

## Reference Manual

Generated by Doxygen 1.8.11



# Contents

<b>1 Modular arbitrary-order ocean-atmosphere model: MAOOAM -- Stochastic Fortran implementation</b>	<b>1</b>
<b>2 Modular arbitrary-order ocean-atmosphere model: The Tangent Linear and Adjoint model</b>	<b>9</b>
<b>3 Modular arbitrary-order ocean-atmosphere model: The MTV and WL parameterizations</b>	<b>11</b>
<b>4 Modular arbitrary-order ocean-atmosphere model: Definition files formats</b>	<b>17</b>
<b>5 Modules Index</b>	<b>19</b>
5.1 Modules List . . . . .	19
<b>6 Data Type Index</b>	<b>21</b>
6.1 Data Types List . . . . .	21
<b>7 File Index</b>	<b>23</b>
7.1 File List . . . . .	23
<b>8 Module Documentation</b>	<b>25</b>
8.1 aotensor_def Module Reference . . . . .	25
8.1.1 Detailed Description . . . . .	26
8.1.2 Function/Subroutine Documentation . . . . .	26
8.1.2.1 a(i) . . . . .	26
8.1.2.2 add_count(i, j, k, v) . . . . .	26
8.1.2.3 coeff(i, j, k, v) . . . . .	26
8.1.2.4 compute_aotensor(func) . . . . .	27
8.1.2.5 init_aotensor . . . . .	27
8.1.2.6 kdelta(i, j) . . . . .	28

8.1.2.7	psi(i) . . . . .	28
8.1.2.8	t(i) . . . . .	28
8.1.2.9	theta(i) . . . . .	28
8.1.3	Variable Documentation . . . . .	29
8.1.3.1	aotensor . . . . .	29
8.1.3.2	count_elems . . . . .	29
8.1.3.3	real_eps . . . . .	29
8.2	corr_tensor Module Reference . . . . .	29
8.2.1	Detailed Description . . . . .	30
8.2.2	Function/Subroutine Documentation . . . . .	30
8.2.2.1	init_corr_tensor . . . . .	30
8.2.3	Variable Documentation . . . . .	31
8.2.3.1	dumb_mat1 . . . . .	31
8.2.3.2	dumb_mat2 . . . . .	31
8.2.3.3	dumb_vec . . . . .	32
8.2.3.4	dy . . . . .	32
8.2.3.5	ddy . . . . .	32
8.2.3.6	expm . . . . .	32
8.2.3.7	ydy . . . . .	32
8.2.3.8	yddy . . . . .	33
8.2.3.9	yy . . . . .	33
8.3	corrmod Module Reference . . . . .	33
8.3.1	Detailed Description . . . . .	34
8.3.2	Function/Subroutine Documentation . . . . .	34
8.3.2.1	corrcomp_from_def(s) . . . . .	34
8.3.2.2	corrcomp_from_fit(s) . . . . .	38
8.3.2.3	corrcomp_from_spline(s) . . . . .	38
8.3.2.4	fs(s, p) . . . . .	39
8.3.2.5	init_corr . . . . .	39
8.3.2.6	splint(xa, ya, y2a, n, x, y) . . . . .	40

8.3.3	Variable Documentation	41
8.3.3.1	corr_i	41
8.3.3.2	corr_i_full	41
8.3.3.3	corr_ij	41
8.3.3.4	corrcomp	42
8.3.3.5	inv_corr_i	42
8.3.3.6	inv_corr_i_full	42
8.3.3.7	khi	42
8.3.3.8	klo	42
8.3.3.9	mean	42
8.3.3.10	mean_full	43
8.3.3.11	nspl	43
8.3.3.12	xa	43
8.3.3.13	y2	43
8.3.3.14	ya	43
8.4	dec_tensor Module Reference	44
8.4.1	Detailed Description	45
8.4.2	Function/Subroutine Documentation	45
8.4.2.1	init_dec_tensor	45
8.4.2.2	init_sub_tensor(t, cst, v)	49
8.4.2.3	reorder(t, cst, v)	49
8.4.2.4	suppress_and(t, cst, v1, v2)	50
8.4.2.5	suppress_or(t, cst, v1, v2)	50
8.4.3	Variable Documentation	51
8.4.3.1	bxxx	51
8.4.3.2	bxyy	51
8.4.3.3	bxyy	52
8.4.3.4	byxx	52
8.4.3.5	byxy	52
8.4.3.6	byyy	52

8.4.3.7	dumb	52
8.4.3.8	ff_tensor	53
8.4.3.9	fs_tensor	53
8.4.3.10	hx	53
8.4.3.11	hy	53
8.4.3.12	lxx	53
8.4.3.13	lxy	54
8.4.3.14	lyx	54
8.4.3.15	lyy	54
8.4.3.16	sf_tensor	54
8.4.3.17	ss_tensor	54
8.4.3.18	ss_tl_tensor	55
8.5	ic_def Module Reference	55
8.5.1	Detailed Description	55
8.5.2	Function/Subroutine Documentation	55
8.5.2.1	load_ic	55
8.5.3	Variable Documentation	57
8.5.3.1	exists	57
8.5.3.2	ic	57
8.6	inprod_analytic Module Reference	57
8.6.1	Detailed Description	59
8.6.2	Function/Subroutine Documentation	59
8.6.2.1	b1(Pi, Pj, Pk)	59
8.6.2.2	b2(Pi, Pj, Pk)	59
8.6.2.3	calculate_a(i, j)	60
8.6.2.4	calculate_b(i, j, k)	60
8.6.2.5	calculate_c_atm(i, j)	60
8.6.2.6	calculate_c_oc(i, j, k)	61
8.6.2.7	calculate_d(i, j)	61
8.6.2.8	calculate_g(i, j, k)	61

8.6.2.9	calculate_k(i, j) . . . . .	62
8.6.2.10	calculate_m(i, j) . . . . .	62
8.6.2.11	calculate_n(i, j) . . . . .	63
8.6.2.12	calculate_o(i, j, k) . . . . .	63
8.6.2.13	calculate_s(i, j) . . . . .	64
8.6.2.14	calculate_w(i, j) . . . . .	64
8.6.2.15	delta(r) . . . . .	64
8.6.2.16	flambda(r) . . . . .	65
8.6.2.17	init_inprod . . . . .	65
8.6.2.18	s1(Pj, Pk, Mj, Hk) . . . . .	66
8.6.2.19	s2(Pj, Pk, Mj, Hk) . . . . .	66
8.6.2.20	s3(Pj, Pk, Hj, Hk) . . . . .	66
8.6.2.21	s4(Pj, Pk, Hj, Hk) . . . . .	67
8.6.3	Variable Documentation . . . . .	67
8.6.3.1	atmos . . . . .	67
8.6.3.2	awavenum . . . . .	67
8.6.3.3	ocean . . . . .	67
8.6.3.4	owavenum . . . . .	67
8.7	int_comp Module Reference . . . . .	68
8.7.1	Detailed Description . . . . .	68
8.7.2	Function/Subroutine Documentation . . . . .	68
8.7.2.1	integrate(func, ss) . . . . .	68
8.7.2.2	midexp(func, aa, bb, s, n) . . . . .	69
8.7.2.3	midpnt(func, a, b, s, n) . . . . .	69
8.7.2.4	polint(xa, ya, n, x, y, dy) . . . . .	70
8.7.2.5	qromb(func, a, b, ss) . . . . .	70
8.7.2.6	qromo(func, a, b, ss, choose) . . . . .	71
8.7.2.7	trapzd(func, a, b, s, n) . . . . .	72
8.8	int_corr Module Reference . . . . .	72
8.8.1	Detailed Description . . . . .	73

---

8.8.2	Function/Subroutine Documentation . . . . .	73
8.8.2.1	comp_corrint . . . . .	73
8.8.2.2	func_ij(s) . . . . .	74
8.8.2.3	func_ijkl(s) . . . . .	75
8.8.2.4	init_corrint . . . . .	75
8.8.3	Variable Documentation . . . . .	75
8.8.3.1	corr2int . . . . .	75
8.8.3.2	corrint . . . . .	76
8.8.3.3	oi . . . . .	76
8.8.3.4	oj . . . . .	76
8.8.3.5	ok . . . . .	76
8.8.3.6	ol . . . . .	76
8.8.3.7	real_eps . . . . .	76
8.9	integrator Module Reference . . . . .	76
8.9.1	Detailed Description . . . . .	77
8.9.2	Function/Subroutine Documentation . . . . .	78
8.9.2.1	init_integrator . . . . .	78
8.9.2.2	step(y, t, dt, res) . . . . .	78
8.9.2.3	tendencies(t, y, res) . . . . .	78
8.9.3	Variable Documentation . . . . .	79
8.9.3.1	buf_f0 . . . . .	79
8.9.3.2	buf_f1 . . . . .	79
8.9.3.3	buf_ka . . . . .	79
8.9.3.4	buf_kb . . . . .	79
8.9.3.5	buf_y1 . . . . .	80
8.10	mar Module Reference . . . . .	80
8.10.1	Detailed Description . . . . .	80
8.10.2	Function/Subroutine Documentation . . . . .	81
8.10.2.1	init_mar . . . . .	81
8.10.2.2	mar_step(x) . . . . .	81

---

8.10.2.3	mar_step_red(xred)	82
8.10.2.4	stoch_vec(dW)	82
8.10.3	Variable Documentation	82
8.10.3.1	buf_y	82
8.10.3.2	dw	83
8.10.3.3	ms	83
8.10.3.4	q	83
8.10.3.5	qred	83
8.10.3.6	rred	83
8.10.3.7	w	83
8.10.3.8	wred	84
8.11	memory Module Reference	84
8.11.1	Detailed Description	84
8.11.2	Function/Subroutine Documentation	85
8.11.2.1	compute_m3(y, dt, dtn, savey, save_ev, evolve, inter, h_int)	85
8.11.2.2	init_memory	86
8.11.2.3	test_m3(y, dt, dtn, h_int)	86
8.11.3	Variable Documentation	87
8.11.3.1	buf_m	87
8.11.3.2	buf_m3	87
8.11.3.3	step	87
8.11.3.4	t_index	87
8.11.3.5	x	88
8.11.3.6	xs	88
8.11.3.7	zs	88
8.12	mtv_int_tensor Module Reference	88
8.12.1	Detailed Description	90
8.12.2	Function/Subroutine Documentation	90
8.12.2.1	init_mtv_int_tensor	90
8.12.3	Variable Documentation	92

---

8.12.3.1	b1	93
8.12.3.2	b2	93
8.12.3.3	btot	93
8.12.3.4	dumb_mat1	93
8.12.3.5	dumb_mat2	93
8.12.3.6	dumb_mat3	94
8.12.3.7	dumb_mat4	94
8.12.3.8	dumb_vec	94
8.12.3.9	h1	94
8.12.3.10	h2	94
8.12.3.11	h3	95
8.12.3.12	htot	95
8.12.3.13	l1	95
8.12.3.14	l2	95
8.12.3.15	l3	95
8.12.3.16	ltot	96
8.12.3.17	mtot	96
8.12.3.18	q1	96
8.12.3.19	q2	96
8.12.3.20	utot	96
8.12.3.21	vtot	97
8.13	params Module Reference	97
8.13.1	Detailed Description	99
8.13.2	Function/Subroutine Documentation	100
8.13.2.1	init_nml	100
8.13.2.2	init_params	100
8.13.3	Variable Documentation	101
8.13.3.1	ams	101
8.13.3.2	betp	101
8.13.3.3	ca	101

---

8.13.3.4 co . . . . .	102
8.13.3.5 cpa . . . . .	102
8.13.3.6 cpo . . . . .	102
8.13.3.7 d . . . . .	102
8.13.3.8 dp . . . . .	102
8.13.3.9 dt . . . . .	103
8.13.3.10 epsa . . . . .	103
8.13.3.11 f0 . . . . .	103
8.13.3.12 g . . . . .	103
8.13.3.13 ga . . . . .	103
8.13.3.14 go . . . . .	104
8.13.3.15 gp . . . . .	104
8.13.3.16 h . . . . .	104
8.13.3.17 k . . . . .	104
8.13.3.18 kd . . . . .	104
8.13.3.19 kdp . . . . .	105
8.13.3.20 kp . . . . .	105
8.13.3.21 l . . . . .	105
8.13.3.22 lambda . . . . .	105
8.13.3.23 lpa . . . . .	105
8.13.3.24 lpo . . . . .	106
8.13.3.25 lr . . . . .	106
8.13.3.26 lsbpa . . . . .	106
8.13.3.27 lsypo . . . . .	106
8.13.3.28 n . . . . .	106
8.13.3.29 natm . . . . .	107
8.13.3.30 nbatm . . . . .	107
8.13.3.31 nboc . . . . .	107
8.13.3.32 ndim . . . . .	107
8.13.3.33 noc . . . . .	107

---

8.13.3.34 nua . . . . .	108
8.13.3.35 nuap . . . . .	108
8.13.3.36 nuo . . . . .	108
8.13.3.37 nuop . . . . .	108
8.13.3.38 oms . . . . .	108
8.13.3.39 phi0 . . . . .	109
8.13.3.40 phi0_npi . . . . .	109
8.13.3.41 pi . . . . .	109
8.13.3.42 r . . . . .	109
8.13.3.43 rp . . . . .	109
8.13.3.44 rr . . . . .	110
8.13.3.45 rra . . . . .	110
8.13.3.46 sb . . . . .	110
8.13.3.47 sbpa . . . . .	110
8.13.3.48 sbpo . . . . .	110
8.13.3.49 sc . . . . .	111
8.13.3.50 scale . . . . .	111
8.13.3.51 sig0 . . . . .	111
8.13.3.52 t_run . . . . .	111
8.13.3.53 t_trans . . . . .	111
8.13.3.54 ta0 . . . . .	112
8.13.3.55 to0 . . . . .	112
8.13.3.56 tw . . . . .	112
8.13.3.57 writeout . . . . .	112
8.14 rk2_mtv_integrator Module Reference . . . . .	112
8.14.1 Detailed Description . . . . .	114
8.14.2 Function/Subroutine Documentation . . . . .	114
8.14.2.1 compg(y) . . . . .	114
8.14.2.2 full_step(y, t, dt, dtn, res) . . . . .	114
8.14.2.3 init_g . . . . .	115

---

---

8.14.2.4	init_integrator	115
8.14.2.5	init_noise	115
8.14.2.6	step(y, t, dt, dtn, res, tend)	116
8.14.3	Variable Documentation	117
8.14.3.1	anoise	117
8.14.3.2	buf_f0	117
8.14.3.3	buf_f1	117
8.14.3.4	buf_g	117
8.14.3.5	buf_y1	117
8.14.3.6	compute_mult	118
8.14.3.7	dw	118
8.14.3.8	dwar	118
8.14.3.9	dwau	118
8.14.3.10	dwmult	118
8.14.3.11	dwor	118
8.14.3.12	dwou	118
8.14.3.13	g	119
8.14.3.14	mult	119
8.14.3.15	noise	119
8.14.3.16	noisemult	119
8.14.3.17	q1fill	119
8.14.3.18	sq2	119
8.15	rk2_ss_integrator Module Reference	120
8.15.1	Detailed Description	120
8.15.2	Function/Subroutine Documentation	121
8.15.2.1	init_ss_integrator	121
8.15.2.2	ss_step(y, ys, t, dt, dtn, res)	121
8.15.2.3	ss_tl_step(y, ys, t, dt, dtn, res)	122
8.15.2.4	tendencies(t, y, res)	122
8.15.2.5	tl_tendencies(t, y, ys, res)	123

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8.15.3 Variable Documentation . . . . .	123
8.15.3.1 anoise . . . . .	123
8.15.3.2 buf_f0 . . . . .	123
8.15.3.3 buf_f1 . . . . .	123
8.15.3.4 buf_y1 . . . . .	123
8.15.3.5 dwar . . . . .	124
8.15.3.6 dwor . . . . .	124
8.16 rk2_stoch_integrator Module Reference . . . . .	124
8.16.1 Detailed Description . . . . .	125
8.16.2 Function/Subroutine Documentation . . . . .	125
8.16.2.1 init_integrator(force) . . . . .	125
8.16.2.2 step(y, t, dt, dtn, res, tend) . . . . .	126
8.16.2.3 tendencies(t, y, res) . . . . .	126
8.16.3 Variable Documentation . . . . .	127
8.16.3.1 anoise . . . . .	127
8.16.3.2 buf_f0 . . . . .	127
8.16.3.3 buf_f1 . . . . .	127
8.16.3.4 buf_y1 . . . . .	127
8.16.3.5 dwar . . . . .	127
8.16.3.6 dwau . . . . .	128
8.16.3.7 dwor . . . . .	128
8.16.3.8 dwou . . . . .	128
8.16.3.9 int_tensor . . . . .	128
8.17 rk2_wl_integrator Module Reference . . . . .	128
8.17.1 Detailed Description . . . . .	129
8.17.2 Function/Subroutine Documentation . . . . .	129
8.17.2.1 compute_m1(y) . . . . .	129
8.17.2.2 compute_m2(y) . . . . .	130
8.17.2.3 full_step(y, t, dt, dtn, res) . . . . .	130
8.17.2.4 init_integrator . . . . .	131

8.17.2.5	step(y, t, dt, dtn, res, tend)	131
8.17.3	Variable Documentation	132
8.17.3.1	anoise	132
8.17.3.2	buf_f0	133
8.17.3.3	buf_f1	133
8.17.3.4	buf_m	133
8.17.3.5	buf_m1	133
8.17.3.6	buf_m2	133
8.17.3.7	buf_m3	133
8.17.3.8	buf_m3s	133
8.17.3.9	buf_y1	133
8.17.3.10	dwar	134
8.17.3.11	dwau	134
8.17.3.12	dwor	134
8.17.3.13	dwou	134
8.17.3.14	x1	134
8.17.3.15	x2	134
8.18	sf_def Module Reference	134
8.18.1	Detailed Description	135
8.18.2	Function/Subroutine Documentation	135
8.18.2.1	load_sf	135
8.18.3	Variable Documentation	137
8.18.3.1	bar	137
8.18.3.2	bau	137
8.18.3.3	bor	137
8.18.3.4	bou	137
8.18.3.5	exists	137
8.18.3.6	ind	137
8.18.3.7	n_res	138
8.18.3.8	n_unres	138

8.18.3.9 rind . . . . .	138
8.18.3.10 sf . . . . .	138
8.18.3.11 sl_ind . . . . .	138
8.18.3.12 sl_rind . . . . .	138
8.19 sigma Module Reference . . . . .	139
8.19.1 Detailed Description . . . . .	139
8.19.2 Function/Subroutine Documentation . . . . .	139
8.19.2.1 compute_mult_sigma(y) . . . . .	139
8.19.2.2 init_sigma(mult, Q1fill) . . . . .	140
8.19.3 Variable Documentation . . . . .	141
8.19.3.1 dumb_mat1 . . . . .	141
8.19.3.2 dumb_mat2 . . . . .	141
8.19.3.3 dumb_mat3 . . . . .	141
8.19.3.4 dumb_mat4 . . . . .	141
8.19.3.5 ind1 . . . . .	141
8.19.3.6 ind2 . . . . .	141
8.19.3.7 n1 . . . . .	142
8.19.3.8 n2 . . . . .	142
8.19.3.9 rind1 . . . . .	142
8.19.3.10 rind2 . . . . .	142
8.19.3.11 sig1 . . . . .	142
8.19.3.12 sig1r . . . . .	142
8.19.3.13 sig2 . . . . .	142
8.20 sqrt_mod Module Reference . . . . .	143
8.20.1 Detailed Description . . . . .	143
8.20.2 Function/Subroutine Documentation . . . . .	143
8.20.2.1 chol(A, sqA, info) . . . . .	143
8.20.2.2 csqrtm_triu(A, sqA, info, bs) . . . . .	144
8.20.2.3 init_sqrt . . . . .	145
8.20.2.4 rsf2csf(T, Z, Tz, Zz) . . . . .	145

