Nonlin. Processes Geophys., 31, 1–6, 2024
https://doi.org/10.5194/npg-31-1-2024

Nonlinear Processes in Geophysics
Open Access
EGU

Brief Communication: A modified Korteweg–de Vries equation for Rossby–Khantadze waves in a sheared zonal flow of the ionospheric E layer

Laila Zafar Kahlon[1], Hassan Amir Shah[1], Tamaz David Kaladze[2,3], Qura Tul Ain[1], and Syed Assad Bukhari[1]

[1]Physics Department, Forman Christian College (a Chartered University), Lahore 54600, Pakistan
[2]I. Vekua Institute of Applied Mathematics, Tbilisi State University, 2 University Street, Tbilisi 0186, Georgia
[3]E. Andronikashvili Institute of Physics, I. Javakhishvili Tbilisi State University, Tbilisi 0128, Georgia

Correspondence: Laila Zafar Kahlon (lailakahlon@fccollege.edu.pk)

Received: 17 May 2023 – Discussion started: 1 June 2023
Revised: 17 November 2023 – Accepted: 19 November 2023 – Published: 9 January 2024

Abstract. The system of non-linear equations for electromagnetic Rossby–Khantadze waves in a weakly ionized conductive ionospheric E-layer plasma with sheared zonal flow is given. Use of multiple-scale analysis allows reduction of an obtained set of equations to a $(1+1)D$ non-linear modified KdV (mKdV) equation with cubic non-linearity describing the propagation of solitary Rossby–Khantadze solitons.

1 Introduction

Different satellite and ground-based investigations indicate the presence of zonal flows in various atmospheric regions around the Earth (Pedlosky, 1987). The reason for the existence of zonal flows is the non-uniform warming of the Earth's atmospheric regions by the Sun. The presence of sheared flow along the meridians with inhomogeneous velocity is closely connected with the ultra-low-frequency perturbations in the ionospheric E and F regions of the ionosphere (Satoh, 2004; Shukla and Stenflo, 2003; Onishchenko et al., 2004; Kaladze et al., 2007, 2008). Effects of sheared flow appear in linear and non-linear properties of the waves, and conditions suitable for that are available in the Earth's ionosphere. This gives rise to a variety of non-linear phenomena like the formation of solitary structures (solitons, vortices, zonal flows, etc.).

Due to a significant role in the global atmospheric circulation, Rossby waves attract special scientific attention in connection with sheared zonal flows. Note that the spatial non-homogeneity of the Coriolis parameter along with the ambient geomagnetic field along the meridians causes the propagation of such coupled Rossby–Khantadze (RK) electromagnetic (EM) waves (see e.g. Kaladze et al., 2011). The generation of sheared RK EM planetary vortices in the ionospheric E region is also discussed (Kaladze et al., 2011, 2014). It was revealed that the propagation of coupled EM RK waves could be self-organized into solitary dipolar vortices, and the possibility of the generation of an intensive magnetic field is shown. In recent decades, several non-linear phenomena related to the excitation of sheared zonal flows by EM Rossby waves have been investigated. Taking into account Reynolds stresses, zonal flow generations by short-wavelength EM Rossby waves were studied (Shukla and Stenflo, 2003; Onishchenko et al., 2004). The zonal flow generation in the ionospheric E layer by Rossby waves was revealed by Kaladze et al. (2007). Such non-linear Rossby wave structures broken into numerous parts depend on the zonal flow energy (Kaladze et al., 2008). Numerical work on EM RK waves with sheared zonal flow in ionospheric E plasma was found as well (Futatani et al., 2013, 2015). In this work, the splitting of vortices, where the energy is transported by sheared flow into multiple pieces, was pointed out. Equatorial Rossby wave solitons under the action of sheared flows were also discussed (Qiang et al., 2001), and the existence of the solitons was confirmed by the observations of the Freja and Viking satellites (Qiang et al., 2001; Bostrom, 1992; Dovner et al., 1994; Lindqvist et al., 1994). Jian et al. (2009) investigated non-linear propagation of Rossby waves in stratified neutral fluids with zonal shear flow and obtained a modified Korteweg–de Vries (mKdV) equation with cubic non-linearity. Generation of the zonal flow along with the magnetic field in the ionospheric E plasma by Rossby–Khantadze EM planetary waves was also discussed (Kaladze et al., 2012; Kahlon and Kaladze, 2015). The possibility of a magnetic field generation of 10^3 nT is predicted. Kaladze et al. (2019) investigated the non-linear interaction of magnetized electrostatic Rossby waves with sheared zonal flows in the Earth's ionospheric E layer and developed the modified Korteweg–de Vries (mKdV) equation with cubic non-linearity describing the propagation of appropriate solitons. Some premises of the possibility of the existence of planetary Rossby waves in the dynamo E area of the weakly ionized ionosphere and corresponding experimental interpretation were discussed by Forbes (1996). Also, Vukcevic and Popovic (2020) pointed out the possibility of many soliton structure formations at different latitudes and in diverse ionospheric layers. Direct observations of such soliton structures from the surface of the Earth or on board the satellites are discussed.

