J. Geophys. Res.*, *90*, 283, 1985.

Winske, D. and Quest K. B., Electron ion beam instabilities: Comparison of one- and two- dimensional simulations, *Geophys. Res. Lett.*, *91*, 8789, 1986.