8.20.2.5 <code>selectev(a, b)</code>	146
8.20.2.6 <code>sqrtn(A, sqA, info, info_triu, bs)</code>	146
8.20.2.7 <code>sqrtn_svd(A, sqA, info, info_triu, bs)</code>	147
8.20.2.8 <code>sqrtn_triu(A, sqA, info, bs)</code>	148
8.20.3 Variable Documentation	149
8.20.3.1 <code>lwork</code>	149
8.20.3.2 <code>real_eps</code>	149
8.20.3.3 <code>work</code>	149
8.21 stat Module Reference	150
8.21.1 Detailed Description	150
8.21.2 Function/Subroutine Documentation	150
8.21.2.1 <code>acc(x)</code>	150
8.21.2.2 <code>init_stat</code>	151
8.21.2.3 <code>iter()</code>	151
8.21.2.4 <code>mean()</code>	151
8.21.2.5 <code>reset</code>	151
8.21.2.6 <code>var()</code>	151
8.21.3 Variable Documentation	152
8.21.3.1 <code>i</code>	152
8.21.3.2 <code>m</code>	152
8.21.3.3 <code>mprev</code>	152
8.21.3.4 <code>mtmp</code>	152
8.21.3.5 <code>v</code>	152
8.22 stoch_mod Module Reference	152
8.22.1 Detailed Description	153
8.22.2 Function/Subroutine Documentation	153
8.22.2.1 <code>gasdev()</code>	153
8.22.2.2 <code>stoch_atm_res_vec(dW)</code>	154
8.22.2.3 <code>stoch_atm_unres_vec(dW)</code>	154
8.22.2.4 <code>stoch_atm_vec(dW)</code>	154

---

8.22.2.5	stoch_oc_res_vec(dW)	155
8.22.2.6	stoch_oc_unres_vec(dW)	155
8.22.2.7	stoch_oc_vec(dW)	155
8.22.2.8	stoch_vec(dW)	156
8.22.3	Variable Documentation	156
8.22.3.1	gset	156
8.22.3.2	iset	156
8.23	stoch_params Module Reference	156
8.23.1	Detailed Description	157
8.23.2	Function/Subroutine Documentation	158
8.23.2.1	init_stoch_params	158
8.23.3	Variable Documentation	158
8.23.3.1	dtn	158
8.23.3.2	dts	158
8.23.3.3	dtsn	158
8.23.3.4	eps_pert	159
8.23.3.5	int_corr_mode	159
8.23.3.6	load_mode	159
8.23.3.7	maxint	159
8.23.3.8	meml	159
8.23.3.9	mems	160
8.23.3.10	mnuti	160
8.23.3.11	mode	160
8.23.3.12	muti	160
8.23.3.13	q_ar	160
8.23.3.14	q_au	161
8.23.3.15	q_or	161
8.23.3.16	q_ou	161
8.23.3.17	t_trans_mem	161
8.23.3.18	t_trans_stoch	161

---

8.23.3.19 <code>tdelta</code>	162
8.23.3.20 <code>x_int_mode</code>	162
8.24 tensor Module Reference	162
8.24.1 Detailed Description	165
8.24.2 Function/Subroutine Documentation	165
8.24.2.1 <code>add_check(t, i, j, k, v, dst)</code>	165
8.24.2.2 <code>add_elem(t, i, j, k, v)</code>	165
8.24.2.3 <code>add_matc_to_tensor(i, src, dst)</code>	166
8.24.2.4 <code>add_matc_to_tensor4(i, j, src, dst)</code>	167
8.24.2.5 <code>add_to_tensor(src, dst)</code>	168
8.24.2.6 <code>add_vec_ijk_to_tensor4(i, j, k, src, dst)</code>	169
8.24.2.7 <code>add_vec_ikl_to_tensor4(i, k, l, src, dst)</code>	170
8.24.2.8 <code>add_vec_ikl_to_tensor4_perm(i, k, l, src, dst)</code>	170
8.24.2.9 <code>add_vec_jk_to_tensor(j, k, src, dst)</code>	171
8.24.2.10 <code>coo_to_mat_i(i, src, dst)</code>	172
8.24.2.11 <code>coo_to_mat_ij(src, dst)</code>	173
8.24.2.12 <code>coo_to_mat_ik(src, dst)</code>	173
8.24.2.13 <code>coo_to_mat_j(j, src, dst)</code>	174
8.24.2.14 <code>coo_to_vec_jk(j, k, src, dst)</code>	174
8.24.2.15 <code>copy_coo(src, dst)</code>	174
8.24.2.16 <code>jsparse_mul(coolist_ijk, arr_j, jcoo_ij)</code>	175
8.24.2.17 <code>jsparse_mul_mat(coolist_ijk, arr_j, jcoo_ij)</code>	176
8.24.2.18 <code>load_tensor4_from_file(s, t)</code>	177
8.24.2.19 <code>load_tensor_from_file(s, t)</code>	178
8.24.2.20 <code>mat_to_coo(src, dst)</code>	179
8.24.2.21 <code>matc_to_coo(src, dst)</code>	179
8.24.2.22 <code>print_tensor(t, s)</code>	180
8.24.2.23 <code>print_tensor4(t)</code>	180
8.24.2.24 <code>scal_mul_coo(s, t)</code>	181
8.24.2.25 <code>simplify(tensor)</code>	181

8.24.2.26 sparse_mul2(coolist_ij, arr_j, res) . . . . .	182
8.24.2.27 sparse_mul2_j(coolist_ijk, arr_j, res) . . . . .	183
8.24.2.28 sparse_mul2_k(coolist_ijk, arr_k, res) . . . . .	183
8.24.2.29 sparse_mul3(coolist_ijk, arr_j, arr_k, res) . . . . .	184
8.24.2.30 sparse_mul3_mat(coolist_ijk, arr_k, res) . . . . .	185
8.24.2.31 sparse_mul3_with_mat(coolist_ijk, mat_jk, res) . . . . .	185
8.24.2.32 sparse_mul4(coolist_ijkl, arr_j, arr_k, arr_l, res) . . . . .	186
8.24.2.33 sparse_mul4_mat(coolist_ijkl, arr_k, arr_l, res) . . . . .	186
8.24.2.34 sparse_mul4_with_mat_jl(coolist_ijkl, mat_jl, res) . . . . .	187
8.24.2.35 sparse_mul4_with_mat_kl(coolist_ijkl, mat_kl, res) . . . . .	188
8.24.2.36 tensor4_empty(t) . . . . .	188
8.24.2.37 tensor4_to_coo4(src, dst) . . . . .	189
8.24.2.38 tensor_empty(t) . . . . .	189
8.24.2.39 tensor_to_coo(src, dst) . . . . .	190
8.24.2.40 write_tensor4_to_file(s, t) . . . . .	191
8.24.2.41 write_tensor_to_file(s, t) . . . . .	191
8.24.3 Variable Documentation . . . . .	191
8.24.3.1 real_eps . . . . .	191
8.25 tl_ad_integrator Module Reference . . . . .	192
8.25.1 Detailed Description . . . . .	192
8.25.2 Function/Subroutine Documentation . . . . .	192
8.25.2.1 ad_step(y, ystar, t, dt, res) . . . . .	192
8.25.2.2 init_tl_ad_integrator . . . . .	193
8.25.2.3 tl_step(y, ystar, t, dt, res) . . . . .	193
8.25.3 Variable Documentation . . . . .	194
8.25.3.1 buf_f0 . . . . .	194
8.25.3.2 buf_f1 . . . . .	194
8.25.3.3 buf_ka . . . . .	194
8.25.3.4 buf_kb . . . . .	194
8.25.3.5 buf_y1 . . . . .	195

---

8.26 tl_ad_tensor Module Reference . . . . .	195
8.26.1 Detailed Description . . . . .	196
8.26.2 Function/Subroutine Documentation . . . . .	196
8.26.2.1 ad(t, ystar, deltay, buf) . . . . .	196
8.26.2.2 ad_add_count(i, j, k, v) . . . . .	196
8.26.2.3 ad_add_count_ref(i, j, k, v) . . . . .	197
8.26.2.4 ad_coeff(i, j, k, v) . . . . .	197
8.26.2.5 ad_coeff_ref(i, j, k, v) . . . . .	198
8.26.2.6 compute_adtensor(func) . . . . .	198
8.26.2.7 compute_adtensor_ref(func) . . . . .	199
8.26.2.8 compute_tltensor(func) . . . . .	199
8.26.2.9 init_adtensor . . . . .	199
8.26.2.10 init_adtensor_ref . . . . .	199
8.26.2.11 init_tltensor . . . . .	200
8.26.2.12 jacobian(ystar) . . . . .	200
8.26.2.13 jacobian_mat(ystar) . . . . .	201
8.26.2.14 tl(t, ystar, deltay, buf) . . . . .	201
8.26.2.15 tl_add_count(i, j, k, v) . . . . .	202
8.26.2.16 tl_coeff(i, j, k, v) . . . . .	202
8.26.3 Variable Documentation . . . . .	203
8.26.3.1 adtensor . . . . .	203
8.26.3.2 count_elems . . . . .	203
8.26.3.3 real_eps . . . . .	203
8.26.3.4 tltensor . . . . .	203
8.27 util Module Reference . . . . .	203
8.27.1 Detailed Description . . . . .	204
8.27.2 Function/Subroutine Documentation . . . . .	204
8.27.2.1 cdiag(A, d) . . . . .	204
8.27.2.2 choldc(a, p) . . . . .	205
8.27.2.3 cprintmat(A) . . . . .	205

---

8.27.2.4 diag(A, d) . . . . .	205
8.27.2.5 floordiv(i, j) . . . . .	205
8.27.2.6 init_one(A) . . . . .	206
8.27.2.7 init_random_seed() . . . . .	206
8.27.2.8 invmat(A) . . . . .	206
8.27.2.9 ireduce(A, Ared, n, ind, rind) . . . . .	206
8.27.2.10 isin(c, s) . . . . .	207
8.27.2.11 mat_contract(A, B) . . . . .	207
8.27.2.12 mat_trace(A) . . . . .	207
8.27.2.13 piksrt(k, arr, par) . . . . .	208
8.27.2.14 printmat(A) . . . . .	208
8.27.2.15 reduce(A, Ared, n, ind, rind) . . . . .	208
8.27.2.16 rstr(x, fm) . . . . .	209
8.27.2.17 str(k) . . . . .	209
8.27.2.18 triu(A, T) . . . . .	209
8.27.2.19 vector_outer(u, v, A) . . . . .	209
8.28 wl_tensor Module Reference . . . . .	209
8.28.1 Detailed Description . . . . .	211
8.28.2 Function/Subroutine Documentation . . . . .	211
8.28.2.1 init_wl_tensor . . . . .	211
8.28.3 Variable Documentation . . . . .	214
8.28.3.1 b1 . . . . .	214
8.28.3.2 b14 . . . . .	214
8.28.3.3 b14def . . . . .	214
8.28.3.4 b2 . . . . .	215
8.28.3.5 b23 . . . . .	215
8.28.3.6 b23def . . . . .	215
8.28.3.7 b3 . . . . .	215
8.28.3.8 b4 . . . . .	215
8.28.3.9 dumb_mat1 . . . . .	215

---

8.28.3.10 dumb_mat2 . . . . .	216
8.28.3.11 dumb_mat3 . . . . .	216
8.28.3.12 dumb_mat4 . . . . .	216
8.28.3.13 dumb_vec . . . . .	216
8.28.3.14 l1 . . . . .	216
8.28.3.15 l2 . . . . .	217
8.28.3.16 l4 . . . . .	217
8.28.3.17 l5 . . . . .	217
8.28.3.18 ldef . . . . .	217
8.28.3.19 ltot . . . . .	217
8.28.3.20 m11 . . . . .	217
8.28.3.21 m12 . . . . .	218
8.28.3.22 m12def . . . . .	218
8.28.3.23 m13 . . . . .	218
8.28.3.24 m1tot . . . . .	218
8.28.3.25 m21 . . . . .	218
8.28.3.26 m21def . . . . .	218
8.28.3.27 m22 . . . . .	219
8.28.3.28 m22def . . . . .	219
8.28.3.29 mdef . . . . .	219
8.28.3.30 mtot . . . . .	219

---

<b>9 Data Type Documentation</b>	<b>221</b>
9.1 <code>inprod_analytic::atm_tensors</code> Type Reference . . . . .	221
9.1.1 Detailed Description . . . . .	221
9.1.2 Member Data Documentation . . . . .	221
9.1.2.1 <code>a</code> . . . . .	221
9.1.2.2 <code>b</code> . . . . .	221
9.1.2.3 <code>c</code> . . . . .	222
9.1.2.4 <code>d</code> . . . . .	222
9.1.2.5 <code>g</code> . . . . .	222
9.1.2.6 <code>s</code> . . . . .	222
9.2 <code>inprod_analytic::atm_wavenum</code> Type Reference . . . . .	222
9.2.1 Detailed Description . . . . .	223
9.2.2 Member Data Documentation . . . . .	223
9.2.2.1 <code>h</code> . . . . .	223
9.2.2.2 <code>m</code> . . . . .	223
9.2.2.3 <code>nx</code> . . . . .	223
9.2.2.4 <code>ny</code> . . . . .	223
9.2.2.5 <code>p</code> . . . . .	223
9.2.2.6 <code>typ</code> . . . . .	223
9.3 <code>tensor::coolist</code> Type Reference . . . . .	224
9.3.1 Detailed Description . . . . .	224
9.3.2 Member Data Documentation . . . . .	224
9.3.2.1 <code>elems</code> . . . . .	224
9.3.2.2 <code>nelems</code> . . . . .	224
9.4 <code>tensor::coolist4</code> Type Reference . . . . .	224
9.4.1 Detailed Description . . . . .	225
9.4.2 Member Data Documentation . . . . .	225
9.4.2.1 <code>elems</code> . . . . .	225
9.4.2.2 <code>nelems</code> . . . . .	225
9.5 <code>tensor::coolist_elem</code> Type Reference . . . . .	225

---

9.5.1	Detailed Description	226
9.5.2	Member Data Documentation	226
9.5.2.1	j	226
9.5.2.2	k	226
9.5.2.3	v	226
9.6	tensor::coolist_elem4 Type Reference	226
9.6.1	Detailed Description	227
9.6.2	Member Data Documentation	227
9.6.2.1	j	227
9.6.2.2	k	227
9.6.2.3	l	227
9.6.2.4	v	227
9.7	inprod_analytic::ocean_tensors Type Reference	227
9.7.1	Detailed Description	228
9.7.2	Member Data Documentation	228
9.7.2.1	c	228
9.7.2.2	k	228
9.7.2.3	m	228
9.7.2.4	n	228
9.7.2.5	o	229
9.7.2.6	w	229
9.8	inprod_analytic::ocean_wavenum Type Reference	229
9.8.1	Detailed Description	229
9.8.2	Member Data Documentation	229
9.8.2.1	h	229
9.8.2.2	nx	229
9.8.2.3	ny	230
9.8.2.4	p	230

---

<b>10 File Documentation</b>	<b>231</b>
10.1 aotensor_def.f90 File Reference . . . . .	231
10.2 corr_tensor.f90 File Reference . . . . .	232
10.3 corrmod.f90 File Reference . . . . .	232
10.4 dec_tensor.f90 File Reference . . . . .	233
10.5 doc/def_doc.md File Reference . . . . .	235
10.6 doc/gen_doc.md File Reference . . . . .	235
10.7 doc/sto_doc.md File Reference . . . . .	235
10.8 doc/tl_ad_doc.md File Reference . . . . .	235
10.9 ic_def.f90 File Reference . . . . .	235
10.10inprod_analytic.f90 File Reference . . . . .	235
10.11int_comp.f90 File Reference . . . . .	237
10.12int_corr.f90 File Reference . . . . .	237
10.13LICENSE.txt File Reference . . . . .	238
10.13.1 Function Documentation . . . . .	239
10.13.1.1 files(the""Software""") . . . . .	240
10.13.1.2 License(MIT) Copyright(c) 2015-2018 Lesley De Cruz and Jonathan Demaeyer Permission is hereby granted . . . . .	240
10.13.2 Variable Documentation . . . . .	240
10.13.2.1 charge . . . . .	240
10.13.2.2 CLAIM . . . . .	240
10.13.2.3 conditions . . . . .	240
10.13.2.4 CONTRACT . . . . .	240
10.13.2.5 copy . . . . .	240
10.13.2.6 distribute . . . . .	240
10.13.2.7 FROM . . . . .	241
10.13.2.8 IMPLIED . . . . .	241
10.13.2.9 KIND . . . . .	241
10.13.2.10 LIABILITY . . . . .	241
10.13.2.11 MERCHANTABILITY . . . . .	241
10.13.2.12 merge . . . . .	241

10.13.2.13	modify	241
10.13.2.14	OTHERWISE	242
10.13.2.15	publish	242
10.13.2.16	restriction	242
10.13.2.17	so	242
10.13.2.18	Software	242
10.13.2.19	sublicense	242
10.13.2.20	use	242
10.14	maooam.f90 File Reference	242
10.14.1	Function/Subroutine Documentation	243
10.14.1.1	maooam	243
10.15	maooam_MTV.f90 File Reference	243
10.15.1	Function/Subroutine Documentation	243
10.15.1.1	maooam_mtv	243
10.16	maooam_stoch.f90 File Reference	243
10.16.1	Function/Subroutine Documentation	244
10.16.1.1	maooam_stoch	244
10.17	maooam_WL.f90 File Reference	244
10.17.1	Function/Subroutine Documentation	244
10.17.1.1	maooam_wl	244
10.18	MAR.f90 File Reference	244
10.19	memory.f90 File Reference	245
10.20	MTV_int_tensor.f90 File Reference	246
10.21	MTV_sigma_tensor.f90 File Reference	247
10.22	params.f90 File Reference	248
10.23	rk2_integrator.f90 File Reference	251
10.24	rk2_MTV_integrator.f90 File Reference	251
10.25	rk2_ss_integrator.f90 File Reference	252
10.26	rk2_stoch_integrator.f90 File Reference	253
10.27	rk2_tl_ad_integrator.f90 File Reference	254

---

10.28rk2_WL_integrator.f90 File Reference . . . . .	254
10.29rk4_integrator.f90 File Reference . . . . .	255
10.30rk4_tL_ad_integrator.f90 File Reference . . . . .	256
10.31sf_def.f90 File Reference . . . . .	256
10.32sqrt_mod.f90 File Reference . . . . .	257
10.33stat.f90 File Reference . . . . .	258
10.34stoch_mod.f90 File Reference . . . . .	258
10.35stoch_params.f90 File Reference . . . . .	259
10.36tensor.f90 File Reference . . . . .	260
10.37test_aotensor.f90 File Reference . . . . .	263
10.37.1 Function/Subroutine Documentation . . . . .	263
10.37.1.1 test_aotensor . . . . .	263
10.38test_corr.f90 File Reference . . . . .	263
10.38.1 Function/Subroutine Documentation . . . . .	264
10.38.1.1 test_corr . . . . .	264
10.39test_corr_tensor.f90 File Reference . . . . .	264
10.39.1 Function/Subroutine Documentation . . . . .	264
10.39.1.1 test_corr_tensor . . . . .	264
10.40test_dec_tensor.f90 File Reference . . . . .	264
10.40.1 Function/Subroutine Documentation . . . . .	264
10.40.1.1 test_dec_tensor . . . . .	264
10.41test_inprod_analytic.f90 File Reference . . . . .	265
10.41.1 Function/Subroutine Documentation . . . . .	265
10.41.1.1 inprod_analytic_test . . . . .	265
10.42test_MAR.f90 File Reference . . . . .	265
10.42.1 Function/Subroutine Documentation . . . . .	265
10.42.1.1 test_mar . . . . .	265
10.43test_memory.f90 File Reference . . . . .	266
10.43.1 Function/Subroutine Documentation . . . . .	266
10.43.1.1 test_memory . . . . .	266

---

10.44test_MTV_int_tensor.f90 File Reference . . . . .	266
10.44.1 Function/Subroutine Documentation . . . . .	266
10.44.1.1 test_mtv_int_tensor . . . . .	266
10.45test_MTV_sigma_tensor.f90 File Reference . . . . .	266
10.45.1 Function/Subroutine Documentation . . . . .	267
10.45.1.1 test_sigma . . . . .	267
10.46test_sqrtm.f90 File Reference . . . . .	267
10.46.1 Function/Subroutine Documentation . . . . .	267
10.46.1.1 test_sqrtm . . . . .	267
10.47test_tl_ad.f90 File Reference . . . . .	267
10.47.1 Function/Subroutine Documentation . . . . .	267
10.47.1.1 test_tl_ad . . . . .	267
10.48test_WL_tensor.f90 File Reference . . . . .	268
10.48.1 Function/Subroutine Documentation . . . . .	268
10.48.1.1 test_wl_tensor . . . . .	268
10.49tl_ad_tensor.f90 File Reference . . . . .	268
10.50util.f90 File Reference . . . . .	269
10.50.1 Function/Subroutine Documentation . . . . .	270
10.50.1.1 lcg(s) . . . . .	270
10.51WL_tensor.f90 File Reference . . . . .	270
<b>Index</b>	<b>273</b>



# Chapter 1

## Modular arbitrary-order ocean-atmosphere model: MAOOAM -- Stochastic Fortran implementation

### About

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This software is provided as supplementary material with:

- De Cruz, L., Demaeyer, J. and Vannitsem, S.: The Modular Arbitrary-Order Ocean-Atmosphere Model: M $\leftrightarrow$ AOOAM v1.0, Geosci. Model Dev., 9, 2793-2808, doi:10.5194/gmd-9-2793-2016, 2016.

for the MAOOAM original code, and with

- Demaeyer, J. and Vannitsem, S.: Comparison of stochastic parameterizations in the framework of a coupled ocean-atmosphere model, Nonlin. Processes Geophys. Discuss., 2018.

for the stochastic part.