In the given paper, we generalize the abovementioned results for the weakly ionized conducting ionospheric E-region plasma by incorporating the streamfunction evolution of the geomagnetic field for electromagnetic RK waves, which to the best of our knowledge was not reported so far and thus provides novelty to this work. In Sect. 2, from the obtained system of non-linear two-dimensional equations by using the multiple-scale analysis and the perturbation approach, we derive a one-dimensional mKdV equation with cubic non-linearity describing solitary Rossby–Khantadze wave dynamics along with zonal (shear) flows. Section 3 includes the discussion of the results.

2 Mathematical preliminaries

We consider the partially ionized E-ionospheric region consisting of a small concentration of electrons, ions, and the bulk of the neutral particles, where such ionospheric plasma is enclosed in a spatially inhomogeneous geomagnetic field $\mathbf{B}_0 = \left(0,\ B_{0y},\ B_{0z}\right)$ and the Earth's angular velocity is $\boldsymbol{\Omega} = \left(0, \Omega_{0y},\ \Omega_{0z}\right)$. In the weakly ionized ionospheric E-layer plasma, we consider the two-dimensional wave motion $\mathbf{v} = (u\mathrm{v}, 0)$, where $u = -\frac{\partial \psi}{\partial y}$, $\mathrm{v} = \frac{\partial \psi}{\partial x}$ and $\psi(x,\ y,\ t)$ is the streamfunction.

We consider a local Cartesian system of coordinates with zonal x, latitudinal y, and z in the local vertical direction. Then, the non-linear behaviour of the sheared electromagnetic Rossby–Khantadze waves can be described by the following two-dimensional system of equations (e.g. Kaladze et al., 2014):

$$\left\{\begin{array}{l} \frac{\partial \Delta\psi}{\partial t} + \beta\frac{\partial \psi}{\partial x} + \mathrm{J}\left(\psi,\ \Delta\psi\right) - \frac{1}{\mu_0 \rho}\beta_{\mathrm{B}}\frac{\partial h}{\partial x} = 0, \\ \frac{\partial h}{\partial t} + \mathrm{J}\left(\psi,\ h\right) + \beta_{\mathrm{B}}\frac{\partial \psi}{\partial x} + c_{\mathrm{B}}\frac{\partial h}{\partial x} = 0. \end{array}\right. \tag{1}$$

The first equation describes the evolution of the z component of the vorticity ($\zeta_z = \mathbf{e}_z \cdot \nabla \times \mathbf{v} = \Delta\psi$) of the single fluid momentum equation under the action of the geomagnetic field, and v is the velocity of the incompressible neutral gas. The second equation is the z component of the perturbed magnetic induction h obtained through Faraday's law, and $\beta = \frac{\partial f}{\partial y} = \frac{2\partial \Omega_{0z}}{\partial y}$ describes the latitudinal inhomogeneity of the angular velocity. Also, the parameter $c_{\mathrm{B}} = \beta_{\mathrm{B}}/en\mu_0$ with $\beta_{\mathrm{B}} = \frac{\partial \mathrm{B}_{0z}}{\partial y}$ describes the latitudinal inhomogeneity in the background magnetic field, n is the number density of the charged particles, μ_0 is the magnetic permeability, $J\left(a, b\right) = \frac{\partial a}{\partial x}\frac{\partial b}{\partial y} - \frac{\partial a}{\partial y}\frac{\partial b}{\partial x}$ is the Jacobian (responsible for the vector non-linearity), and $\Delta = \partial_x^2 + \partial_y^2$. Note that the small concentration of charged particles (compared to the neutral particles) only gives the contribution in the inductive current (Kaladze et al., 2013a, b). It should also be noted that the ambient magnetic field and the Coriolis parameter are spatially inhomogeneous (Kaladze et al., 2014). Details on system (1) can be found in Kaladze et al. (2012).

The boundary conditions that are fulfilled for this system are given as

$$\psi\left(0\right) = \psi\left(1\right) = 0, \tag{2}$$

which represents the flow's edges, specifically in the southern and northern directions (Pedlosky, 1987; Satoh, 2004).

Perturbation and weakly non-linear approach

The background streamfunction is considered in the following manner:

$$\Psi\left(y\right) = -\int\left[U\left(y\right) - c_0\right]\mathrm{d}y. \tag{3}$$

Here, $U(y)$ describes the basic background flow with c_0 as a constant eigenvalue. The whole streamfunction ψ is considered the sum of the background (zonal flow) streamfunction $\Psi(y)$ and a disturbed ψ' streamfunction. This assumption makes it a weakly non-linear system that is the subject of this study. While the perturbed magnetic field is also characterized by a small parameter ε. Therefore, the streamfunction and the magnetic perturbations take the forms

$$\begin{aligned} &\psi = \Psi\left(y\right) + \varepsilon\psi' = -\int\left[U\left(y\right) - c_0\right]\mathrm{d}y + \varepsilon\psi', \\ &h = \varepsilon h', \end{aligned} \tag{4}$$

where $\varepsilon \ll 1$ is a small parameter indicating that the perturbed quantities are small compared to the background parameters.

Inserting Eq. (4) into Eq. (1) gives the following.

$$\left\{\begin{array}{l} \frac{\partial \Delta\psi'}{\partial t} + \left(U\left(y\right) - c_0\right)\frac{\partial \Delta\psi'}{\partial x} + \left(\beta - U''\right)\frac{\partial \psi'}{\partial x} \\ + \frac{\beta_{\mathrm{B}}}{\mu_0 \rho}\frac{\partial h'}{\partial x} + \varepsilon J\left(\psi',\ \Delta\psi'\right) = 0 \\ \frac{\partial h'}{\partial t} + \varepsilon\mathrm{J}\left(\psi',\ h'\right) + \left(U\left(y\right) - c_0\right)\frac{\partial h'}{\partial x} + \beta_{\mathrm{B}}\frac{\partial \psi'}{\partial x} \\ + c_{\mathrm{B}}\frac{\partial h'}{\partial x} = 0 \end{array}\right. \tag{5}$$

$U'' = \frac{d^2U}{dy^2}$.

By using the multiple-scale analysis, we obtain the asymptotic solution where we take the spatial and temporal parameters as $X = \varepsilon x$ and time $T = \varepsilon^3 t$ respectively. Further, by eliminating h' from 5(b) into 5(a), we get the single equation for ψ':

$$L_0(\psi) + \varepsilon^2 L_1(\psi) + \varepsilon J\left(\psi, \frac{\partial^2\psi}{\partial y^2}\right) + \varepsilon^3 J\left(\psi, \frac{\partial^2\psi}{\partial X^2}\right) + \varepsilon^4 \frac{\partial^3\psi}{\partial T \partial X^2} = 0. \quad (6)$$

In Eq. (6), the prime on the perturbed streamfunction is dropped, and the following linear differential operators are introduced:

$$L_0 = \left[(U - c_0)\frac{\partial^2}{\partial y^2} + p(y) + \frac{\alpha(y)}{U - c_0 + c_\mathrm{B}}\right]\frac{\partial}{\partial x},$$
$$L_1 = \frac{\partial}{\partial T}\frac{\partial^2}{\partial y^2} + (U - c_0)\frac{\partial^3}{\partial X^3}, \quad (7)$$

where $\alpha(y) = \frac{\beta_\mathrm{B}^2}{\mu_0 \rho}$ and $p(y) = \beta - U''$. Here the parameter α takes into account the spatial inhomogeneity of the background magnetic field which was not considered before in Kaladze et al. (2019).