**Please cite both articles if you use (a part of) this software for a publication.**

The authors would appreciate it if you could also send a reprint of your paper to [lesley.decruz@meteo.be](mailto:lesley.decruz@meteo.be), [jonathan.demaeyer@meteo.be](mailto:jonathan.demaeyer@meteo.be) and [svn@meteo.be](mailto:svn@meteo.be).

Consult the MAOOAM [code repository](#) for updates, and [our website](#) for additional resources.

A pdf version of this manual is available [here](#).

## Installation

The program can be installed with Makefile. We provide configuration files for two compilers : gfortran and ifort.

By default, gfortran is selected. To select one or the other, simply modify the Makefile accordingly or pass the CO←MPILER flag to make. If gfortran is selected, the code should be compiled with gfortran 4.7+ (allows for allocatable arrays in namelists). If ifort is selected, the code has been tested with the version 14.0.2 and we do not guarantee compatibility with older compiler version.

To install, unpack the archive in a folder or clone with git:

```
1 git clone https://github.com/Climdyn/MAOOAM.git
2 cd MAOOAM
```

and run:

```
1 make
```

By default, the inner products of the basis functions, used to compute the coefficients of the ODEs, are not stored in memory. If you want to enable the storage in memory of these inner products, run make with the following flag:

```
1 make RES=store
```

Depending on the chosen resolution, storing the inner products may result in a huge memory usage and is not recommended unless you need them for a specific purpose.

Remark: The command "make clean" removes the compiled files.

## Description of the files

The model tendencies are represented through a tensor called aotensor which includes all the coefficients. This tensor is computed once at the program initialization.

The following files are part of the MAOOAM model alone:

- [maooam.f90](#) : Main program.
- [aotensor\\_def.f90](#) : Tensor aotensor computation module.
- [IC\\_def.f90](#) : A module which loads the user specified initial condition.
- [inprod\\_analytic.f90](#) : Inner products computation module.
- [rk2\\_integrator.f90](#) : A module which contains the Heun integrator for the model equations.
- [rk4\\_integrator.f90](#) : A module which contains the RK4 integrator for the model equations.
- Makefile : The Makefile.
- [params.f90](#) : The model parameters module.
- [tl\\_ad\\_tensor.f90](#) : Tangent Linear (TL) and Adjoint (AD) model tensors definition module
- [rk2\\_tl\\_ad\\_integrator.f90](#) : Heun Tangent Linear (TL) and Adjoint (AD) model integrators module
- [rk4\\_tl\\_ad\\_integrator.f90](#) : RK4 Tangent Linear (TL) and Adjoint (AD) model integrators module

- `test_tl_ad.f90` : Tests for the Tangent Linear (TL) and Adjoint (AD) model versions
- `README.md` : A read me file.
- `LICENSE.txt` : The license text of the program.
- `util.f90` : A module with various useful functions.
- `tensor.f90` : Tensor utility module.
- `stat.f90` : A module for statistic accumulation.
- `params.nml` : A namelist to specify the model parameters.
- `int_params.nml` : A namelist to specify the integration parameters.
- `modeselection.nml` : A namelist to specify which spectral decomposition will be used.

with the addition of the files:

- `maooam_stoch.f90` : Stochastic implementation of MAOOAM.
- `maooam_MTV.f90` : Main program - MTV implementation for MAOOAM.
- `maooam_WL.f90` : Main program - WL implementation for MAOOAM.
- `corrmod.f90` : Unresolved variables correlation matrix initialization module.
- `corr_tensor.f90` : Correlations and derivatives for the memory term of the WL parameterization.
- `dec_tensor.f90` : Tensor resolved-unresolved components decomposition module.
- `int_comp.f90` : Utility module containing the routines to perform the integration of functions.
- `int_corr.f90` : Module to compute or load the integrals of the correlation matrices.
- `MAR.f90` : Multidimensional AutoRegressive (MAR) module to generate the correlation for the WL parameterization.
- `memory.f90` : WL parameterization memory term  $M_3$  computation module.
- `MTV_int_tensor.f90` : MTV tensors computation module.
- `MTV_sigma_tensor.f90` : MTV noise sigma matrices computation module.
- `WL_tensor.f90` : WL tensors computation module.
- `rk2_stoch_integrator.f90` : Stochastic RK2 integration routines module.
- `rk2_ss_integrator.f90` : Stochastic uncoupled resolved nonlinear and tangent linear RK2 dynamics integration module.
- `rk2_MTV_integrator.f90` : MTV RK2 integration routines module.
- `rk2_WL_integrator.f90` : WL RK2 integration routines module.
- `sf_def.f90` : Module to select the resolved-unresolved components.
- `SF.nml` : A namelist to select the resolved-unresolved components.
- `sqrt_mod.f90` : Utility module with various routine to compute matrix square root.
- `stoch_mod.f90` : Utility module containing the stochastic related routines.
- `stoch_params.f90` : Stochastic models parameters module.
- `stoch_params.nml` : A namelist to specify the stochastic models parameters.

which belong specifically to the stochastic implementation.

## MAOOAM Usage

The user first has to fill the params.nml and int\_params.nml namelist files according to their needs. Indeed, model and integration parameters can be specified respectively in the params.nml and int\_params.nml namelist files. Some examples related to already published article are available in the params folder.

The modeselection.nml namelist can then be filled :

- NBOC and NBATM specify the number of blocks that will be used in respectively the ocean and the atmosphere. Each block corresponds to a given x and y wavenumber.
- The OMS and AMS arrays are integer arrays which specify which wavenumbers of the spectral decomposition will be used in respectively the ocean and the atmosphere. Their shapes are OMS(NBOC,2) and AMS(NB $\leftarrow$ ATM,2).
- The first dimension specifies the number attributed by the user to the block and the second dimension specifies the x and the y wavenumbers.
- The VDDG model, described in Vannitsem et al. (2015) is given as an example in the archive.
- Note that the variables of the model are numbered according to the chosen order of the blocks.

The Makefile allows to change the integrator being used for the time evolution. The user should modify it according to its need. By default a RK2 scheme is selected.

Finally, the IC.nml file specifying the initial condition should be defined. To obtain an example of this configuration file corresponding to the model you have previously defined, simply delete the current IC.nml file (if it exists) and run the program :

```
./maooam
```

It will generate a new one and start with the 0 initial condition. If you want another initial condition, stop the program, fill the newly generated file and restart :

```
./maooam
```

It will generate two files :

- evol\_field.dat : the recorded time evolution of the variables.
- mean\_field.dat : the mean field (the climatology)

The tangent linear and adjoint models of MAOOAM are provided in the [tl\\_ad\\_tensor](#), [rk2\\_tl\\_ad\\_integrator](#) and [rk4\\_tl\\_ad\\_integrator](#) modules. It is documented [here](#).

## Stochastic code usage

The user first has to fill the MAOOAM model namelist files according to their needs (see the previous section). Additional namelist files for the fine tuning of the parameterization must then be filled, and some "definition" files (with the extension .def) must be provided. An example is provided with the code.

Full details over the parameterization options and definition files are available [here](#).

The program "maooam\_stoch" will generate the evolution of the full stochastic dynamics with the command:

```
./maooam_stoch
```

or any other dynamics if specified as an argument (see the header of `maooam_stoch.f90`). It will generate two files :

- `evol_field.dat` : the recorded time evolution of the variables.
- `mean_field.dat` : the mean field (the climatology)

The program "maooam\_MTV" will generate the evolution of the MTV parameterization evolution, with the command:

```
./maooam_MTV
```

It will generate three files :

- `evol_MTV.dat` : the recorded time evolution of the variables.
- `ptend_MTV.dat` : the recorded time evolution of the tendencies (used for debugging).
- `mean_field_MTV.dat` : the mean field (the climatology)

The program "maooam\_WL" will generate the evolution of the MTV parameterization evolution, with the command:

```
./maooam_WL
```

It will generate three files :

- `evol_WL.dat` : the recorded time evolution of the variables.
- `ptend_WL.dat` : the recorded time evolution of the tendencies (used for debugging).
- `mean_field_WL.dat` : the mean field (the climatology)

## MAOOAM Implementation notes

As the system of differential equations is at most bilinear in  $z_j$  ( $j = 1..n$ ),  $z$  being the array of variables, it can be expressed as a tensor contraction :

$$\frac{dz_i}{dt} = \sum_{j,k=0}^{ndim} \mathcal{T}_{i,j,k} z_k z_j$$

with  $z_0 = 1$ .

The tensor `aotensor_def::aotensor` is the tensor  $\mathcal{T}$  that encodes the differential equations is composed so that:

- $\mathcal{T}_{i,j,k}$  contains the contribution of  $dz_i/dt$  proportional to  $z_j z_k$ .
- Furthermore,  $z_0$  is always equal to 1, so that  $\mathcal{T}_{i,0,0}$  is the constant contribution to  $dz_i/dt$
- $\mathcal{T}_{i,j,0} + \mathcal{T}_{i,0,j}$  is the contribution to  $dz_i/dt$  which is linear in  $z_j$ .

Ideally, the tensor `aotensor_def::aotensor` is composed as an upper triangular matrix (in the last two coordinates).

The tensor for this model is composed in the `aotensor_def` module and uses the inner products defined in the `inprod_analytic` module.

## Stochastic code implementation notes

A stochastic version of MAOOAM and two stochastic parameterization methods (MTV and WL) are provided with this code.

The stochastic version of MAOOAM is given by

$$\frac{d\mathbf{z}}{dt} = f(\mathbf{z}) + \mathbf{q} \cdot d\mathbf{W}(t)$$

where  $d\mathbf{W}$  is a vector of standard Gaussian White noise and where several choice for  $f(\mathbf{z})$  are available. For instance, the default choice is to use the full dynamics:

$$f(\mathbf{z}) = \sum_{j,k=0}^{ndim} \mathcal{T}_{i,j,k} z_k z_j.$$

The implementation uses the tensorial framework described above and add some noise to it. This stochastic version is further detailed [here](#).

The MTV parameterization for MAOOAM is given by

$$\frac{dx}{dt} = F_x(\mathbf{x}) + \frac{1}{\delta} R(\mathbf{x}) + G(\mathbf{x}) + \sqrt{2} \sigma(\mathbf{x}) \cdot d\mathbf{W}$$

where  $x$  is the set of resolved variables and  $d\mathbf{W}$  is a vector of standard Gaussian White noise.  $F_x$  is the set of tendencies of resolved system alone and  $\delta$  is the timescale separation parameter.

The WL parameterizations for MAOOAM is given by

$$\frac{dx}{dt} = F_x(\mathbf{x}) + \varepsilon M_1(\mathbf{x}) + \varepsilon^2 M_2(\mathbf{x}, t) + \varepsilon^2 M_3(\mathbf{x}, t)$$

where  $\varepsilon$  is the resolved-unresolved components coupling strength and where the different terms  $M_i$  account for different effect.

The implementation for these two approaches uses the tensorial framework described above, with the addition of new tensors to account for the terms  $R, G, \sigma, M_1, M_2, M_3$ . They are detailed more completely [here](#).

## Final Remarks

The authors would like to thank Kris for help with the lua2fortran project. It has greatly reduced the amount of (error-prone) work.

No animals were harmed during the coding process.



## Chapter 2

# Modular arbitrary-order ocean-atmosphere model: The Tangent Linear and Adjoint model

### Description :

The Tangent Linear and Adjoint model model are implemented in the same way as the nonlinear model, with a tensor storing the different terms. The Tangent Linear (TL) tensor  $\mathcal{T}_{i,j,k}^{TL}$  is defined as:

$$\mathcal{T}_{i,j,k}^{TL} = \mathcal{T}_{i,k,j} + \mathcal{T}_{i,j,k}$$

while the Adjoint (AD) tensor  $\mathcal{T}_{i,j,k}^{AD}$  is defined as:

$$\mathcal{T}_{i,j,k}^{AD} = \mathcal{T}_{j,k,i} + \mathcal{T}_{j,i,k}.$$

where  $\mathcal{T}_{i,j,k}$  is the tensor of the nonlinear model.

These two tensors are used to compute the trajectories of the models, with the equations

$$\frac{d\delta z_i}{dt} = \sum_{j=1}^{ndim} \sum_{k=0}^{ndim} \mathcal{T}_{i,j,k}^{TL} y_k^* \delta z_j.$$

$$-\frac{d\delta z_i}{dt} = \sum_{j=1}^{ndim} \sum_{k=0}^{ndim} \mathcal{T}_{i,j,k}^{AD} y_k^* \delta z_j.$$

where  $y^*$  is the point where the Tangent model is defined (with  $z_0^* = 1$ ).

### Implementation :

The two tensors are implemented in the module `tl_ad_tensor` and must be initialized (after calling `params::init_<→params` and `aotensor_def::aotensor`) by calling `tl_ad_tensor::init_tltensor()` and `tl_ad_tensor::init_adtensor()`. The tendencies are then given by the routine `tl(t,ystar,deltay,buf)` and `ad(t,ystar,deltay,buf)`. An integrator with the Heun method is available in the module `rk2_tl_ad_integrator` and a fourth-order Runge-Kutta integrator in `rk4_tl_ad_integrator`. An example on how to use it can be found in the test file `test_tl_ad.f90`



## Chapter 3

# Modular arbitrary-order ocean-atmosphere model: The MTV and WL parameterizations

### The stochastic version of MAOOAM

The stochastic version of MAOOAM is given by

$$\frac{dz}{dt} = f(z) + q \cdot dW(t)$$

where  $dW$  is a vector of standard Gaussian White noise and where several choice for  $f(z)$  are available. For instance, the default choice is to use the full dynamics:

$$f(z) = \sum_{j,k=0}^{ndim} \mathcal{T}_{i,j,k} z_k z_j.$$

The implementation uses thus the tensorial framework of MAOOAM and add some noise to it. To study parameterization methods in MAOOAM, the models variables  $z$  is divided in two components: the resolved component  $x$  and the unresolved component  $y$  (see below for more details).

Since MAOOAM is a ocean-atmosphere model, it can be decomposed further into oceanic and atmospheric components:

$$z = \{x_a, x_o, y_a, y_o\}$$

and in the present implementation, the noise amplitude can be set in each component:

$$\frac{dx_a}{dt} = f_{x,a}(z) + q_{x,a} \cdot dW_{x,a}(t)$$

$$\frac{dx_o}{dt} = f_{x,o}(z) + q_{x,o} \cdot dW_{x,o}(t)$$

$$\frac{dy_a}{dt} = f_{y,a}(z) + q_{y,a} \cdot dW_{y,a}(t)$$

$$\frac{dy_o}{dt} = f_{y,o}(z) + q_{y,o} \cdot dW_{y,o}(t)$$

through the parameters `stoch_params::q_ar`, `stoch_params::q_au`, `stoch_params::q_or` and `stoch_params::q_ou`.

## The resolved-unresolved components

Due to the decomposition into resolved variables  $\mathbf{x}$  and unresolved variables  $\mathbf{y}$ , the equation of the MAOOAM model can be rewritten:

$$\begin{aligned}\frac{d\mathbf{x}}{dt} &= \mathbf{H}^x + \mathbf{L}^{xx} \cdot \mathbf{x} + \mathbf{L}^{xy} \cdot \mathbf{y} + \mathbf{B}^{xxx} : \mathbf{x} \otimes \mathbf{x} + \mathbf{B}^{xxy} : \mathbf{x} \otimes \mathbf{y} + \mathbf{B}^{xyy} : \mathbf{y} \otimes \mathbf{y} + \mathbf{q}_x \cdot d\mathbf{W}_x \\ \frac{d\mathbf{y}}{dt} &= \mathbf{H}^y + \mathbf{L}^{yx} \cdot \mathbf{x} + \mathbf{L}^{yy} \cdot \mathbf{y} + \mathbf{B}^{yxx} : \mathbf{x} \otimes \mathbf{x} + \mathbf{B}^{yxy} : \mathbf{x} \otimes \mathbf{y} + \mathbf{B}^{yyy} : \mathbf{y} \otimes \mathbf{y} + \mathbf{q}_y \cdot d\mathbf{W}_y\end{aligned}$$

where  $\mathbf{q}_x = \{\mathbf{q}_{x,a}, \mathbf{q}_{x,o}\}$  and  $\mathbf{q}_y = \{\mathbf{q}_{y,a}, \mathbf{q}_{y,o}\}$ . We have thus also  $d\mathbf{W}_x = \{d\mathbf{W}_{x,a}, d\mathbf{W}_{x,o}\}$  and  $d\mathbf{W}_y = \{d\mathbf{W}_{y,a}, d\mathbf{W}_{y,o}\}$ . The various terms of the equations above are accessible in the `dec_tensor` module. To specify which variables belong to the resolved (unresolved) component, the user must fill the SF.nml namelist file by setting the component of the vector `sf_def::sf` to 0 (1). This file must be filled before starting any of the stochastic and parameterization codes. If this file is not present, launch one of the programs. It will generate a new SF.nml file and then abort.

The purpose of the parameterization is to reduce the  $\mathbf{x}$  equation by closing it while keeping the statistical properties of the full system. To apply the parameterizations proposed in this implementation, we consider a modified version of the equation above:

$$\begin{aligned}\frac{d\mathbf{x}}{dt} &= F_x(\mathbf{x}) + \mathbf{q}_x \cdot d\mathbf{W}_x + \frac{\varepsilon}{\delta} \Psi_x(\mathbf{x}, \mathbf{y}) \\ \frac{d\mathbf{y}}{dt} &= \frac{1}{\delta^2} \left( F_y(\mathbf{y}) + \delta \mathbf{q}_y \cdot d\mathbf{W}_y \right) + \frac{\varepsilon}{\delta} \Psi_y(\mathbf{x}, \mathbf{y})\end{aligned}$$

where  $\varepsilon$  is the resolved-unresolved components coupling strength given by the parameter `stoch_params::eps_pert`.  $\delta$  is the timescale separation parameter given by the parameter `stoch_params::tdelta`. By setting those to 1, one recover the first equations above.

The function  $\Psi_x$  includes all the  $\mathbf{x}$  terms, and thus  $F_x$  and  $\Psi_x$  are unequivocally defined. On the other hand, depending on the value of the parameter `stoch_params::mode`, the terms regrouped in the function  $F_y$  can be different. Indeed, if `stoch_params::mode` is set to

- 'qfst', then:

$$F_y(\mathbf{y}) = \mathbf{B}^{yyy} : \mathbf{y} \otimes \mathbf{y}$$

- 'ures', then:

$$F_y(\mathbf{y}) = \mathbf{H}^y + \mathbf{L}^{yy} \cdot \mathbf{y} + \mathbf{B}^{yyy} : \mathbf{y} \otimes \mathbf{y}$$

However, for the WL parameterization, this parameter must be set to 'ures' by definition. See the article accompanying this code for more details.