Furthermore, we expand the streamfunction ψ (in series with respect to the ε) as

$$\psi = \psi_0 + \varepsilon \psi_1 + \varepsilon^2 \psi_2 + \ldots. \quad (8)$$

By inserting Eq. (8) into Eq. (6), we obtain from the lowest-order $O(\varepsilon^0)$ the following equation:

$$L_0[\psi_0] = 0, \quad \text{with} \quad \psi_0 = 0 \text{ for } y = 0, 1. \quad (9)$$

The above Eq. (9) is a linear differential equation. By performing a separation of the variable method for $\psi_0 = A(X, T)\,\Phi_0(y)$ in this form and substituting it into Eq. (7), we get the following equation with the boundary conditions:

$$\left(\frac{d^2}{dy^2} + \frac{p(y)}{U - c_0} + \frac{\alpha(y)}{(U - c_0)(U - c_0 + c_\mathrm{B})}\right)\Phi_0 = 0,$$
$$\text{with } \Phi_0(0) = \Phi_0(1) = 0. \quad (10)$$

Here we consider $U - c_0 \neq 0$ and $U - c_0 + c_\mathrm{B} \neq 0$. This is an eigenvalue problem for eigenvalue c_0. By specifying $p(y)$ and $\alpha(y)$, $\Phi_0(y)$ can be found. Since $p(y)$ and $\alpha(y)$ are dependent on the variable y, it is not easy to solve this eigenvalue problem analytically. From the lowest-order $r\,O(\varepsilon^0)$, we see that the problem is time-independent but cannot be solved analytically as we have not substituted any definite dependence on y for the parameters $p(y)$ and $\alpha(y)$. Thus, in order to get more details about the amplitude of these waves, we go to the next order. In other words, with O (ε^1) from Eqs. (7) and (8), we obtain

$$L_0[\psi_1] = -J\left(\psi_0, \frac{\partial^2\psi_0}{\partial y^2}\right) \equiv F_1$$
$$= A\frac{\partial A}{\partial X}\left(\frac{p(y)}{U - c_0} + \frac{\alpha}{(U - c_0)(U - c_0 + c_\mathrm{B})}\right)_y \Phi_0^2. \quad (11)$$

Furthermore, we carry out a separation of variables in the following manner, i.e. $\psi_1 = \frac{1}{2}A^2(X, T)\,\Phi_1(y)$ for non-singular neutral solutions in Eq. (11).

$$\left(\frac{d^2}{dy^2} + \frac{p(y)}{U - c_0} + \frac{\alpha}{(U - c_0)(U - c_0 + c_\mathrm{B})}\right)\Phi_1$$
$$= \left(\frac{p(y)}{U - c_0} + \frac{\alpha}{(U - c_0)(U - c_0 + c_\mathrm{B})}\right)_y \frac{\Phi_0^2}{(U - c_0)} \quad (12)$$

The given boundary conditions are $\Phi_1(0) = \Phi_1(1) = 0$. To get the amplitude, we solve Eqs. (7) and (8) in the next order, i.e. $O(\varepsilon^2)$, which gives

$$L_0[\psi_2] = -L_1[\psi_0] - \mathrm{J}\left(\psi_0, \frac{\partial^2\psi_1}{\partial y^2}\right) - \mathrm{J}\left(\psi_1, \frac{\partial^2\psi_0}{\partial y^2}\right) \equiv F_2, \quad (13)$$

with $\psi_2(0) = \psi_2(1) = 0$.

Here, it is pointed out that the dispersion effect, given in the definition of L_1, competes with the weakly non-linear effect, which appears through the Jacobian in Eq. (9).

Furthermore, we again perform a separation of variables, $\psi_2 = B(X, T)\,\Phi_2(y)$, and multiply Eq. (13) by ψ_0 and integrate over y, which yields

$$\int_0^1 dy \frac{F_2}{U - c_0}\Phi_0 = 0. \quad (14)$$

By substituting F_2 and using $\psi_1 = \frac{1}{2}A^2(X, T)\,\Phi_1(y)$ into Eq. (14), we get the modified KdV (mKdV) equation (Kaladze et al., 2019):

$$\frac{\partial A}{\partial T} + N A^2 \frac{\partial A}{\partial X} + D \frac{\partial^3 A}{\partial X^3} = 0. \quad (15)$$

This equation has a cubic non-linearity, whereas the standard KdV equation has a quadratic non-linearity.

In Eq. (5) above,

$$N = \frac{I_2}{I_0}, \qquad D = -\frac{I_1}{I_0}, \quad (16)$$

where

$$\begin{cases} I_0 = \int_0^1 \mathrm{d}y\, \Phi_0^2(y) \left[\frac{p(y)}{(U(y)-c_0)^2} + \frac{\alpha}{(U(y)-c_0)^2 (U(y)-c_0+c_\mathrm{B})} \right], \\ I_1 = \int_0^1 \mathrm{d}y\, \Phi_0^2(y), \\ I_2 = \int_0^1 \mathrm{d}y\, \frac{\Phi_0^2(y)}{U(y)-c_0}, \\ \left\{ \begin{array}{l} \frac{3}{2} \left(\frac{p(y)}{U(y)-c_0} + \frac{\alpha}{(U(y)-c_0)(U(y)-c_0+c_\mathrm{B})} \right)_y \Phi_1(y) \\ -\frac{1}{2} \Phi_0^2(y) \left[\left(\frac{p(y)}{U(y)-c_0} + \frac{\alpha}{(U(y)-c_0)(U(y)-c_0+c_\mathrm{B})} \right)_y \frac{1}{U(y)-c_0} \right]_y \end{array} \right\}, \end{cases} \tag{17}$$

Kaladze et al. (2019) and Jian et al. (2009) also obtained the same mKdV Eq. (15) with cubic non-linearity for Rossby waves and pointed out that the background flow shear is a necessary condition for the existence of solitary waves, whereas in this work, we get the mKdV for the Rossby–Khantadze waves where the coefficients have been modified by the inclusion of inhomogeneity in the geomagnetic field. Moreover, the effects of the shear basic flow on the spatial structure, propagation velocity, and wave width of solitary Rossby waves have been studied. We would like to point out here the meridional dependence of functions $\beta(y)$, $\alpha(y)$, and $U(y)$ that appears in the coefficients N and D.

Amid numerous exact solutions of mKdV Eq. (15) (see e.g. Wazwaz, 2009), we are interested in a soliton-like traveling wave solution. The one-soliton solution of Eq. (15) is

$$A(X,T) = \pm\sqrt{\frac{6c}{N}}\,\mathrm{sech}\left(\sqrt{\frac{c}{D}}(X-cT)\right), \tag{18}$$

where c is the traveling wave velocity and the coefficients N and D are defined by Eqs. (16)–(17). In order for a wave to have an exact solitary solution associated with it, one needs to a robust equation like KdV. A modified KdV, too, has infinite conservation laws associated with it and hence is integrable and contains one- and N-soliton solutions. Shown in the above equation is the one-soliton solution of the mKdV. One can use Hirota's method, where by using a suitable transformation, one converts the non-linear equation into a bilinear equation, and then by using Hirota's differential operator and solving the subsequent equation, one can obtain a multi-soliton solution. Some types of mKdV spatially periodic solutions (cnoidal solutions) are discussed (Kevrekidis et al., 2004). It was noted that the mKdV equation with a non-linear term may have an alternate sign. Properties of such differences are also discussed.