## The MTV parameterization

This parameterization is also called homogenization. Its acronym comes from the names of the authors that proposed this approach for climate modes: Majda, Timofeyev and Vanden Eijnden (Majda et al., 2001). It is given by

$$\frac{d\mathbf{x}}{dt} = F_X(\mathbf{x}) + \frac{1}{\delta} R(\mathbf{x}) + G(\mathbf{x}) + \sqrt{2} \sigma(\mathbf{x}) \cdot d\mathbf{W}$$

where  $\mathbf{x}$  is the set of resolved variables and  $d\mathbf{W}$  is a vector of standard Gaussian White noise.  $F_x$  is the set of tendencies of resolved system alone and  $\delta$  is the timescale separation parameter.

## Correlations specification

The ingredients needed to compute the terms  $R, G, \sigma$  of this parametrization are the unresolved variables covariance matrix and the integrated correlation matrices. The unresolved variables covariance matrix is given by

$$\sigma_y = \langle \mathbf{y} \otimes \mathbf{y} \rangle$$

and is present in the implementation through the matrices `corrmod::corr_i` and `corrmod::corr_i_full`. Their inverses are also available through `corrmod::inv_corr_i` and `corrmod::inv_corr_i_full`. The integrated correlation matrices are given by

$$\Sigma = \int_0^\infty ds \langle \mathbf{y} \otimes \mathbf{y}^s \rangle$$

$$\Sigma_2 = \int_0^\infty ds (\langle \mathbf{y} \otimes \mathbf{y}^s \rangle \otimes \langle \mathbf{y} \otimes \mathbf{y}^s \rangle)$$

and is present in the implementation through the matrices `int_corr::corrint` and `int_corr::corr2int`.

These matrices are computed from the correlation matrix  $\langle \mathbf{y} \otimes \mathbf{y}^s \rangle$  which is accessible through the function `corrmod::corrcomp`. For instance, the covariance matrix  $\sigma_y$  is then simply the correlation matrix at the lagtime 0, and  $\Sigma$  and  $\Sigma_2$  can be computed via integration over the lagtime.

There exists three different ways to load the correlation matrix, specified by the value of the parameters `stoch_params::load_mode` and `stoch_params::int_corr_mode`. The `stoch_params::load_mode` specify how the correlation matrix is loaded can take three different values:

- 'defi': from an analytical definition encoded in the corrmod module function `corrmod::corrcomp_from_def`.
- 'spli': from a spline definition file 'corrspline.def'.
- 'expo': from a fit with exponentials definition file 'correxpo.def'

The `stoch_params::int_corr_mode` specify how the correlation are integrated and can take two different values:

- 'file': Integration results provided by files 'corrint.def' and 'corr2int.def'
- 'prog': Integration computed directly by the program with the correlation matrix. Write 'corrint.def' and 'corr2int.def' files to be reused later.

These parameters can be set up in the namelist file `stoch_params.nml`. Examples of the ".def" files specifying the integrals are provided with the code.

## Other MTV setup parameters

Some additional parameters complete the options possible for the MTV parameters :

- `stoch_params::mnuti` : Multiplicative noise update time interval – Time interval over which the matrix  $\sigma(x)$  is updated.
- `stoch_params::t_trans_stoch` : Transient period of the stochastic model.
- `stoch_params::maxint` : Specify the upper limit of the numerical integration if `stoch_params::int_corr_mode` is set to 'prog'.

## Definition files

The following definition files are needed by the parameterization, depending on the value of the parameters described above. Examples of those files are joined to the code. The files include:

- 'mean.def' : Mean  $\langle \mathbf{y} \rangle$  of the unresolved variables.
- 'correxp.def': Coefficients  $a_k$  of the fit of the elements of the correlations matrix  $\langle \mathbf{y} \otimes \mathbf{y}^s \rangle$  with the function

$$a_4 + a_0 \exp\left(-\frac{s}{a_1}\right) \cos(a_2 s + a_3)$$

where  $t$  is the lag-time and  $\tau$  is the decorrelation time. Used if `stoch_params::load_mode` is set to 'expo'.

- 'corrspline.def': Coefficients  $b_k$  of the spline used to model the elements of the correlation matrix  $\langle \mathbf{y} \otimes \mathbf{y}^s \rangle$ . Used if `stoch_params::load_mode` is set to 'spli'.
- 'corrint.def': File holding the matrix  $\Sigma$ . Used if `stoch_params::int_corr_mode` is set to 'file'.
- 'corr2int.def': File holding the matrix  $\Sigma_2$ .

The various terms are then constructed according to these definition files. More details on the format of the definition files can be found [here](#).

## The WL parameterization

This parameterization is based on the Ruelle response theory. Its acronym comes from the names of the authors that proposed this approach: Wouters and Lucarini (Wouters and Lucarini, 2012). It is given by

$$\frac{d\mathbf{x}}{dt} = F_x(\mathbf{x}) + \varepsilon M_1(\mathbf{x}) + \varepsilon^2 M_2(\mathbf{x}, t) + \varepsilon^2 M_3(\mathbf{x}, t)$$

where  $\varepsilon$  is the resolved-unresolved components coupling strength and where the different terms  $M_i$  account for average, correlation and memory effects.

### Correlations specification

The ingredients needed to compute the  $M_i$  terms of this parametrization are the unresolved variable covariance matrix  $\langle \mathbf{y} \otimes \mathbf{y} \rangle$  and correlation matrix  $\langle \mathbf{y} \otimes \mathbf{y}^s \rangle$ . The unresolved variables covariance matrix is given by

$$\boldsymbol{\sigma}_y = \langle \mathbf{y} \otimes \mathbf{y} \rangle$$

and is present in the implementation through the matrices `corrmod::corr_i` and `corrmod::corr_i_full`. Their inverses are also available through `corrmod::inv_corr_i` and `corrmod::inv_corr_i_full`.

The correlation matrix  $\langle \mathbf{y} \otimes \mathbf{y}^s \rangle$  is accessible through the function `corrmod::corrcomp`.

As for the MTV case, there exists three different ways to load the correlation matrix, specified by the value of the parameters `stoch_params::load_mode` and `stoch_params::int_corr_mode`. The `stoch_params::load_mode` specify how the correlation matrix is loaded can take three different values:

- 'defi': from an analytical definition encoded in the corrmod module function `corrmod::corrcomp_from_def` .
- 'spli': from a spline definition file 'corrspline.def' .
- 'expo': from a fit with exponentials definition file 'correxp.def'

The correlation term  $M_2$  is emulated by an order  $m$  multidimensional AutoRegressive (MAR) process:

$$\mathbf{u}_n = \sum_{i=1}^m \mathbf{u}_{n-i} \cdot \mathbf{W}_i + \mathbf{Q} \cdot \boldsymbol{\xi}_n$$

of which the  $\mathbf{W}_i$  and  $\mathbf{Q}$  matrices are also needed (the  $\boldsymbol{\xi}_n$  are vectors of standard Gaussian white noise). It is implemented in the MAR module.

## Other WL setup parameters

Some additional parameters complete the options possible for the WL parameters :

- `stoch_params::muti` : Memory term  $M_3$  update time interval.
- `stoch_params::t_trans_stoch` : Transient period of the stochastic model.
- `stoch_params::meml` : Time over which the memory kernel is numerically integrated.
- `stoch_params::t_trans_mem` : Transient period of the stochastic model to initialize the memory term.
- `stoch_params::dts` : Intrinsic resolved dynamics time step.
- `stoch_params::x_int_mode` : Integration mode for the resolved component (not used for the moment – must be set to 'reso').

Note that the `stoch_params::mode` must absolutely be set to 'ures', by definition.

## Definition files

The following definition files are needed by the parameterization, depending on the value of the parameters described above. Examples of those files are joined to the code. The files include:

- 'correxp.def': Coefficients  $a_k$  of the fit of the elements of the correlations matrix  $\langle \mathbf{y} \otimes \mathbf{y}^s \rangle$  with the function
$$a_4 + a_0 \exp\left(-\frac{s}{a_1}\right) \cos(a_2 s + a_3)$$
where  $t$  is the lag-time and  $\tau$  is the decorrelation time. Used if `stoch_params::load_mode` is set to 'expo'.
- 'corrspline.def': Coefficients  $b_k$  of the spline used to model the elements of the correlation matrix  $\langle \mathbf{y} \otimes \mathbf{y}^s \rangle$ . Used if `stoch_params::load_mode` is set to 'spli'.
- 'MAR\_R\_params.def': File specifying the  $\mathbf{R} = \mathbf{Q}^2$  matrix for the MAR.
- 'MAR\_W\_params.def': File specifying the  $\mathbf{W}_i$  matrices for the MAR.

The various terms are then constructed according to these definition files. More details on the format of the definition files can be found [here](#).

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Please see the main article for the full list of references:

- Demaeyer, J. and Vannitsem, S.: Comparison of stochastic parameterizations in the framework of a coupled ocean-atmosphere model, *Nonlin. Processes Geophys. Discuss.*, 2018.



## Chapter 4

# Modular arbitrary-order ocean-atmosphere model: Definition files formats

This page describes the format of the definition files needed by the stochastic model.

### MTV parameterization

The following definition files are needed by the MTV parameterization. Examples of those files are joined to the code. The files include:

- 'mean.def' : Mean  $\langle \mathbf{y} \rangle$  of the unresolved variables.
  - **Format:** one line per  $\langle y_i \rangle$  value
- 'correxp.def': Coefficients  $a_k$  of the fit of the elements of the correlations matrix  $\langle \mathbf{y} \otimes \mathbf{y}^s \rangle$  with the function

$$a_4 + a_0 \exp\left(-\frac{s}{a_1}\right) \cos(a_2 s + a_3)$$

where  $t$  is the lag-time and  $\tau$  is the decorrelation time.

- **Format:** First line is two numbers: the number of unresolved variables and the value of `stoch_params::maxint` to be used (range of validity of the fit).

Then each line specify the fit of an element  $i, j$  of the matrix  $\langle \mathbf{y} \otimes \mathbf{y}^s \rangle$  as follow:

$$i, j, a_0, a_1, a_2, a_3$$

- Used if `stoch_params::load_mode` is set to 'expo'.
- 'corrspline.def': Coefficients  $b_k$  of the spline used to model the elements of the correlation matrix  $\langle \mathbf{y} \otimes \mathbf{y}^s \rangle$ .
  - **Format:** First line is two numbers: the number of unresolved variables and the number of points used. Second line is the times  $\tau_k$  of the points in timeunits.  
Then  $i \times j$  sequences of 3 lines occurs as follow:
    1.  $i, j$
    2. Values of  $\langle \mathbf{y} \otimes \mathbf{y}^s \rangle_{i,j}$  at  $\tau_k$
    3. Coefficients  $b_k$  of the spline giving the second derivative of the interpolating function at  $\tau$
  - Used if `stoch_params::load_mode` is set to 'spli'.
- 'corrint.def': File holding the matrix  $\Sigma = \int_0^\infty ds \langle \mathbf{y} \otimes \mathbf{y}^s \rangle$ .

- **Format:** Matrix in a Fortran-contiguous format
- Used if `stoch_params::int_corr_mode` is set to 'file'.
- 'corr2int.def': File holding the matrix  $\Sigma_2 = \int_0^\infty ds (\langle \mathbf{y} \otimes \mathbf{y}^s \rangle \otimes \langle \mathbf{y} \otimes \mathbf{y}^s \rangle)$ .
- **Format:** Matrix in a sparse format, `params::ndim` sequences with
  1. a first line with the first index  $i$  of the matrix and then the number of entries the sub-matrix  $\Sigma_{2,i,\dots}$ .  
has
  2. a list of the entries of the matrix in the format:

$$i, j, k, l, v$$

where  $v$  is the value of the entry

## WL parameterization

The following definition files are needed by the parameterization, depending on the value of the parameters described above. Examples of those files are joined to the code. The files include:

- 'mean.def' : Mean  $\langle \mathbf{y} \rangle$  of the unresolved variables.
  - **Format:** one line per  $\langle y_i \rangle$  value
- 'correxp.def': Coefficients  $a_k$  of the fit of the elements of the correlations matrix  $\langle \mathbf{y} \otimes \mathbf{y}^s \rangle$  with the function

$$a_4 + a_0 \exp\left(-\frac{s}{a_1}\right) \cos(a_2 s + a_3)$$

where  $t$  is the lag-time and  $\tau$  is the decorrelation time.

- **Format:** First line is two numbers: the number of unresolved variables and the value of `stoch_params::maxint` to be used (range of validity of the fit).
- Then each line specify the fit of an element  $i, j$  of the matrix  $\langle \mathbf{y} \otimes \mathbf{y}^s \rangle$  as follow:

$$i, j, a_0, a_1, a_2, a_3$$

- Used if `stoch_params::load_mode` is set to 'expo'.
- 'corrspline.def': Coefficients  $b_k$  of the spline used to model the elements of the correlation matrix  $\langle \mathbf{y} \otimes \mathbf{y}^s \rangle$ .
  - **Format:** First line is two numbers: the number of unresolved variables and the number of points used. Second line is the times  $\tau_k$  of the points in timeunits.
  - Then  $i \times j$  sequences of 3 lines occurs as follow:
    1.  $i, j$
    2. Values of  $\langle \mathbf{y} \otimes \mathbf{y}^s \rangle_{i,j}$  at  $\tau_k$
    3. Coefficients  $b_k$  of the spline giving the second derivative of the interpolating function at  $\tau$
  - Used if `stoch_params::load_mode` is set to 'spli'.
- 'MAR\_R\_params.def': File specifying the  $\mathbf{R} = Q^2$  matrix for the MAR.
  - **Format:** Matrix in a Fortran-contiguous format
- 'MAR\_W\_params.def': File specifying the  $\mathbf{W}_i$  matrices for the MAR.
  - **Format:** Matrix in a Fortran-contiguous format

# Chapter 5

## Modules Index

### 5.1 Modules List

Here is a list of all modules with brief descriptions:

<a href="#">aotensor_def</a>	The equation tensor for the coupled ocean-atmosphere model with temperature which allows for an extensible set of modes in the ocean and in the atmosphere . . . . .	25
<a href="#">corr_tensor</a>	Module to compute the correlations and derivatives used to compute the memory term of the WL parameterization . . . . .	29
<a href="#">corrmod</a>	Module to initialize the correlation matrix of the unresolved variables . . . . .	33
<a href="#">dec_tensor</a>	The resolved-unresolved components decomposition of the tensor . . . . .	44
<a href="#">ic_def</a>	Module to load the initial condition . . . . .	55
<a href="#">inprod_analytic</a>	Inner products between the truncated set of basis functions for the ocean and atmosphere streamfunction fields. These are partly calculated using the analytical expressions from Cehelsky, P., & Tung, K. K. : Theories of multiple equilibria and weather regimes-A critical reexamination. Part II: Baroclinic two-layer models. Journal of the atmospheric sciences, 44(21), 3282-3303, 1987 . . . . .	57
<a href="#">int_comp</a>	Utility module containing the routines to perform the integration of functions . . . . .	68
<a href="#">int_corr</a>	Module to compute or load the integrals of the correlation matrices . . . . .	72
<a href="#">integrator</a>	Module with the integration routines . . . . .	76
<a href="#">mar</a>	Multidimensional Autoregressive module to generate the correlation for the WL parameterization . . . . .	80
<a href="#">memory</a>	Module that compute the memory term $M_3$ of the WL parameterization . . . . .	84
<a href="#">mtv_int_tensor</a>	The MTV tensors used to integrate the MTV model . . . . .	88
<a href="#">params</a>	The model parameters module . . . . .	97
<a href="#">rk2_mtv_integrator</a>	Module with the MTV rk2 integration routines . . . . .	112

<a href="#">rk2_ss_integrator</a>	Module with the stochastic uncoupled resolved nonlinear and tangent linear rk2 dynamics integration routines . . . . .	120
<a href="#">rk2_stoch_integrator</a>	Module with the stochastic rk2 integration routines . . . . .	124
<a href="#">rk2_wl_integrator</a>	Module with the WL rk2 integration routines . . . . .	128
<a href="#">sf_def</a>	Module to select the resolved-unresolved components . . . . .	134
<a href="#">sigma</a>	The MTV noise sigma matrices used to integrate the MTV model . . . . .	139
<a href="#">sqrt_mod</a>	Utility module with various routine to compute matrix square root . . . . .	143
<a href="#">stat</a>	Statistics accumulators . . . . .	150
<a href="#">stoch_mod</a>	Utility module containing the stochastic related routines . . . . .	152
<a href="#">stoch_params</a>	The stochastic models parameters module . . . . .	156
<a href="#">tensor</a>	Tensor utility module . . . . .	162
<a href="#">tl_ad_integrator</a>	Tangent Linear (TL) and Adjoint (AD) model versions of MAOOAM. Integrators module . . . . .	192
<a href="#">tl_ad_tensor</a>	Tangent Linear (TL) and Adjoint (AD) model versions of MAOOAM. Tensors definition module .	195
<a href="#">util</a>	Utility module . . . . .	203
<a href="#">wl_tensor</a>	The WL tensors used to integrate the model . . . . .	209

# Chapter 6

## Data Type Index

### 6.1 Data Types List

Here are the data types with brief descriptions:

<a href="#">inprod_analytic::atm_tensors</a>	Type holding the atmospheric inner products tensors . . . . .	221
<a href="#">inprod_analytic::atm_wavenum</a>	Atmospheric bloc specification type . . . . .	222
<a href="#">tensor::coolist</a>	Coordinate list. Type used to represent the sparse tensor . . . . .	224
<a href="#">tensor::coolist4</a>	4d coordinate list. Type used to represent the rank-4 sparse tensor . . . . .	224
<a href="#">tensor::coolist_elem</a>	Coordinate list element type. Elementary elements of the sparse tensors . . . . .	225
<a href="#">tensor::coolist_elem4</a>	4d coordinate list element type. Elementary elements of the 4d sparse tensors . . . . .	226
<a href="#">inprod_analytic::ocean_tensors</a>	Type holding the oceanic inner products tensors . . . . .	227
<a href="#">inprod_analytic::ocean_wavenum</a>	Oceanic bloc specification type . . . . .	229



# Chapter 7

## File Index

### 7.1 File List

Here is a list of all files with brief descriptions:

aotensor_def.f90	231
corr_tensor.f90	232
corrmod.f90	232
dec_tensor.f90	233
ic_def.f90	235
inprod_analytic.f90	235
int_comp.f90	237
int_corr.f90	237
maooam.f90	242
maooam_MTV.f90	243
maooam_stoch.f90	243
maooam_WL.f90	244
MAR.f90	244
memory.f90	245
MTV_int_tensor.f90	246
MTV_sigma_tensor.f90	247
params.f90	248
rk2_integrator.f90	251
rk2_MTV_integrator.f90	251
rk2_ss_integrator.f90	252
rk2_stoch_integrator.f90	253
rk2_tl_ad_integrator.f90	254
rk2_WL_integrator.f90	254
rk4_integrator.f90	255
rk4_tl_ad_integrator.f90	256
sf_def.f90	256
sqrt_mod.f90	257
stat.f90	258
stoch_mod.f90	258
stoch_params.f90	259
tensor.f90	260
test_aotensor.f90	263
test_corr.f90	263
test_corr_tensor.f90	264
test_dec_tensor.f90	264

test_inprod_analytic.f90	265
test_MAR.f90	265
test_memory.f90	266
test_MTV_int_tensor.f90	266
test_MTV_sigma_tensor.f90	266
test_sqrtm.f90	267
test_tl_ad.f90	267
test_WL_tensor.f90	268
tl_ad_tensor.f90	268
util.f90	269
WL_tensor.f90	270

# Chapter 8

## Module Documentation

### 8.1 aotensor\_def Module Reference

The equation tensor for the coupled ocean-atmosphere model with temperature which allows for an extensible set of modes in the ocean and in the atmosphere.