3 Discussion

In the present paper, we have studied the non-linear dynamics of large-scale electromagnetic Rossby–Khantadze waves with zonal flows in E-ionospheric plasma. Both the latitudinal inhomogeneities in the angular velocity of the Earth's rotation and the geomagnetic field are taken into account. The latitudinal inhomogeneity of the magnetic field is responsible for coupled Rossby–Khantadze waves. Such coupling results in an appearance of the dispersion of Khantadze waves. To derive the non-linear modified KdV value, we used the multiple-scale analysis technique. From the lowest order of $O(\varepsilon^0)$, we get an eigenvalue problem with a constant eigenvalue c_0 along with the boundary conditions. The parameters $p(y)$ and $\alpha(y)$ are dependent on the variable y, making it impossible to solve this eigenvalue problem analytically. From the next order $O(\varepsilon^1)$, by using separation of variable techniques and after doing some mathematical manipulations, we arrive at the mKdV in Eq. (15) with cubic non-linearity of the $(1+1)$ dimension. The traveling wave solitary solution of this equation is given by Eq. (18), where the parameter $\sqrt{\frac{6c}{N}}$ describes the amplitude of solitary RK structures. The obtained coefficients N and D depend on the spatially inhomogeneous Coriolis force $\alpha(y)$ and the background magnetic field $\beta(y)$ respectively.

In anticipation of the future for the experimental observations of RK vortical motions in the weakly ionized ionospheric E layer, we expect the following characteristics. Apart from the ordinary Rossby waves, electromagnetic RK perturbations generated by the latitudinal gradient of the geomagnetic field represent the variation of the vertical electric field $\mathbf{E}_v = \mathbf{v}_D \times \mathbf{B}_0$, where $\mathbf{v}_D = \mathbf{E} \times \mathbf{B}_0 / B_0^2$ is the electron drift velocity. RK waves propagate along the latitude with a velocity of $|c_\mathrm{B}| \approx 2$–$20\,\mathrm{km\,s^{-1}}$. Frequency ($\omega = k_x c_\mathrm{B}$) and phase velocity c_B depend on the number density of the charged particles and vary by 1 order of magnitude under the daytime and nighttime conditions (which is so suitable for experimental observations). Such perturbations have a relatively high frequency $\left(10^4 - 10^{-1}\right)\mathrm{s^{-1}}$ and have wavelengths $\sim 10^3\,\mathrm{km}$. Compared with the ordinary Rossby waves, electromagnetic RK waves accompanied by the strong pulsations of the geomagnetic field are 20–80 nT. Note the Khantadze waves at the middle and moderate latitudes observed at the launching of spacecrafts (Burmaka et al., 2006) and by the world network of ionospheric and magnetic observations (Sharadze et al., 1988, 1989; Sharadze, 1991; Alperovich and Fedorov, 2007). Forbes (1996) provides data analyses for discussing the penetration of Rossby-type planetary wave effects into the ionospheric dynamo E region (100–170 km) and the electrodynamic interactions which ensue there.

RK waves are mainly of a zonal type and are mainly observed during magnetic storms along with substorms, artificial explosions, earthquakes, etc. They give valuable information on large-scale synoptic processes and external sources as well as dynamical processes in the ionosphere. Therefore, theoretical investigations of electromagnetic Rossby-type oscillations will provide valuable information for further ionospheric experimental investigations.

Data availability. No data sets were used in this article.

Author contributions. LZK: conceptualization (equal); formal analysis (equal); investigation (equal); methodology (equal); writing; – original draft (equal); supervision (equal); writing – review and editing (equal). HAS: methodology (equal); investigation (equal); supervision (equal); writing – review and editing (equal). TDK: conceptualization (equal); investigation (equal); methodology (equal); writing – review and editing (equal). QTA: formal analysis (equal); methodology (equal); writing; – original draft (equal). SAB: investigation (equal); writing; – original draft (equal); writing – review and editing (equal).

Competing interests. The contact author has declared that none of the authors has any competing interests.

Disclaimer. Publisher's note: Copernicus Publications remains neutral with regard to jurisdictional claims made in the text, published maps, institutional affiliations, or any other geographical representation in this paper. While Copernicus Publications makes every effort to include appropriate place names, the final responsibility lies with the authors.

Acknowledgements. We are thankful to the editor Victor Shrira and anonymous reviewers for their valuable suggestions to improve the manuscript.

Review statement. This paper was edited by Victor Shrira and reviewed by Yu Qin and two anonymous referees.

References

Alperovich, L. S. and Fedorov, E. N.: Hydromagnetic Waves in the Magnetosphere and the Ionosphere, Springer, https://doi.org/10.1007/978-1-4020-6637-5, 2007.

Bostrom, R.: Observations of weak double layers on auroral field lines, IEEE T. Plasma Sci., 20, 756–763, https://doi.org/10.1109/27.199524, 1992.

Burmaka, V. P., Lysenko, V. N., Chernogor, L. F., and Chernyak, Y. V.: Wave-like process in the ionospheric F region that accompanied rocket launches from the Baikonur Site, Geomagn. Aeronomy, 46, 742–759, https://doi.org/10.1134/S0016793206060107, 2006.

Dovner, P. O., Eriksson, A. I., Bostrom, R., and Holback, B.: Freja multiprobe observations of electrostatic solitary structures, Geophys. Res. Lett., 21, 1827–1830, https://doi.org/10.1029/94GL00886, 1994.

Forbes, J. M.: Planetary waves in the thermosphere-ionosphere system, J. Geomagn. Geoelectr., 48, 91–98, 1996.

Futatani, S., Horton, W., and Kaladze, T. D.: Nonlinear propagation of Rossby-Khantadze electromagnetic planetary waves in the ionospheric E-layer, Phys. Plasmas, 20, 102903, https://doi.org/10.1063/1.4826592, 2013.

Futatani, S., Horton, W., Kahlon, L. Z., and Kaladze, T. D.: Rossby-Khantadze electromagnetic planetary waves driven by sheared zonal winds in the E-layer ionosphere, Phys. Plasmas, 22, 012906, https://doi.org/10.1063/1.4906362, 2015.

Jian, S., Lian-Gui, Y., Chao-Jiu, D. A., and Hui-Qin, Z.: mKdV equation for the amplitude of solitary Rossby waves in stratified shear flows with a zonal shear flow, Atmospheric Oceanic Science Letters, 2, 18–23, https://doi.org/10.1080/16742834.2009.11446771, 2009.

Kahlon, L. Z. and Kaladze, T. D.: Generation of zonal flow and magnetic field in the ionospheric E-layer, J. Plasma Phys., 81, 905810512, https://doi.org/10.1017/S002237781500080X, 2015.

Kaladze, T. D., Wu, D. J., Pokhotelov, O. A., Sagdeev, R. Z., Stenflo, L., and Shukla, P. K.: Rossby-wave driven zonal flows in the ionospheric E-layer, J. Plasma Phys., 73, 131–140, https://doi.org/10.1017/S0022377806004351, 2007.

Kaladze, T. D., Pokhotelov, O. A., Stenflo, L., Rogava, J., Tsamalashvili, L. V., and Tsik-Lauri, M.: Zonal flow interaction with Rossby waves in the Earth's atmosphere: a numerical simulation, Phys. Lett. A, 372, 5177–5180, https://doi.org/10.1016/j.physleta.2008.06.008, 2008.

Kaladze, T. D., Tsamalashvili, L. V., and Kahlon, L. Z.: Rossby-Khantadze electromagnetic planetary vortical motions in the ionospheric E-layer, J. Plasma Phys., 77, 813–828, https://doi.org/10.1017/S0022377811000237, 2011.

Kaladze, T. D., Kahlon, L. Z., and Tsamalashvili, L. V.: Excitation of zonal flow and magnetic field by Rossby-Khantadze electromagnetic planetary waves in the ionospheric E-layer, Phys. Plasmas, 19, 022902, https://doi.org/10.1063/1.3681370, 2012.

Kaladze, T. D., Horton, W., Kahlon, L. Z., Pokhotelov, O., and Onishchenko, O.: Zonal flows and magnetic fields driven by large-amplitude Rossby-Alfvén-Khantadze waves in the E-layer ionosphere, J. Geophys. Res.-Space, 118, 7822–7833, https://doi.org/10.1002/2013JA019415, 2013a.