#### Functions/Subroutines

- integer function `psi` (i)  
*Translate the  $\psi_{o,i}$  coefficients into effective coordinates.*
- integer function `theta` (i)  
*Translate the  $\theta_{o,i}$  coefficients into effective coordinates.*
- integer function `a` (i)  
*Translate the  $\psi_{o,i}$  coefficients into effective coordinates.*
- integer function `t` (i)  
*Translate the  $\delta T_{o,i}$  coefficients into effective coordinates.*
- integer function `kdelta` (i, j)  
*Kronecker delta function.*
- subroutine `coeff` (i, j, k, v)  
*Subroutine to add element in the aotensor  $\mathcal{T}_{i,j,k}$  structure.*
- subroutine `add_count` (i, j, k, v)  
*Subroutine to count the elements of the aotensor  $\mathcal{T}_{i,j,k}$ . Add +1 to `count_elems(i)` for each value that is added to the tensor i-th component.*
- subroutine `compute_aotensor` (func)  
*Subroutine to compute the tensor aotensor.*
- subroutine, public `init_aotensor`  
*Subroutine to initialise the aotensor tensor.*

#### Variables

- integer, dimension(:), allocatable `count_elems`  
*Vector used to count the tensor elements.*
- real(kind=8), parameter `real_eps` = 2.2204460492503131e-16  
*Epsilon to test equality with 0.*
- type(`coolist`), dimension(:), allocatable, public `aotensor`  
 $\mathcal{T}_{i,j,k}$  - Tensor representation of the tendencies.

### 8.1.1 Detailed Description

The equation tensor for the coupled ocean-atmosphere model with temperature which allows for an extensible set of modes in the ocean and in the atmosphere.

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#### Remarks

Generated Fortran90/95 code from aotensor.lua

### 8.1.2 Function/Subroutine Documentation

#### 8.1.2.1 integer function aotensor\_def::a ( integer i ) [private]

Translate the  $\psi_{o,i}$  coefficients into effective coordinates.

Definition at line 76 of file aotensor\_def.f90.

```
76      INTEGER :: i,a
77      a = i + 2 * natm
```

#### 8.1.2.2 subroutine aotensor\_def::add\_count ( integer, intent(in) i, integer, intent(in) j, integer, intent(in) k, real(kind=8), intent(in) v ) [private]

Subroutine to count the elements of the aotensor  $\mathcal{T}_{i,j,k}$ . Add +1 to count\_elems(i) for each value that is added to the tensor i-th component.

#### Parameters

<i>i</i>	tensor <i>i</i> index
<i>j</i>	tensor <i>j</i> index
<i>k</i>	tensor <i>k</i> index
<i>v</i>	value that will be added

Definition at line 124 of file aotensor\_def.f90.

```
124      INTEGER, INTENT(IN) :: i,j,k
125      REAL(KIND=8), INTENT(IN) :: v
126      IF (abs(v) .ge. real_eps) count_elems(i)=count_elems(i)+1
```

#### 8.1.2.3 subroutine aotensor\_def::coeff ( integer, intent(in) i, integer, intent(in) j, integer, intent(in) k, real(kind=8), intent(in) v ) [private]

Subroutine to add element in the aotensor  $\mathcal{T}_{i,j,k}$  structure.

### Parameters

<i>i</i>	tensor <i>i</i> index
<i>j</i>	tensor <i>j</i> index
<i>k</i>	tensor <i>k</i> index
<i>v</i>	value to add

Definition at line 99 of file aotensor\_def.f90.

```

99      INTEGER, INTENT(IN) :: i,j,k
100     REAL(KIND=8), INTENT(IN) :: v
101     INTEGER :: n
102     IF (.NOT. ALLOCATED(aotensor)) stop "*** coeff routine : tensor not yet allocated ***"
103     IF (.NOT. ALLOCATED(aotensor(i)%elems)) stop "*** coeff routine : tensor not yet allocated ***"
104     IF (abs(v) .ge. real_eps) THEN
105       n=(aotensor(i)%nelems)+1
106       IF (j .LE. k) THEN
107         aotensor(i)%elems(n)%j=j
108         aotensor(i)%elems(n)%k=k
109       ELSE
110         aotensor(i)%elems(n)%j=k
111         aotensor(i)%elems(n)%k=j
112       END IF
113       aotensor(i)%elems(n)%v=v
114       aotensor(i)%nelems=n
115     END IF

```

#### 8.1.2.4 subroutine aotensor\_def::compute\_aotensor( external func ) [private]

Subroutine to compute the tensor [aotensor](#).

### Parameters

<i>func</i>	External function to be used
-------------	------------------------------

Definition at line 132 of file aotensor\_def.f90.

#### 8.1.2.5 subroutine, public aotensor\_def::init\_aotensor( )

Subroutine to initialise the [aotensor](#) tensor.

### Remarks

This procedure will also call [params::init\\_params\(\)](#) and [inprod\\_analytic::init\\_inprod\(\)](#). It will finally call [inprod\\_analytic::deallocate\\_inprod\(\)](#) to remove the inner products, which are not needed anymore at this point.

Definition at line 203 of file aotensor\_def.f90.

```

203     INTEGER :: i
204     INTEGER :: allocstat
205
206     CALL init_params ! Iniatialise the parameter
207
208     CALL init_inprod ! Initialise the inner product tensors
209
210     ALLOCATE(aotensor(ndim),count_elems(ndim), stat=allocstat)
211     IF (allocstat /= 0) stop "*** Not enough memory ! ***"

```

```

212      count_elems=0
213
214      CALL compute_aotensor(add_count)
215
216      DO i=1,ndim
217          ALLOCATE(aotensor(i)%elems(count_elems(i)), stat=allocstat)
218          IF (allocstat /= 0) stop "*** Not enough memory ! ***"
219      END DO
220
221      DEALLOCATE(count_elems, stat=allocstat)
222      IF (allocstat /= 0) stop "*** Deallocation problem ! ***"
223
224      CALL compute_aotensor(coeff)
225
226      CALL simplify(aotensor)
227

```

#### 8.1.2.6 integer function aotensor\_def::kdelta ( integer $i$ , integer $j$ ) [private]

Kronecker delta function.

Definition at line 88 of file aotensor\_def.f90.

```

88      INTEGER :: i,j,kdelta
89      kdelta=0
90      IF (i == j) kdelta = 1

```

#### 8.1.2.7 integer function aotensor\_def::psi ( integer $i$ ) [private]

Translate the  $\psi_{a,i}$  coefficients into effective coordinates.

Definition at line 64 of file aotensor\_def.f90.

```

64      INTEGER :: i,psi
65      psi = i

```

#### 8.1.2.8 integer function aotensor\_def::t ( integer $i$ ) [private]

Translate the  $\delta T_{o,i}$  coefficients into effective coordinates.

Definition at line 82 of file aotensor\_def.f90.

```

82      INTEGER :: i,t
83      t = i + 2 * natm + noc

```

#### 8.1.2.9 integer function aotensor\_def::theta ( integer $i$ ) [private]

Translate the  $\theta_{a,i}$  coefficients into effective coordinates.

Definition at line 70 of file aotensor\_def.f90.

```

70      INTEGER :: i,theta
71      theta = i + natm

```

### 8.1.3 Variable Documentation

#### 8.1.3.1 type(coolist), dimension(:), allocatable, public aotensor\_def::aotensor

$\mathcal{T}_{i,j,k}$  - Tensor representation of the tendencies.

Definition at line 45 of file aotensor\_def.f90.

```
45      TYPE(coolist), DIMENSION(:), ALLOCATABLE, PUBLIC :: aotensor
```

#### 8.1.3.2 integer, dimension(:), allocatable aotensor\_def::count\_elems [private]

Vector used to count the tensor elements.

Definition at line 37 of file aotensor\_def.f90.

```
37      INTEGER, DIMENSION(:), ALLOCATABLE :: count_elems
```

#### 8.1.3.3 real(kind=8), parameter aotensor\_def::real\_eps = 2.2204460492503131e-16 [private]

Epsilon to test equality with 0.

Definition at line 40 of file aotensor\_def.f90.

```
40      REAL(KIND=8), PARAMETER :: real_eps = 2.2204460492503131e-16
```

## 8.2 corr\_tensor Module Reference

Module to compute the correlations and derivatives used to compute the memory term of the WL parameterization.

### Functions/Subroutines

- subroutine, public init\_corr\_tensor

*Subroutine to initialise the correlations tensors.*

## Variables

- type([coolist](#)), dimension(:, :, :), allocatable, public **yy**  
*Coolist holding the  $\langle Y \otimes Y^s \rangle$  terms.*
- type([coolist](#)), dimension(:, :, :), allocatable, public **dy**  
*Coolist holding the  $\langle \partial_Y \otimes Y^s \rangle$  terms.*
- type([coolist](#)), dimension(:, :, :), allocatable, public **ydy**  
*Coolist holding the  $\langle Y \otimes \partial_Y \otimes Y^s \rangle$  terms.*
- type([coolist](#)), dimension(:, :, :), allocatable, public **dyy**  
*Coolist holding the  $\langle \partial_Y \otimes Y^s \otimes Y^s \rangle$  terms.*
- type([coolist4](#)), dimension(:, :, :), allocatable, public **ydyy**  
*Coolist holding the  $\langle Y \otimes \partial_Y \otimes Y^s \otimes Y^s \rangle$  terms.*
- real(kind=8), dimension(:, :), allocatable **dumb\_vec**  
*Dumb vector to be used in the calculation.*
- real(kind=8), dimension(:, :, :), allocatable **dumb\_mat1**  
*Dumb matrix to be used in the calculation.*
- real(kind=8), dimension(:, :, :), allocatable **dumb\_mat2**  
*Dumb matrix to be used in the calculation.*
- real(kind=8), dimension(:, :, :), allocatable **expm**  
*Matrix holding the product  $\text{inv\_corr\_i} * \text{corr\_ij}$  at time  $s$ .*

### 8.2.1 Detailed Description

Module to compute the correlations and derivatives used to compute the memory term of the WL parameterization.

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#### Remarks

### 8.2.2 Function/Subroutine Documentation

#### 8.2.2.1 subroutine, public corr\_tensor::init\_corr\_tensor( )

Subroutine to initialise the correlations tensors.

Definition at line 45 of file corr\_tensor.f90.

```

45      INTEGER :: i,j,m,allocstat
46
47      CALL init_corr
48
49      print*, 'Computing the time correlation tensors...'
50
51      ALLOCATE(yy(ndim,mems),dy(ndim,mems), dyy(ndim,mems), stat=allocstat)
52      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
53
54      ALLOCATE(ydy(ndim,mems), ydyy(ndim,mems), stat=allocstat)
55      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
56
57      ALLOCATE(dumb_vec(ndim), stat=allocstat)

```

```

58      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
59
60      ALLOCATE(dumb_mat1(ndim,ndim), dumb_mat2(ndim,ndim), stat=allocstat)
61      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
62
63      ALLOCATE(expm(n_unres,n_unres), stat=allocstat)
64      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
65
66      DO m=1,mems
67          CALL corrcomp((m-1)*muti)
68
69          ! YY
70          CALL ireduce(dumb_mat2,corr_ij,n_unres,ind,rind)
71          CALL matc_to_coo(dumb_mat2,yy(:,m))
72
73          ! dy
74          expm=matmul(inv_corr_i,corr_ij)
75          CALL ireduce(dumb_mat2,expm,n_unres,ind,rind)
76          CALL matc_to_coo(dumb_mat2,dy(:,m))
77
78          ! YdY
79          DO i=1,n_unres
80              CALL ireduce(dumb_mat2,mean(i)*expm,n_unres,ind,rind)
81              CALL add_matc_to_tensor(ind(i),dumb_mat2,ydy(:,m))
82          ENDDO
83
84          ! dYY
85          dumb_vec(1:n_unres)=matmul(mean,expm)
86          DO i=1,n_unres
87              CALL vector_outer(expm(i,:),dumb_vec(1:n_unres),dumb_mat2(1:n_unres,1:n_unres))
88              CALL ireduce(dumb_mat1,dumb_mat2+transpose(dumb_mat2),n_unres,ind,rind)
89              CALL add_matc_to_tensor(ind(i),dumb_mat1,dyy(:,m))
90          ENDDO
91
92          ! YdYY
93          DO i=1,n_unres
94              DO j=1,n_unres
95                  CALL vector_outer(corr_ij(i,:),expm(j,:),dumb_mat2(1:n_unres,1:n_unres))
96                  CALL ireduce(dumb_mat1,dumb_mat2+transpose(dumb_mat2),n_unres,ind,rind)
97                  CALL add_matc_to_tensor4(ind(i),ind(j),dumb_mat1,ydyy(:,m))
98              ENDDO
99          ENDDO
100     ENDDO
101
102    DEALLOCATE(dumb_mat1, dumb_mat2, stat=allocstat)
103    IF (allocstat /= 0) stop "*** Problem to deallocate ! ***"
104
105    DEALLOCATE(dumb_vec, stat=allocstat)
106    IF (allocstat /= 0) stop "*** Problem to deallocate ! ***"
107
108

```

### 8.2.3 Variable Documentation

#### 8.2.3.1 real(kind=8), dimension(:,:), allocatable corr\_tensor::dumb\_mat1 [private]

Dumb matrix to be used in the calculation.

Definition at line 37 of file corr\_tensor.f90.

```
37  REAL(KIND=8), DIMENSION(:,:), ALLOCATABLE :: dumb_mat1 !< Dumb matrix to be used in the calculation
```

#### 8.2.3.2 real(kind=8), dimension(:,:), allocatable corr\_tensor::dumb\_mat2 [private]

Dumb matrix to be used in the calculation.

Definition at line 38 of file corr\_tensor.f90.

```
38  REAL(KIND=8), DIMENSION(:,:), ALLOCATABLE :: dumb_mat2 !< Dumb matrix to be used in the calculation
```

### 8.2.3.3 `real(kind=8), dimension(:,), allocatable corr_tensor::dumb_vec` [private]

Dumb vector to be used in the calculation.

Definition at line 36 of file `corr_tensor.f90`.

```
36    REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: dumb_vec !< Dumb vector to be used in the calculation
```

### 8.2.3.4 `type(coolist), dimension(:,:,), allocatable, public corr_tensor::dy`

Coolist holding the  $\langle \partial_Y \otimes Y^s \rangle$  terms.

Definition at line 31 of file `corr_tensor.f90`.

```
31    TYPE(coolist), DIMENSION(:,:,:), ALLOCATABLE, PUBLIC :: dy !< Coolist holding the \f$ \langle \partial_Y \otimes Y^s \rangle terms
```

### 8.2.3.5 `type(coolist), dimension(:,:,), allocatable, public corr_tensor::dyy`

Coolist holding the  $\langle \partial_Y \otimes Y^s \otimes Y^s \rangle$  terms.

Definition at line 33 of file `corr_tensor.f90`.

```
33    TYPE(coolist), DIMENSION(:,:,:,:), ALLOCATABLE, PUBLIC :: dyy !< Coolist holding the \f$ \langle \partial_Y \otimes Y^s \otimes Y^s \rangle terms
```

### 8.2.3.6 `real(kind=8), dimension(:, :, ), allocatable corr_tensor::expm` [private]

Matrix holding the product `inv_corr_ij*corr_ij` at time  $s$ .

Definition at line 39 of file `corr_tensor.f90`.

```
39    REAL(KIND=8), DIMENSION(:, :, ), ALLOCATABLE :: expm !< Matrix holding the product inv_corr_i*corr_ij at time \f\$s\f$
```

### 8.2.3.7 `type(coolist), dimension(:,:,), allocatable, public corr_tensor::ydy`

Coolist holding the  $\langle Y \otimes \partial_Y \otimes Y^s \rangle$  terms.

Definition at line 32 of file `corr_tensor.f90`.

```
32    TYPE(coolist), DIMENSION(:,:,:,:), ALLOCATABLE, PUBLIC :: ydy !< Coolist holding the \f$ \langle Y \otimes \partial_Y \otimes Y^s \rangle terms
```

### 8.2.3.8 type(coolist4), dimension(:, :, ), allocatable, public corr\_tensor::ydy

Coolist holding the  $\langle Y \otimes \partial_Y \otimes Y^s \otimes Y^s \rangle$  terms.

Definition at line 34 of file corr\_tensor.f90.

```
34  TYPE(coolist4), DIMENSION(:, :, ), ALLOCATABLE, PUBLIC :: ydy !< Coolist holding the \f$\langle Y \otimes Y \otimes Y^s \otimes Y^s \rangle\f$ terms
```

### 8.2.3.9 type(coolist), dimension(:, :, ), allocatable, public corr\_tensor::yy

Coolist holding the  $\langle Y \otimes Y^s \rangle$  terms.

Definition at line 30 of file corr\_tensor.f90.

```
30  TYPE(coolist), DIMENSION(:, :, ), ALLOCATABLE, PUBLIC :: yy !< Coolist holding the \f$\langle Y \otimes Y^s \rangle\f$ terms
```

## 8.3 corrmod Module Reference

Module to initialize the correlation matrix of the unresolved variables.

### Functions/Subroutines

- subroutine, public [init\\_corr](#)

*Subroutine to initialise the computation of the correlation.*

- subroutine [corrcomp\\_from\\_def](#) (s)

*Subroutine to compute the correlation of the unresolved variables  $\langle Y \otimes Y^s \rangle$  at time s from the definition given inside the module.*

- subroutine [corrcomp\\_from\\_spline](#) (s)

*Subroutine to compute the correlation of the unresolved variables  $\langle Y \otimes Y^s \rangle$  at time s from the spline representation.*

- subroutine [splint](#) (xa, ya, y2a, n, x, y)

*Routine to compute the spline representation parameters.*

- real(kind=8) function [fs](#) (s, p)

*Exponential fit function.*

- subroutine [corrcomp\\_from\\_fit](#) (s)

*Subroutine to compute the correlation of the unresolved variables  $\langle Y \otimes Y^s \rangle$  at time s from the exponential representation.*

## Variables

- real(kind=8), dimension(:,), allocatable, public **mean**  
*Vector holding the mean of the unresolved dynamics (reduced version)*
- real(kind=8), dimension(:,), allocatable, public **mean\_full**  
*Vector holding the mean of the unresolved dynamics (full version)*
- real(kind=8), dimension(:,,:), allocatable, public **corr\_i\_full**  
*Covariance matrix of the unresolved variables (full version)*
- real(kind=8), dimension(:,,:), allocatable, public **inv\_corr\_i\_full**  
*Inverse of the covariance matrix of the unresolved variables (full version)*
- real(kind=8), dimension(:,,:), allocatable, public **corr\_i**  
*Covariance matrix of the unresolved variables (reduced version)*
- real(kind=8), dimension(:,,:), allocatable, public **inv\_corr\_i**  
*Inverse of the covariance matrix of the unresolved variables (reduced version)*
- real(kind=8), dimension(:,,:), allocatable, public **corr\_ij**  
*Matrix holding the correlation matrix at a given time.*
- real(kind=8), dimension(:,:,:), allocatable **y2**  
*Vector holding coefficient of the spline and exponential correlation representation.*
- real(kind=8), dimension(:,:,:,:), allocatable **ya**  
*Vector holding coefficient of the spline and exponential correlation representation.*
- real(kind=8), dimension(:,), allocatable **xa**  
*Vector holding coefficient of the spline and exponential correlation representation.*
- integer **nspl**  
*Integers needed by the spline representation of the correlation.*
- integer **klo**
- integer **khi**
- procedure(**corrcomp\_from\_spline**), pointer, public **corrcomp**  
*Pointer to the correlation computation routine.*

### 8.3.1 Detailed Description

Module to initialize the correlation matrix of the unresolved variables.

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#### Remarks

### 8.3.2 Function/Subroutine Documentation

#### 8.3.2.1 subroutine **corrmod::corrcomp\_from\_def** ( real(kind=8), intent(in) **s** ) [private]

Subroutine to compute the correlation of the unresolved variables  $\langle Y \otimes Y^s \rangle$  at time  $s$  from the definition given inside the module.

## Parameters

<b>s</b>	time <i>s</i> at which the correlation is computed
----------	--

Definition at line 148 of file corrmod.f90.

```

148      REAL(KIND=8), INTENT(IN) :: s
149      REAL(KIND=8) :: y
150      INTEGER :: i,j
151
152      ! Definition of the corr_ij matrix as a function of time
153
154      corr_ij(10,10)=((7.66977252307669*cos(0.039597160512071454*s))/exp(0.06567483898489704*s) + (1.02409061
73830213*cos(&
155          &0.07283568782600224*s))/exp(0.017262015588746404*s) - (0.6434985372062336*sin(0.03959716051207145
4*s&
156          &))/exp(0.06567483898489704*s) + (0.6434985372062335*sin(0.07283568782600224*s))/exp(0.01726201558
8746404*s))
157      corr_ij(10,9)=((0.6434985372062321*cos(0.039597160512071454*s))/exp(0.06567483898489704*s) - (0.6434985
372062324*cos(
158          &s(0.07283568782600224*s))/exp(0.017262015588746404*s) + (7.669772523076694*sin(0.0395971605120714
54*s
159          &s))/exp(0.06567483898489704*s) + (1.024090617383021*sin(0.07283568782600224*s))/exp(0.01726201558
8746404*s))
160      corr_ij(10,8)=0
161      corr_ij(10,7)=0
162      corr_ij(10,6)=0
163      corr_ij(10,5)=((-2.236364132659011*cos(0.039597160512071454*s))/exp(0.06567483898489704*s) + (6.9528041
48086198*cos(
164          &(0.07283568782600224*s))/exp(0.017262015588746404*s) - (1.4494534432272481*sin(0.0395971605120714
54*s
165          &s))/exp(0.06567483898489704*s) - (0.6818177416446283*sin(0.07283568782600224*s))/exp(0.0172620155
88746404*s))
166      corr_ij(10,4)=((1.4494534432272483*cos(0.039597160512071454*s))/exp(0.06567483898489704*s) + (0.6818177
416446293*cos(
167          &s(0.07283568782600224*s))/exp(0.017262015588746404*s) - (2.2363641326590127*sin(0.039597160512071
454*s
168          &s))/exp(0.06567483898489704*s) + (6.952804148086195*sin(0.07283568782600224*s))/exp(0.0172620155
88746404*s))
169      corr_ij(10,3)=0
170      corr_ij(10,2)=0
171      corr_ij(10,1)=0
172      corr_ij(9,10)=((-0.6434985372062344*cos(0.039597160512071454*s))/exp(0.06567483898489704*s) + (0.643498
537206234*cos(
173          &s(0.07283568782600224*s))/exp(0.017262015588746404*s) - (7.669772523076689*sin(0.0395971605120714
54*s
174          &s))/exp(0.06567483898489704*s) - (1.0240906173830204*sin(0.07283568782600224*s))/exp(0.0172620155
88746404*s))
175      corr_ij(9,9)=((7.66977252307669*cos(0.039597160512071454*s))/exp(0.06567483898489704*s) + (1.0240906173
830204*cos(
176          &0.07283568782600224*s))/exp(0.017262015588746404*s) - (0.643498537206233*sin(0.03959716051207145
*s)&
177          &)/exp(0.06567483898489704*s) + (0.6434985372062327*sin(0.07283568782600224*s))/exp(0.017262015588
746404*s))
178      corr_ij(9,8)=0
179      corr_ij(9,7)=0
180      corr_ij(9,6)=0
181      corr_ij(9,5)=((-1.4494534432272477*cos(0.039597160512071454*s))/exp(0.06567483898489704*s) - (0.6818177
416446249*c(
182          &s(0.07283568782600224*s))/exp(0.017262015588746404*s) + (2.2363641326590105*sin(0.03959716051207
145*s
183          &4*s))/exp(0.06567483898489704*s) - (6.952804148086195*sin(0.07283568782600224*s))/exp(0.017262015
588746404*s))
184      corr_ij(9,4)=((-2.2363641326590127*cos(0.039597160512071454*s))/exp(0.06567483898489704*s) + (6.9528041
48086194*c(
185          &s(0.07283568782600224*s))/exp(0.017262015588746404*s) - (1.4494534432272486*sin(0.039597160512071
454*s
186          &s))/exp(0.06567483898489704*s) - (0.6818177416446249*sin(0.07283568782600224*s))/exp(0.017262015
588746404*s))
187      corr_ij(9,3)=0
188      corr_ij(9,2)=0
189      corr_ij(9,1)=0
190      corr_ij(8,10)=0
191
192      corr_ij(8,9)=0
193      corr_ij(8,8)=(9.135647293470983/exp(0.05076718239027029*s) + 2.2233889637758932/exp(0.01628546700064885
4*s))
194      corr_ij(8,7)=0
195      corr_ij(8,6)=0
196      corr_ij(8,5)=0
197      corr_ij(8,4)=0
198      corr_ij(8,3)=(-5.938566084855411/exp(0.05076718239027029*s) + 11.97129741027622/exp(0.01628546700064885

```

```

4*s))
199 corr_ij(8,2)=0
200 corr_ij(8,1)=0
201 corr_ij(7,10)=0
202
203 corr_ij(7,9)=0
204 corr_ij(7,8)=0
205 corr_ij(7,7)=((11.518026982819887*cos(0.029341409726871926*s))/exp(0.04435489221745234*s) + (0.05351107
793747755*c
206 &os(0.11425932545092894*s))/exp(0.019700737327669783*s) - (0.14054811601869432*sin(0.0293414097268
719&
207 &26*s))/exp(0.04435489221745234*s) + (0.14054811601869702*sin(0.11425932545092894*s))/exp(0.019700
737327669783*s))
208 corr_ij(7,6)=((0.14054811601869532*cos(0.029341409726871926*s))/exp(0.04435489221745234*s) - (0.1405481
1601869702*&
209 &cos(0.11425932545092894*s))/exp(0.019700737327669783*s) + (11.518026982819887*sin(0.0293414097268
719&
210 &26*s))/exp(0.04435489221745234*s) + (0.0535110779374777*sin(0.11425932545092894*s))/exp(0.0197007
37327669783*s))
211 corr_ij(7,5)=0
212 corr_ij(7,4)=0
213 corr_ij(7,3)=0
214 corr_ij(7,2)=((-0.732907009016115*cos(0.029341409726871926*s))/exp(0.04435489221745234*s) + (2.72884503
1386875*cos&
215 &(0.11425932545092894*s))/exp(0.019700737327669783*s) - (2.4717920234033532*sin(0.0293414097268719
26*&
216 &s))/exp(0.04435489221745234*s) - (0.24003801347124257*sin(0.11425932545092894*s))/exp(0.019700737
327669783*s))
217 corr_ij(7,1)=((2.4717920234033532*cos(0.029341409726871926*s))/exp(0.04435489221745234*s) + (0.24003801
34712426*cos&
218 &s(0.11425932545092894*s))/exp(0.019700737327669783*s) - (0.7329070090161153*sin(0.029341409726871
926&
219 &s))/exp(0.04435489221745234*s) + (2.728845031386876*sin(0.11425932545092894*s))/exp(0.0197007373
27669783*s))
220 corr_ij(6,10)=0
221
222 corr_ij(6,9)=0
223 corr_ij(6,8)=0
224 corr_ij(6,7)=((-0.1405481160186977*cos(0.029341409726871926*s))/exp(0.04435489221745234*s) + (0.1405481
1601869713*&
225 &cos(0.11425932545092894*s))/exp(0.019700737327669783*s) - (11.518026982819885*sin(0.0293414097268
719&
226 &26*s))/exp(0.04435489221745234*s) - (0.05351107793747755*sin(0.11425932545092894*s))/exp(0.019700
737327669783*s))
227 corr_ij(6,6)=((11.518026982819885*cos(0.029341409726871926*s))/exp(0.04435489221745234*s) + (0.05351107
793747768*c
228 &os(0.11425932545092894*s))/exp(0.019700737327669783*s) - (0.14054811601869832*sin(0.0293414097268
719&
229 &26*s))/exp(0.04435489221745234*s) + (0.14054811601869707*sin(0.11425932545092894*s))/exp(0.0197007373
737327669783*s))
230 corr_ij(6,5)=0
231 corr_ij(6,4)=0
232 corr_ij(6,3)=0
233 corr_ij(6,2)=((-2.471792023403353*cos(0.029341409726871926*s))/exp(0.04435489221745234*s) - (0.24003801
34712425*cos&
234 &s(0.11425932545092894*s))/exp(0.019700737327669783*s) + (0.7329070090161155*sin(0.029341409726871
926&
235 &s))/exp(0.04435489221745234*s) - (2.7288450313868755*sin(0.11425932545092894*s))/exp(0.019700737
327669783*s))
236 corr_ij(6,1)=((-0.7329070090161154*cos(0.029341409726871926*s))/exp(0.04435489221745234*s) + (2.7288450
31386876*cos&
237 &s(0.11425932545092894*s))/exp(0.019700737327669783*s) - (2.4717920234033524*sin(0.029341409726871
926&
238 &s))/exp(0.04435489221745234*s) - (0.24003801347124343*sin(0.11425932545092894*s))/exp(0.019700737
7327669783*s))
239 corr_ij(5,10)=((0.5794534449999711*cos(0.039597160512071454*s))/exp(0.06567483898489704*s) + (4.1369865
70427212*cos&
240 &(0.07283568782600224*s))/exp(0.017262015588746404*s) - (1.0360597341248128*sin(0.0395971605120714
54*&
241 &s))/exp(0.06567483898489704*s) + (3.167330918996692*sin(0.07283568782600224*s))/exp(0.01726201558
8746404*s))
242 corr_ij(5,9)=((1.0360597341248134*cos(0.039597160512071454*s))/exp(0.06567483898489704*s) - (3.16733091
89966856*cos&
243 &s(0.07283568782600224*s))/exp(0.017262015588746404*s) + (0.5794534449999746*sin(0.039597160512071
454&
244 &s))/exp(0.06567483898489704*s) + (4.1369865704272115*sin(0.07283568782600224*s))/exp(0.017262015
588746404*s))
245 corr_ij(5,8)=0
246 corr_ij(5,7)=0
247 corr_ij(5,6)=0
248 corr_ij(5,5)=((-0.37825091063447547*cos(0.039597160512071454*s))/exp(0.06567483898489704*s) + (30.09469
0926061638*&
249 &cos(0.07283568782600224*s))/exp(0.017262015588746404*s) + (0.16085380971100194*sin(0.039597160512
071&
250 &454*s))/exp(0.06567483898489704*s) - (0.1608538097109995*sin(0.07283568782600224*s))/exp(0.017262
015588746404*s))
251 corr_ij(5,4)=((-0.16085380971100238*cos(0.039597160512071454*s))/exp(0.06567483898489704*s) + (0.160853

```

```

80971100127&
252   &*cos(0.07283568782600224*s))/exp(0.017262015588746404*s) - (0.37825091063447586*sin(0.03959716051
207&   &1454*s))/exp(0.06567483898489704*s) + (30.09469092606163*sin(0.07283568782600224*s))/exp(0.017262
253   015588746404*s))
254   corr_ij(5,3)=0
255   corr_ij(5,2)=0
256   corr_ij(5,1)=0
257   corr_ij(4,10)=((-1.0360597341248106*cos(0.039597160512071454*s))/exp(0.06567483898489704*s) + (3.167330
918996689*c&
258   &s(0.07283568782600224*s))/exp(0.017262015588746404*s) - (0.5794534449999716*sin(0.039597160512071
454&   454*s))/exp(0.06567483898489704*s) - (4.1369865704272115*sin(0.07283568782600224*s))/exp(0.017262015
588746404*s))
260   corr_ij(4,9)=((0.5794534449999711*cos(0.039597160512071454*s))/exp(0.06567483898489704*s) + (4.13698657
04272115*c&
261   &s(0.07283568782600224*s))/exp(0.017262015588746404*s) - (1.0360597341248114*sin(0.039597160512071
454&   454*s))/exp(0.06567483898489704*s) + (3.1673309189966843*sin(0.07283568782600224*s))/exp(0.017262015
588746404*s))
263   corr_ij(4,8)=0
264   corr_ij(4,7)=0
265   corr_ij(4,6)=0
266   corr_ij(4,5)=((0.16085380971100194*cos(0.039597160512071454*s))/exp(0.06567483898489704*s) - (0.1608538
0971100371*&
267   &cos(0.07283568782600224*s))/exp(0.017262015588746404*s) + (0.37825091063447497*sin(0.039597160512
071&   071*s)
268   &454*s))/exp(0.06567483898489704*s) - (30.094690926061617*sin(0.07283568782600224*s))/exp(0.017262
015588746404*s))
269   corr_ij(4,4)=((-0.37825091063447536*cos(0.039597160512071454*s))/exp(0.06567483898489704*s) + (30.09469
0926061617*&
270   &cos(0.07283568782600224*s))/exp(0.017262015588746404*s) + (0.16085380971100172*sin(0.039597160512
071&
271   &454*s))/exp(0.06567483898489704*s) - (0.16085380971100616*sin(0.07283568782600224*s))/exp(0.01726
2015588746404*s))
272   corr_ij(4,3)=0
273   corr_ij(4,2)=0
274   corr_ij(4,1)=0
275   corr_ij(3,10)=0
276
277   corr_ij(3,9)=0
278   corr_ij(3,8)=(0.24013456462471527/exp(0.05076718239027029*s) + 5.792596760796093/exp(0.0162854670006488
54*s))
279   corr_ij(3,7)=0
280   corr_ij(3,6)=0
281   corr_ij(3,5)=0
282   corr_ij(3,4)=0
283   corr_ij(3,3)=(-0.15609785880208227/exp(0.05076718239027029*s) + 31.18882918422289/exp(0.016285467000648
854*s))
284   corr_ij(3,2)=0
285   corr_ij(3,1)=0
286   corr_ij(2,10)=0
287
288   corr_ij(2,9)=0
289   corr_ij(2,8)=0
290   corr_ij(2,7)=((1.6172201305728584*cos(0.029341409726871926*s))/exp(0.04435489221745234*s) + (0.37871789
179790255*c&
291   &os(0.11425932545092894*s))/exp(0.019700737327669783*s) + (1.2889451151208258*sin(0.02934140972687
192&
292   &6*s))/exp(0.04435489221745234*s) + (1.4228849217537705*sin(0.11425932545092894*s))/exp(0.01970073
7327669783*s))
293   corr_ij(2,6)=((-1.2889451151208255*cos(0.029341409726871926*s))/exp(0.04435489221745234*s) - (1.4228849
217537702*c&
294   &os(0.11425932545092894*s))/exp(0.019700737327669783*s) + (1.6172201305728586*sin(0.02934140972687
192&
295   &6*s))/exp(0.04435489221745234*s) + (0.3787178917979035*sin(0.11425932545092894*s))/exp(0.01970073
7327669783*s))
296   corr_ij(2,5)=0
297   corr_ij(2,4)=0
298   corr_ij(2,3)=0
299   corr_ij(2,2)=((0.1789135645266575*cos(0.029341409726871926*s))/exp(0.04435489221745234*s) + (26.8170244
57844113*c&
300   &s(0.11425932545092894*s))/exp(0.019700737327669783*s) - (0.4268927977731004*sin(0.029341409726871
926&
301   &*s))/exp(0.04435489221745234*s) + (0.4268927977730982*sin(0.11425932545092894*s))/exp(0.01970073
327669783*s))
302   corr_ij(2,1)=((0.4268927977731007*cos(0.029341409726871926*s))/exp(0.04435489221745234*s) - (0.42689279
777309963*c&
303   &os(0.11425932545092894*s))/exp(0.019700737327669783*s) + (0.17891356452665746*sin(0.0293414097268
719&
304   &26*s))/exp(0.04435489221745234*s) + (26.81702445784412*sin(0.11425932545092894*s))/exp(0.01970073
7327669783*s))
305   corr_ij(1,10)=0
306
307   corr_ij(1,9)=0
308   corr_ij(1,8)=0
309   corr_ij(1,7)=((1.288945115120824*cos(0.029341409726871926*s))/exp(0.04435489221745234*s) + (1.422884921

```

```

310      7537711*cos&
311      &(0.11425932545092894*s))/exp(0.019700737327669783*s) - (1.617220130572856*sin(0.029341409726871926*s)
312      6*s&
313      &))/exp(0.04435489221745234*s) - (0.3787178917979028*sin(0.11425932545092894*s))/exp(0.019700737327669783*s))
314      corr_ij(1,6)=((1.6172201305728564*cos(0.029341409726871926*s))/exp(0.04435489221745234*s) + (0.37871789179790377*c&
315      &os(0.11425932545092894*s))/exp(0.019700737327669783*s) + (1.2889451151208242*sin(0.029341409726871926*s)
316      &6*s))/exp(0.04435489221745234*s) + (1.4228849217537711*sin(0.11425932545092894*s))/exp(0.0197007327669783*s))
317      corr_ij(1,5)=0
318      corr_ij(1,4)=0
319      corr_ij(1,3)=0
320      corr_ij(1,2)=((-0.4268927977731002*cos(0.029341409726871926*s))/exp(0.04435489221745234*s) + (0.426892797731002
321      *c&
322      &os(0.11425932545092894*s))/exp(0.019700737327669783*s) - (0.1789135645266573*sin(0.029341409726871926*s)
323      &6*s))/exp(0.04435489221745234*s) - (26.81702445784412*sin(0.11425932545092894*s))/exp(0.0197007327669783*s))
324      corr_ij(1,1)=((0.1789135645266574*cos(0.029341409726871926*s))/exp(0.04435489221745234*s) + (26.817024457844113*c
325      &s(0.11425932545092894*s))/exp(0.019700737327669783*s) - (0.42689279777310024*sin(0.029341409726871926*s)
326      &6*s))/exp(0.04435489221745234*s) + (0.4268927977730997*sin(0.11425932545092894*s))/exp(0.0197007327669783*s))
327      corr_ij=q_au**2*corr_ij

```

8.3.2.2 subroutine corrmod::corrcomp\_from\_fit ( real(kind=8), intent(in) s ) [private]

Subroutine to compute the correlation of the unresolved variables  $\langle Y \otimes Y^s \rangle$  at time  $s$  from the exponential representation.

## Parameters

$s$  time  $s$  at which the correlation is computed

Definition at line 399 of file corrmod.f90.

```

399      REAL(KIND=8), INTENT(IN) :: s
400      REAL(KIND=8) :: y
401      INTEGER :: i,j
402
403      corr_ij=0.d0
404      DO i=1,n_unres
405          DO j=1,n_unres
406              corr_ij(i,j)=fs(s,ya(i,j,:))
407          END DO
408      END DO

```

**8.3.2.3 subroutine corrmod::corrcomp\_from\_spline ( real(kind=8), intent(in) s ) [private]**

Subroutine to compute the correlation of the unresolved variables  $\langle Y \otimes Y^s \rangle$  at time  $s$  from the spline representation.

## Parameters

$s$  time  $s$  at which the correlation is computed

Definition at line 333 of file corrmod.f90.

333       REAL (KIND=8), INTENT(IN) :: S

```

334      REAL(KIND=8) :: y
335      INTEGER :: i,j
336      corr_ij=0.d0
337      DO i=1,n_unres
338          DO j=1,n_unres
339              CALL splint(xa,ya(i,j,:),y2(i,j,:),nspl,s,y)
340              corr_ij(i,j)=y
341          END DO
342      END DO

```

### 8.3.2.4 real(kind=8) function corrmod::fs ( real(kind=8), intent(in) s, real(kind=8), dimension(5), intent(in) p ) [private]

Exponential fit function.

#### Parameters

<i>s</i>	time <i>s</i> at which the function is evaluated
<i>p</i>	vector holding the coefficients of the fit function

Definition at line 388 of file corrmod.f90.

```

388      REAL(KIND=8), INTENT(IN) :: s
389      REAL(KIND=8), DIMENSION(5), INTENT(IN) :: p
390      REAL(KIND=8) :: fs
391      fs=p(1)*exp(-s/p(2))*cos(p(3)*s+p(4))
392      RETURN

```

### 8.3.2.5 subroutine, public corrmod::init\_corr ( )

Subroutine to initialise the computation of the correlation.