Kaladze, T. D., Horton, W., Kahlon, L. Z., Pokhotelov, O., and Onishchenko, O.: Generation of zonal flow and magnetic field by coupled Rossby-Alfvén-Khantadze waves in the Earth's ionospheric E-layer, Phys. Scripta, 88, 065501, https://doi.org/10.1088/0031-8949/88/06/065501, 2013b.

Kaladze, T., Kahlon, L., Horton, W., Pokhotelov, O., and Onishchenko, O.: Shear flow driven Rossby-Khantadze electromagnetic planetary vortices in the ionospheric E-layer, Europhys. Lett., 106, 29001, https://doi.org/10.1209/0295-5075/106/29001, 2014.

Kaladze, T., Tsamalashvili, L., Kaladze, D., Ozcan, O., Yesil, A., and Inc, M.: Modified KdV equation for magnetized Rossby waves in a zonal flow of the ionospheric E-layer, Phys. Lett. A, 383, 125888, https://doi.org/10.1016/j.physleta.2019.125888, 2019.

Kevrekidis, P. G., Khare, A., Saxena, A., and Herring, G.: On some classes of mKdV periodic solutions, J. Phys. A-Math. Gen., 37, 10959, https://doi.org/10.1088/0305-4470/37/45/014, 2004.

Lindqvist, P. A., Marklud, G. T., and Blomberg, L. G.: Plasma characteristics determined by the Freja electric field instrument, Space Sci. Rev., 70, 593–602, https://doi.org/10.1007/BF00756888, 1994.

Onishchenko, O. G., Pokhotelov, O. A., Sagdeev, R. Z., Shukla, P. K., and Stenflo, L.: Generation of zonal flows by Rossby waves in the atmosphere, Nonlin. Processes Geophys., 11, 241–244, https://doi.org/10.5194/npg-11-241-2004, 2004.

Pedlosky, J.: Geophysical Fluid Dynamics, Springer-Verlag, New York, https://doi.org/10.1007/978-1-4612-4650-3, 1987.

Qiang, Z., Zuntao, F., and Shikuo, L.: Equatorial envelope Rossby solitons in a shear flow, Adv. Atmos. Sci., 18, 418–428, https://doi.org/10.1007/BF02919321, 2001.

Satoh, M.: Atmospheric Circulation Dynamics and General Circulation Models, Springer, New York, https://doi.org/10.1007/978-3-642-13574-3, 2004.

Shukla, P. K. and Stenflo, L.: Generation of zonal flows by Rossby waves, Phys. Lett. A, 307, 154–157, https://doi.org/10.1016/S0375-9601(02)01675-4, 2003.

Sharadze, Z. S., Japaridze, G. A., Kikvilashvili, G. B., and Liadze, Z. L.: Wave disturbances of non-acoustical nature in the middle-latitude ionosphere, Geomagn. Aeronomy, 28, 446–451, 1988 (in Russian).

Sharadze, Z. S., Mosashvili, N. V., Pushkova, G. N., and Yudovich, L. A.: Long-period-wave disturbances in E-region of the ionosphere, Geomagn. Aeronomy, 29, 1032–1034, 1989 (in Russian).

Sharadze, Z. S.: Phenomena in the middle-latitude ionosphere, PhD Thesis, Moscow, 1991 (in Russian).

Vukcevic, M. and Popovic, L. Č.: Solitons in the ionosphere-Advantages and perspectives, Proceedings of the XII Serbian-Bulgarian Astronomical Conference, (XII SBAC) Sokobanja, Serbia, 25–29 September 2020, edited by: Popović, L. Č., Srećković, V. A., Dimitrijević, M. S., and Kovačević, A., Publ. Astron. Soc. “Rudjer Bošković” No 20, 85–91, https://doi.org/10.3390/app11167194, 2020.

Wazwaz, A.-M.: Partial differential equations and solitary waves theory, Springer, https://doi.org/10.1007/978-3-642-00251-9, 2009.