Definition at line 46 of file corrmod.f90.

```

46      INTEGER :: allocstat,i,j,k,nf
47      REAL(KIND=8), DIMENSION(5) :: dumb
48      LOGICAL :: ex
49
50      ! Selection of the loading mode
51      SELECT CASE (load_mode)
52      CASE ('defi')
53          corrcmp => corrcmp_from_def
54      CASE ('spli')
55          INQUIRE(file='corrspline.def',exist=ex)
56          IF (.not.ex) stop "*** File corrspline.def not found ! ***"
57          OPEN(20,file='corrspline.def',status='old')
58          READ(20,*) nf,nspl
59          IF (nf /= n_unres) stop "*** Dimension in files corrspline.def and sf.nml do not correspond ! ***"
60          ALLOCATE(xa(nspl), ya(n_unres,n_unres,nspl), y2(n_unres,n_unres,nspl),
61 stat=allocstat)
62          IF (allocstat /= 0) stop "*** Not enough memory ! ***"
63          READ(20,*) xa
64          maxint=xa(nspl)/2
65          DO k=1,n_unres*n_unres
66              READ(20,*) i,j
67              READ(20,*) ya(i,j,:)
68              READ(20,*) y2(i,j,:)
69          ENDDO
70          CLOSE(20)
71          corrcmp => corrcmp_from_spline
72          klo=1
73          khi=nspl
74      CASE ('expo')
75          INQUIRE(file='correxpo.def',exist=ex)
76          IF (.not.ex) stop "*** File correxpodef not found ! ***"
77          OPEN(20,file='correxpo.def',status='old')

```

```

77      READ(20,*) nf,maxint
78      IF (nf /= n_unres) stop "*** Dimension in files correxp0.def and sf.nml do not correspond ! ***"
79      ALLOCATE(ya(n_unres,n_unres,5), stat=allocstat)
80      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
81      DO k=1,n_unres*n_unres
82          READ(20,*) i,j,dumb
83          ya(i,j,:)=dumb
84      ENDDO
85      CLOSE(20)
86      corrcmp => corrcmp_from_fit
87      CASE DEFAULT
88          stop '*** LOAD_MODE variable not properly defined in corrm0.nml ***'
89      END SELECT
90
91      ALLOCATE(mean(n_unres),mean_full(0:ndim), stat=allocstat)
92      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
93
94      ALLOCATE(inv_corr_i(n_unres,n_unres), stat=allocstat)
95      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
96
97      ALLOCATE(corr_i(n_unres,n_unres), stat=allocstat)
98      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
99
100     ALLOCATE(corr_ij(n_unres,n_unres), stat=allocstat)
101     IF (allocstat /= 0) stop "*** Not enough memory ! ***"
102
103     ALLOCATE(corr_i_full(ndim,ndim), stat=allocstat)
104     IF (allocstat /= 0) stop "*** Not enough memory ! ***"
105
106     ALLOCATE(inv_corr_i_full(ndim,ndim), stat=allocstat)
107     IF (allocstat /= 0) stop "*** Not enough memory ! ***"
108
109     corr_ij=0.d0
110
111     CALL corrcmp(0.d0)
112     corr_i=corr_ij
113     inv_corr_i=invmat(corr_i)
114
115     corr_i_full=0.d0
116     DO i=1,n_unres
117         DO j=1,n_unres
118             corr_i_full(ind(i),ind(j))=corr_i(i,j)
119         ENDDO
120     ENDDO
121
122     inv_corr_i_full=0.d0
123     DO i=1,n_unres
124         DO j=1,n_unres
125             inv_corr_i_full(ind(i),ind(j))=inv_corr_i(i,j)
126         ENDDO
127     ENDDO
128
129     mean=0.d0
130     INQUIRE(file='mean.def',exist=ex)
131     IF (ex) THEN
132         OPEN(20,file='mean.def',status='old')
133         READ(20,*) mean
134         CLOSE(20)
135     ENDIF
136
137     mean_full=0.d0
138     DO i=1,n_unres
139         mean_full(ind(i))=mean(i)
140     ENDDO
141

```

### 8.3.2.6 subroutine corrm0::splint ( real(kind=8), dimension(n), intent(in) xa, real(kind=8), dimension(n), intent(in) ya, real(kind=8), dimension(n), intent(in) y2a, integer, intent(in) n, real(kind=8), intent(in) x, real(kind=8), intent(out) y ) [private]

Routine to compute the spline representation parameters.

Definition at line 347 of file corrm0.f90.

```

347     INTEGER, INTENT(IN) :: n
348     REAL(KIND=8), INTENT(IN), DIMENSION(n) :: xa,y2a,ya
349     REAL(KIND=8), INTENT(IN) :: x
350     REAL(KIND=8), INTENT(OUT) :: y

```

```

351      INTEGER :: k
352      REAL(KIND=8) :: a,b,h
353      if ((khi-klo.gt.1).or.(xa(klo).gt.x).or.(xa(khi).lt.x)) then
354          if ((khi-klo.eq.1).and.(xa(klo).lt.x)) then
355              khi=klo
356              DO WHILE (xa(khi).lt.x)
357                  khi=khi+1
358              END DO
359              khi=khi-1
360          else
361              khi=n
362              klo=1
363          DO WHILE (khi-klo.gt.1)
364              k=(khi+klo)/2
365              if(xa(k).gt.x)then
366                  khi=k
367              else
368                  klo=k
369              endif
370          END DO
371      end if
372      !    print*, "search",x,khi-klo,xa(klo),xa(khi)
373      ! else
374      !    print*, "ok",x,khi-klo,xa(klo),xa(khi)
375      endif
376      h=xa(khi)-xa(klo)
377      if (h.eq.0.) stop 'bad xa input in splint'
378      a=(xa(khi)-x)/h
379      b=(x-xa(klo))/h
380      y=a*y(a(klo))+b*y(a(khi))+((a**3-a)*y2(a(klo))+(b**3-b)*y2(a(khi)))*(h**2)/6.
381      return

```

### 8.3.3 Variable Documentation

#### 8.3.3.1 real(kind=8), dimension(:,:), allocatable, public corrmod::corr\_i

Covariance matrix of the unresolved variables (reduced version)

Definition at line 30 of file corrmod.f90.

```

30  REAL(KIND=8), DIMENSION(:,:), ALLOCATABLE, PUBLIC :: corr_i !< Covariance matrix of the unresolved
variables (reduced version)

```

#### 8.3.3.2 real(kind=8), dimension(:,:), allocatable, public corrmod::corr\_i\_full

Covariance matrix of the unresolved variables (full version)

Definition at line 28 of file corrmod.f90.

```

28  REAL(KIND=8), DIMENSION(:,:), ALLOCATABLE, PUBLIC :: corr_i_full !< Covariance matrix of the unresolved
variables (full version)

```

#### 8.3.3.3 real(kind=8), dimension(:,:), allocatable, public corrmod::corr\_ij

Matrix holding the correlation matrix at a given time.

Definition at line 32 of file corrmod.f90.

```

32  REAL(KIND=8), DIMENSION(:,:), ALLOCATABLE, PUBLIC :: corr_ij !< Matrix holding the correlation matrix at
a given time

```

### 8.3.3.4 procedure(corrcomp\_from\_spline), pointer, public corrmod::corrcomp

Pointer to the correlation computation routine.

Definition at line 41 of file corrmod.f90.

```
41 PROCEDURE(corrcomp_from_spline), POINTER, PUBLIC :: corrcomp
```

### 8.3.3.5 real(kind=8), dimension(:,:), allocatable, public corrmod::inv\_corr\_i

Inverse of the covariance matrix of the unresolved variables (reduced version)

Definition at line 31 of file corrmod.f90.

```
31 REAL(KIND=8), DIMENSION(:,:), ALLOCATABLE, PUBLIC :: inv_corr_i !< Inverse of the covariance matrix of  
the unresolved variables (reduced version)
```

### 8.3.3.6 real(kind=8), dimension(:,:), allocatable, public corrmod::inv\_corr\_i\_full

Inverse of the covariance matrix of the unresolved variables (full version)

Definition at line 29 of file corrmod.f90.

```
29 REAL(KIND=8), DIMENSION(:,:), ALLOCATABLE, PUBLIC :: inv_corr_i_full !< Inverse of the covariance matrix  
of the unresolved variables (full version)
```

### 8.3.3.7 integer corrmod::khi [private]

Definition at line 38 of file corrmod.f90.

### 8.3.3.8 integer corrmod::klo [private]

Definition at line 38 of file corrmod.f90.

### 8.3.3.9 real(kind=8), dimension(:), allocatable, public corrmod::mean

Vector holding the mean of the unresolved dynamics (reduced version)

Definition at line 26 of file corrmod.f90.

```
26 REAL(KIND=8), DIMENSION(:), ALLOCATABLE, PUBLIC :: mean !< Vector holding the mean of the unresolved  
dynamics (reduced version)
```

**8.3.3.10 real(kind=8), dimension(:), allocatable, public corrmod::mean\_full**

Vector holding the mean of the unresolved dynamics (full version)

Definition at line 27 of file corrmod.f90.

```
27    REAL(KIND=8), DIMENSION(:), ALLOCATABLE, PUBLIC :: mean_full !< Vector holding the mean of the unresolved
dynamics (full version)
```

**8.3.3.11 integer corrmod::nspl [private]**

Integers needed by the spline representation of the correlation.

Definition at line 38 of file corrmod.f90.

```
38    INTEGER :: nspl,klo,khi
```

**8.3.3.12 real(kind=8), dimension(:), allocatable corrmod::xa [private]**

Vector holding coefficient of the spline and exponential correlation representation.

Definition at line 35 of file corrmod.f90.

```
35    REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: xa !< Vector holding coefficient of the spline and exponential
correlation representation
```

**8.3.3.13 real(kind=8), dimension(:,:,), allocatable corrmod::y2 [private]**

Vector holding coefficient of the spline and exponential correlation representation.

Definition at line 33 of file corrmod.f90.

```
33    REAL(KIND=8), DIMENSION(:,:,:), ALLOCATABLE :: y2 !< Vector holding coefficient of the spline and
exponential correlation representation
```

**8.3.3.14 real(kind=8), dimension(:,:,), allocatable corrmod::ya [private]**

Vector holding coefficient of the spline and exponential correlation representation.

Definition at line 34 of file corrmod.f90.

```
34    REAL(KIND=8), DIMENSION(:,:,:), ALLOCATABLE :: ya !< Vector holding coefficient of the spline and
exponential correlation representation
```

## 8.4 dec\_tensor Module Reference

The resolved-unresolved components decomposition of the tensor.

### Functions/Subroutines

- subroutine `suppress_and` (t, cst, v1, v2)  
*Subroutine to suppress from the tensor  $t_{ijk}$  components satisfying  $SF(j)=v1$  and  $SF(k)=v2$ .*
- subroutine `suppress_or` (t, cst, v1, v2)  
*Subroutine to suppress from the tensor  $t_{ijk}$  components satisfying  $SF(j)=v1$  or  $SF(k)=v2$ .*
- subroutine `reorder` (t, cst, v)  
*Subroutine to reorder the tensor  $t_{ijk}$  components : if  $SF(j)=v$  then it return  $t_{ikj}$ .*
- subroutine `init_sub_tensor` (t, cst, v)  
*Subroutine that suppress all the components of a tensor  $t_{ijk}$  where if  $SF(i)=v$ .*
- subroutine, public `init_dec_tensor`  
*Subroutine that initialize and compute the decomposed tensors.*

### Variables

- type(`coolist`), dimension(:), allocatable, public `ff_tensor`  
*Tensor holding the part of the unresolved tensor involving only unresolved variables.*
- type(`coolist`), dimension(:), allocatable, public `sf_tensor`  
*Tensor holding the part of the resolved tensor involving unresolved variables.*
- type(`coolist`), dimension(:), allocatable, public `ss_tensor`  
*Tensor holding the part of the resolved tensor involving only resolved variables.*
- type(`coolist`), dimension(:), allocatable, public `fs_tensor`  
*Tensor holding the part of the unresolved tensor involving resolved variables.*
- type(`coolist`), dimension(:), allocatable, public `hx`  
*Tensor holding the constant part of the resolved tendencies.*
- type(`coolist`), dimension(:), allocatable, public `lxx`  
*Tensor holding the linear part of the resolved tendencies involving the resolved variables.*
- type(`coolist`), dimension(:), allocatable, public `lxy`  
*Tensor holding the linear part of the resolved tendencies involving the unresolved variables.*
- type(`coolist`), dimension(:), allocatable, public `bxxx`  
*Tensor holding the quadratic part of the resolved tendencies involving resolved variables.*
- type(`coolist`), dimension(:), allocatable, public `bxyy`  
*Tensor holding the quadratic part of the resolved tendencies involving both resolved and unresolved variables.*
- type(`coolist`), dimension(:), allocatable, public `hy`  
*Tensor holding the constant part of the unresolved tendencies.*
- type(`coolist`), dimension(:), allocatable, public `lyx`  
*Tensor holding the linear part of the unresolved tendencies involving the resolved variables.*
- type(`coolist`), dimension(:), allocatable, public `lyy`  
*Tensor holding the linear part of the unresolved tendencies involving the unresolved variables.*
- type(`coolist`), dimension(:), allocatable, public `byxx`  
*Tensor holding the quadratic part of the unresolved tendencies involving resolved variables.*
- type(`coolist`), dimension(:), allocatable, public `byxy`  
*Tensor holding the quadratic part of the unresolved tendencies involving both resolved and unresolved variables.*

- type(coolist), dimension(:), allocatable, public **byyy**  
*Tensor holding the quadratic part of the unresolved tendencies involving unresolved variables.*
- type(coolist), dimension(:), allocatable, public **ss\_tl\_tensor**  
*Tensor of the tangent linear model tendencies of the resolved component alone.*
- type(coolist), dimension(:), allocatable **dumb**  
*Dumb coolist to make the computations.*

## 8.4.1 Detailed Description

The resolved-unresolved components decomposition of the tensor.

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### Remarks

## 8.4.2 Function/Subroutine Documentation

### 8.4.2.1 subroutine, public dec\_tensor::init\_dec\_tensor( )

Subroutine that initialize and compute the decomposed tensors.

Definition at line 195 of file dec\_tensor.f90.

```

195      USE params, only:ndim
196      USE aotensor_def, only:aotensor
197      USE sf_def, only: load_sf
198      USE tensor, only:copy_coo,add_to_tensor,
199      scal_mul_coo
200      USE tl_ad_tensor, only: init_tltensor,tltensor
201      USE stoch_params, only: init_stoch_params,mode,
202      tdelta,eps_pert
203      INTEGER :: allocstat
204      CALL init_stoch_params
205      CALL init_tltensor ! and tl tensor
206      CALL load_sf ! Load the resolved-unresolved decomposition
207      ! Allocating the returned arrays
208
209      ALLOCATE(ff_tensor(ndim),fs_tensor(ndim),sf_tensor(ndim),ss_tensor(
210      ndim), stat=allocstat)
211      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
212
213      ALLOCATE(ss_tl_tensor(ndim), stat=allocstat)
214      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
215
216      ALLOCATE(hx(ndim),lxx(ndim),lxy(ndim),bxxx(ndim),bxxy(ndim),bxyy(
217      ndim), stat=allocstat)
218      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
219
220      ALLOCATE(hy(ndim),lyx(ndim),lyy(ndim),byxx(ndim),byxy(ndim),byyy(
221      ndim), stat=allocstat)
222      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
223
224      ! General decomposition
225      ! ff tensor
226      ALLOCATE(dumb(ndim), stat=allocstat)
227      IF (allocstat /= 0) stop "*** Not enough memory ! ***"

```

```

227
228 IF (mode.ne.'qfst') THEN
229   CALL copy_coo(aotensor,dumb) !Copy the tensors
230   CALL init_sub_tensor(dumb,0,0)
231   CALL suppress_or(dumb,1,0,0) ! Clear entries with resolved variables
232   CALL copy_coo(dumb,ff_tensor)
233 ELSE
234   CALL copy_coo(aotensor,dumb) !Copy the tensors
235   CALL init_sub_tensor(dumb,0,0)
236   CALL suppress_or(dumb,0,0,0) ! Clear entries with resolved variables and linear and constant terms
237   CALL copy_coo(dumb,ff_tensor)
238 ENDIF
239
240 allocstat=0
241 DEALLOCATE(dumb, stat=allocstat)
242 IF (allocstat /= 0) stop "*** Problem to deallocate ! ***"
243
244 ! fs tensor
245 ALLOCATE(dumb(ndim), stat=allocstat)
246 IF (allocstat /= 0) stop "*** Not enough memory ! ***"
247
248 IF (mode.ne.'qfst') THEN
249   CALL copy_coo(aotensor,dumb) !Copy the tensors
250   CALL init_sub_tensor(dumb,0,0)
251   CALL suppress_and(dumb,1,1,1) ! Clear entries with only unresolved variables and constant
252   CALL copy_coo(dumb,fs_tensor)
253 ELSE
254   CALL copy_coo(aotensor,dumb) !Copy the tensors
255   CALL init_sub_tensor(dumb,0,0)
256   CALL suppress_and(dumb,0,1,1) ! Clear entries with only quadratic unresolved variables
257   CALL copy_coo(dumb,fs_tensor)
258 ENDIF
259
260 allocstat=0
261 DEALLOCATE(dumb, stat=allocstat)
262 IF (allocstat /= 0) stop "*** Problem to deallocate ! ***"
263
264 ! sf tensor
265 ALLOCATE(dumb(ndim), stat=allocstat)
266 IF (allocstat /= 0) stop "*** Not enough memory ! ***"
267
268
269 CALL copy_coo(aotensor,dumb) !Copy the tensors
270 CALL init_sub_tensor(dumb,1,1)
271 CALL suppress_and(dumb,0,0,0) ! Clear entries with only unresolved variables and constant
272 CALL copy_coo(dumb,sf_tensor)
273
274 allocstat=0
275 DEALLOCATE(dumb, stat=allocstat)
276 IF (allocstat /= 0) stop "*** Problem to deallocate ! ***"
277
278 ! ss tensor
279 ALLOCATE(dumb(ndim), stat=allocstat)
280 IF (allocstat /= 0) stop "*** Not enough memory ! ***"
281
282
283 CALL copy_coo(aotensor,dumb) !Copy the tensors
284 CALL init_sub_tensor(dumb,1,1)
285 CALL suppress_or(dumb,0,1,1) ! Clear entries with only unresolved variables and constant
286 CALL copy_coo(dumb,ss_tensor)
287
288 allocstat=0
289 DEALLOCATE(dumb, stat=allocstat)
290 IF (allocstat /= 0) stop "*** Problem to deallocate ! ***"
291
292 ! ss tangent linear tensor
293
294 ALLOCATE(dumb(ndim), stat=allocstat)
295 IF (allocstat /= 0) stop "*** Not enough memory ! ***"
296
297 CALL copy_coo(tltensor,dumb) !Copy the tensors
298 CALL init_sub_tensor(dumb,1,1)
299 CALL suppress_or(dumb,0,1,1) ! Clear entries with only unresolved variables and constant
300 CALL copy_coo(dumb,ss_tl_tensor)
301
302 allocstat=0
303 DEALLOCATE(dumb, stat=allocstat)
304 IF (allocstat /= 0) stop "*** Problem to deallocate ! ***"
305
306 ! Multiply the aotensor part that need to be by the perturbation and time
307 ! separation parameter
308
309 ALLOCATE(dumb(ndim), stat=allocstat)
310 IF (allocstat /= 0) stop "*** Not enough memory ! ***"
311
312 CALL copy_coo(ss_tensor,dumb)
313 CALL scal_mul_coo(1.d0/tdelta**2,ff_tensor)

```

```

314     CALL scal_mul_coo(eps_pert/tdelta,fs_tensor)
315     CALL add_to_tensor(ff_tensor,dumb)
316     CALL add_to_tensor(fs_tensor,dumb)
317     CALL scal_mul_coo(eps_pert/tdelta,sf_tensor)
318     CALL add_to_tensor(sf_tensor,dumb)
319
320     allocstat=0
321     DEALLOCATE(aotensor, stat=allocstat)
322     IF (allocstat /= 0) stop "*** Problem to deallocate ! ***"
323
324     ALLOCATE(aotensor(ndim), stat=allocstat)
325     IF (allocstat /= 0) stop "*** Not enough memory ! ***"
326
327     CALL copy_coo(dumb,aotensor)
328
329     allocstat=0
330     DEALLOCATE(dumb, stat=allocstat)
331     IF (allocstat /= 0) stop "*** Problem to deallocate ! ***"
332
333     ! MTV decomposition
334     ! Unresolved tensors
335
336     ! Hy tensor
337     ALLOCATE(dumb(ndim), stat=allocstat)
338     IF (allocstat /= 0) stop "*** Not enough memory ! ***"
339
340     CALL copy_coo(aotensor,dumb) !Copy the tensors
341     CALL init_sub_tensor(dumb,0,0)
342     CALL suppress_or(dumb,0,1,1) ! Clear entries with unresolved variables
343     CALL suppress_or(dumb,1,0,0) ! Suppress linear and nonlinear resolved terms
344     CALL copy_coo(dumb,hy)
345
346     allocstat=0
347     DEALLOCATE(dumb, stat=allocstat)
348     IF (allocstat /= 0) stop "*** Problem to deallocate ! ***"
349
350     ! Lyx tensor
351     ALLOCATE(dumb(ndim), stat=allocstat)
352     IF (allocstat /= 0) stop "*** Not enough memory ! ***"
353
354     CALL copy_coo(aotensor,dumb) !Copy the tensors
355     CALL init_sub_tensor(dumb,0,0)
356     CALL suppress_or(dumb,0,1,1) ! Clear entries with unresolved variables
357     CALL suppress_and(dumb,1,1,1) ! Clear constant entries
358     CALL suppress_and(dumb,1,0,0) ! Clear entries with nonlinear resolved terms
359     CALL reorder(dumb,1,0) ! Resolved variables must be the third (k) index
360     CALL copy_coo(dumb,lyx)
361
362     allocstat=0
363     DEALLOCATE(dumb, stat=allocstat)
364     IF (allocstat /= 0) stop "*** Problem to deallocate ! ***"
365
366     ! Lyy tensor
367     ALLOCATE(dumb(ndim), stat=allocstat)
368     IF (allocstat /= 0) stop "*** Not enough memory ! ***"
369
370     CALL copy_coo(aotensor,dumb) !Copy the tensors
371     CALL init_sub_tensor(dumb,0,0)
372     CALL suppress_or(dumb,1,0,0) ! Clear entries with resolved variables
373     CALL suppress_and(dumb,0,1,1) ! Clear entries with nonlinear unresolved terms
374     CALL suppress_and(dumb,0,0,0) ! Clear constant entries
375     CALL reorder(dumb,0,1) ! Unresolved variables must be the third (k) index
376     CALL copy_coo(dumb,lyy)
377
378     allocstat=0
379     DEALLOCATE(dumb, stat=allocstat)
380     IF (allocstat /= 0) stop "*** Problem to deallocate ! ***"
381
382     ! Byxy tensor
383     ALLOCATE(dumb(ndim), stat=allocstat)
384     IF (allocstat /= 0) stop "*** Not enough memory ! ***"
385
386     CALL copy_coo(aotensor,dumb) !Copy the tensors
387     CALL init_sub_tensor(dumb,0,0)
388     CALL suppress_and(dumb,1,1,1) ! Clear constant or linear terms and nonlinear unresolved only entries
389     CALL suppress_and(dumb,0,0,0) ! Clear entries with only resolved variables and constant
390     CALL reorder(dumb,0,1) ! Unresolved variables must be the third (k) index
391     CALL copy_coo(dumb,byxy)
392
393     allocstat=0
394     DEALLOCATE(dumb, stat=allocstat)
395     IF (allocstat /= 0) stop "*** Problem to deallocate ! ***"
396
397     ! Byyy tensor
398     ALLOCATE(dumb(ndim), stat=allocstat)
399     IF (allocstat /= 0) stop "*** Not enough memory ! ***"
400

```

```

401 CALL copy_coo(aotensor,dumb) !Copy the tensors
402 CALL init_sub_tensor(dumb,0,0)
403 CALL suppress_or(dumb,0,0,0) ! Clear entries with resolved variables and linear and constant terms
404 CALL copy_coo(dumb,byyy)
405
406 allocstat=0
407 DEALLOCATE(dumb, stat=allocstat)
408 IF (allocstat /= 0) stop "*** Problem to deallocate ! ***"
409
410 ! Byxx tensor
411 ALLOCATE(dumb(ndim), stat=allocstat)
412 IF (allocstat /= 0) stop "*** Not enough memory ! ***"
413
414 CALL copy_coo(aotensor,dumb) !Copy the tensors
415 CALL init_sub_tensor(dumb,0,0)
416 CALL suppress_or(dumb,1,1,1) ! Clear entries with unresolved variables and linear and constant terms
417 CALL copy_coo(dumb,byxx)
418
419 allocstat=0
420 DEALLOCATE(dumb, stat=allocstat)
421 IF (allocstat /= 0) stop "*** Problem to deallocate ! ***"
422
423 ! Resolved tensors
424
425 ! Hx tensor
426 ALLOCATE(dumb(ndim), stat=allocstat)
427 IF (allocstat /= 0) stop "*** Not enough memory ! ***"
428
429 CALL copy_coo(aotensor,dumb) !Copy the tensors
430 CALL init_sub_tensor(dumb,1,1)
431 CALL suppress_or(dumb,1,0,0) ! Clear entries with resolved variables
432 CALL suppress_or(dumb,0,1,1) ! Suppress linear and nonlinear unresolved terms
433 CALL copy_coo(dumb,hx)
434
435 allocstat=0
436 DEALLOCATE(dumb, stat=allocstat)
437 IF (allocstat /= 0) stop "*** Problem to deallocate ! ***"
438
439 ! Lxy tensor
440 ALLOCATE(dumb(ndim), stat=allocstat)
441 IF (allocstat /= 0) stop "*** Not enough memory ! ***"
442
443 CALL copy_coo(aotensor,dumb) !Copy the tensors
444 CALL init_sub_tensor(dumb,1,1)
445 CALL suppress_or(dumb,1,0,0) ! Clear entries with resolved variables
446 CALL suppress_and(dumb,0,0,0) ! Clear constant entries
447 CALL suppress_and(dumb,0,1,1) ! Clear entries with nonlinear unresolved terms
448 CALL reorder(dumb,0,1) ! Resolved variables must be the third (k) index
449 CALL copy_coo(dumb,lxy)
450
451 allocstat=0
452 DEALLOCATE(dumb, stat=allocstat)
453 IF (allocstat /= 0) stop "*** Problem to deallocate ! ***"
454
455 ! Lxx tensor
456 ALLOCATE(dumb(ndim), stat=allocstat)
457 IF (allocstat /= 0) stop "*** Not enough memory ! ***"
458
459 CALL copy_coo(aotensor,dumb) !Copy the tensors
460 CALL init_sub_tensor(dumb,1,1)
461 CALL suppress_or(dumb,0,1,1) ! Clear entries with unresolved variables
462 CALL suppress_and(dumb,1,0,0) ! Clear entries with nonlinear resolved terms
463 CALL suppress_and(dumb,1,1,1) ! Clear constant entries
464 CALL reorder(dumb,1,0) ! Resolved variables must be the third (k) index
465 CALL copy_coo(dumb,lxx)
466
467 allocstat=0
468 DEALLOCATE(dumb, stat=allocstat)
469 IF (allocstat /= 0) stop "*** Problem to deallocate ! ***"
470
471 ! Bxy tensor
472 ALLOCATE(dumb(ndim), stat=allocstat)
473 IF (allocstat /= 0) stop "*** Not enough memory ! ***"
474
475 CALL copy_coo(aotensor,dumb) !Copy the tensors
476 CALL init_sub_tensor(dumb,1,1)
477 CALL suppress_and(dumb,1,1,1) ! Clear constant or linear terms and nonlinear unresolved only entries
478 CALL suppress_and(dumb,0,0,0) ! Clear entries with only resolved variables and constant
479 CALL reorder(dumb,0,1) ! Unresolved variables must be the third (k) index
480 CALL copy_coo(dumb,bxyy)
481
482 allocstat=0
483 DEALLOCATE(dumb, stat=allocstat)
484 IF (allocstat /= 0) stop "*** Problem to deallocate ! ***"
485
486 ! Bxxx tensor
487 ALLOCATE(dumb(ndim), stat=allocstat)

```

```

488     IF (allocstat /= 0) stop "*** Not enough memory ! ***"
489
490     CALL copy_coo(aotensor,dumb) !Copy the tensors
491     CALL init_sub_tensor(dumb,1,1)
492     CALL suppress_or(dumb,1,1,1) ! Clear entries with unresolved variables and linear and constant terms
493     CALL copy_coo(dumb,bxxx)
494
495     allocstat=0
496     DEALLOCATE(dumb, stat=allocstat)
497     IF (allocstat /= 0) stop "*** Problem to deallocate ! ***"
498
499     ! Bxyy tensor
500     ALLOCATE(dumb(ndim), stat=allocstat)
501     IF (allocstat /= 0) stop "*** Not enough memory ! ***"
502
503     CALL copy_coo(aotensor,dumb) !Copy the tensors
504     CALL init_sub_tensor(dumb,1,1)
505     CALL suppress_or(dumb,0,0,0) ! Clear entries with resolved variables and linear and constant terms
506     CALL copy_coo(dumb,bxyy)
507
508     allocstat=0
509     DEALLOCATE(dumb, stat=allocstat)
510     IF (allocstat /= 0) stop "*** Problem to deallocate ! ***"
511
512
513

```

#### 8.4.2.2 subroutine dec\_tensor::init\_sub\_tensor ( type(coolist), dimension(ndim), intent(inout) t, integer, intent(in) cst, integer, intent(in) v )

Subroutine that suppress all the components of a tensor  $t_{ijk}$  where if SF(i)=v.

##### Parameters

<i>t</i>	tensor over which the routine acts
<i>cst</i>	constant which controls if the 0 index is taken as a unresolved or a resolved one
<i>v</i>	constant of the conditional (0 to suppress resolved, 1 for unresolved)

Definition at line 174 of file dec\_tensor.f90.

```

174     USE params, only:ndim
175     USE sf_def, only: sf
176     TYPE(coolist), DIMENSION(ndim), INTENT(INOUT) :: t
177     INTEGER, INTENT(IN) :: cst,v
178     INTEGER :: i
179
180     sf(0)=cst ! control wether 0 index is considered unresolved or not
181     DO i=1,ndim
182       IF (sf(i)==v) t(i)%nelems=0
183     ENDDO
184

```

#### 8.4.2.3 subroutine dec\_tensor::reorder ( type(coolist), dimension(ndim), intent(inout) t, integer, intent(in) cst, integer, intent(in) v )

Subroutine to reorder the tensor  $t_{ijk}$  components : if SF(j)=v then it return  $t_{ikj}$ .

##### Parameters

<i>t</i>	tensor over which the routine acts
<i>cst</i>	constant which controls if the 0 index is taken as a unresolved or a resolved one
<i>v</i>	constant of the conditional (0 to invert resolved, 1 for unresolved)

Definition at line 148 of file dec\_tensor.f90.

```

148 USE params, only:ndim
149 USE sf_def, only: sf
150 TYPE(coolist), DIMENSION(ndim), INTENT(INOUT) :: t
151 INTEGER, INTENT(IN) :: cst,v
152 INTEGER :: i,n,li,lippi
153
154 sf(0)=cst ! control wether 0 index is considered unresolved or not
155 DO i=1,ndim
156
157     n=t(i)%nelems
158     DO li=1,n
159         IF (sf(t(i)%elems(li)%j)==v) THEN
160             lippi=t(i)%elems(li)%j
161             t(i)%elems(li)%j=t(i)%elems(li)%k
162             t(i)%elems(li)%k=lippi
163     ENDIF
164     ENDDO
165 ENDDO
166

```

#### 8.4.2.4 subroutine dec\_tensor::suppress\_and ( type(coolist), dimension(ndim), intent(inout) t, integer, intent(in) cst, integer, intent(in) v1, integer, intent(in) v2 ) [private]

Subroutine to suppress from the tensor  $t_{ijk}$  components satisfying SF(j)=v1 and SF(k)=v2.

##### Parameters

<i>t</i>	tensor over which the routine acts
<i>cst</i>	constant which controls if the 0 index is taken as a unresolved or a resolved one
<i>v1</i>	first constant of the conditional (0 to suppress resolved, 1 for unresolved)
<i>v2</i>	second constant of the conditional (0 to suppress resolved, 1 for unresolved)

Definition at line 77 of file dec\_tensor.f90.

```

77 USE params, only:ndim
78 USE sf_def, only: sf
79 TYPE(coolist), DIMENSION(ndim), INTENT(INOUT) :: t
80 INTEGER, INTENT(IN) :: cst,v1,v2
81 INTEGER :: i,n,li,lippi
82
83 sf(0)=cst ! control wether 0 index is considered unresolved or not
84 DO i=1,ndim
85     n=t(i)%nelems
86     DO li=1,n
87         ! Clear entries with only resolved variables and shift rest of the items one place down.
88         ! Make sure not to skip any entries while shifting!
89
90         DO WHILE ((sf(t(i)%elems(li)%j)==v1).and.(sf(t(i)%elems(li)%k)==v2))
91             !print*, i,li,t(i)%nelems,n
92             DO lippi=li+1,n
93                 t(i)%elems(lippi-1)%j=t(i)%elems(lippi)%j
94                 t(i)%elems(lippi-1)%k=t(i)%elems(lippi)%k
95                 t(i)%elems(lippi-1)%v=t(i)%elems(lippi)%v
96             ENDDO
97             t(i)%nelems=t(i)%nelems-1
98             IF (li>t(i)%nelems) EXIT
99         ENDDO
100        IF (li>t(i)%nelems) EXIT
101    ENDDO
102 ENDDO
103

```

#### 8.4.2.5 subroutine dec\_tensor::suppress\_or ( type(coolist), dimension(ndim), intent(inout) t, integer, intent(in) cst, integer, intent(in) v1, integer, intent(in) v2 )

Subroutine to suppress from the tensor  $t_{ijk}$  components satisfying SF(j)=v1 or SF(k)=v2.

## Parameters

<i>t</i>	tensor over which the routine acts
<i>cst</i>	constant which controls if the 0 index is taken as a unresolved or a resolved one
<i>v1</i>	first constant of the conditional (0 to suppress resolved, 1 for unresolved)
<i>v2</i>	second constant of the conditional (0 to suppress resolved, 1 for unresolved)

Definition at line 113 of file dec\_tensor.f90.

```

113      USE params, only:ndim
114      USE sf_def, only: sf
115      TYPE(coolist), DIMENSION(ndim), INTENT(INOUT) :: t
116      INTEGER, INTENT(IN) :: cst,v1,v2
117      INTEGER :: i,n,li,liii
118
119      sf(0)=cst ! control wether 0 index is considered unresolved or not
120      DO i=1,ndim
121          n=t(i)%nelems
122          DO li=1,n
123              ! Clear entries with only resolved variables and shift rest of the items one place down.
124              ! Make sure not to skip any entries while shifting!
125
126              DO WHILE ((sf(t(i)%elems(li)%j)==v1).or.(sf(t(i)%elems(li)%k)==v2))
127                  !print*, i,li,t(i)%nelems,n
128                  DO liii=li+1,n
129                      t(i)%elems(lilli-1)%j=t(i)%elems(lilli)%j
130                      t(i)%elems(lilli-1)%k=t(i)%elems(lilli)%k
131                      t(i)%elems(lilli-1)%v=t(i)%elems(lilli)%v
132                  ENDDO
133                  t(i)%nelems=t(i)%nelems-1
134                  IF (li>t(i)%nelems) exit
135              ENDDO
136              IF (li>t(i)%nelems) exit
137          ENDDO
138      ENDDO
139

```

## 8.4.3 Variable Documentation

### 8.4.3.1 type(coolist), dimension(:), allocatable, public dec\_tensor::bxxx

Tensor holding the quadratic part of the resolved tendencies involving resolved variables.

Definition at line 39 of file dec\_tensor.f90.

```

39      TYPE(coolist), DIMENSION(:), ALLOCATABLE, PUBLIC :: bxxx !< Tensor holding the quadratic part of
the resolved tendencies involving resolved variables

```

### 8.4.3.2 type(coolist), dimension(:), allocatable, public dec\_tensor::bxyy

Tensor holding the quadratic part of the resolved tendencies involving both resolved and unresolved variables.

Definition at line 40 of file dec\_tensor.f90.

```

40      TYPE(coolist), DIMENSION(:), ALLOCATABLE, PUBLIC :: bxyy !< Tensor holding the quadratic part of
the resolved tendencies involving both resolved and unresolved variables

```

#### 8.4.3.3 type(coolist), dimension(:), allocatable, public dec\_tensor::bxyy

Tensor holding the quadratic part of the resolved tendencies involving unresolved variables.

Definition at line 41 of file dec\_tensor.f90.

```
41  TYPE(coolist), DIMENSION(:), ALLOCATABLE, PUBLIC :: bxyy !< Tensor holding the quadratic part of
   the resolved tendencies involving unresolved variables
```

#### 8.4.3.4 type(coolist), dimension(:), allocatable, public dec\_tensor::byxx

Tensor holding the quadratic part of the unresolved tendencies involving resolved variables.

Definition at line 46 of file dec\_tensor.f90.

```
46  TYPE(coolist), DIMENSION(:), ALLOCATABLE, PUBLIC :: byxx !< Tensor holding the quadratic part of
   the unresolved tendencies involving resolved variables
```

#### 8.4.3.5 type(coolist), dimension(:), allocatable, public dec\_tensor::byxy

Tensor holding the quadratic part of the unresolved tendencies involving both resolved and unresolved variables.

Definition at line 47 of file dec\_tensor.f90.

```
47  TYPE(coolist), DIMENSION(:), ALLOCATABLE, PUBLIC :: byxy !< Tensor holding the quadratic part of
   the unresolved tendencies involving both resolved and unresolved variables
```

#### 8.4.3.6 type(coolist), dimension(:), allocatable, public dec\_tensor::byyy

Tensor holding the quadratic part of the unresolved tendencies involving unresolved variables.

Definition at line 48 of file dec\_tensor.f90.

```
48  TYPE(coolist), DIMENSION(:), ALLOCATABLE, PUBLIC :: byyy !< Tensor holding the quadratic part of
   the unresolved tendencies involving unresolved variables
```

#### 8.4.3.7 type(coolist), dimension(:), allocatable dec\_tensor::dumb [private]

Dumb coolist to make the computations.

Definition at line 53 of file dec\_tensor.f90.

```
53  TYPE(coolist), DIMENSION(:), ALLOCATABLE :: dumb !< Dumb coolist to make the computations
```

**8.4.3.8 type(coolist), dimension(:), allocatable, public dec\_tensor::ff\_tensor**

Tensor holding the part of the unresolved tensor involving only unresolved variables.

Definition at line 31 of file dec\_tensor.f90.

```
31   TYPE(coolist), DIMENSION(:), ALLOCATABLE, PUBLIC :: ff_tensor !< Tensor holding the part of the
unresolved tensor involving only unresolved variables
```

**8.4.3.9 type(coolist), dimension(:), allocatable, public dec\_tensor::fs\_tensor**

Tensor holding the part of the unresolved tensor involving resolved variables.

Definition at line 34 of file dec\_tensor.f90.

```
34   TYPE(coolist), DIMENSION(:), ALLOCATABLE, PUBLIC :: fs_tensor !< Tensor holding the part of the
unresolved tensor involving resolved variables
```

**8.4.3.10 type(coolist), dimension(:), allocatable, public dec\_tensor::hx**

Tensor holding the constant part of the resolved tendencies.

Definition at line 36 of file dec\_tensor.f90.

```
36   TYPE(coolist), DIMENSION(:), ALLOCATABLE, PUBLIC :: hx !< Tensor holding the constant part of the
resolved tendencies
```

**8.4.3.11 type(coolist), dimension(:), allocatable, public dec\_tensor::hy**

Tensor holding the constant part of the unresolved tendencies.

Definition at line 43 of file dec\_tensor.f90.

```
43   TYPE(coolist), DIMENSION(:), ALLOCATABLE, PUBLIC :: hy !< Tensor holding the constant part of the
unresolved tendencies
```

**8.4.3.12 type(coolist), dimension(:), allocatable, public dec\_tensor::lxx**

Tensor holding the linear part of the resolved tendencies involving the resolved variables.

Definition at line 37 of file dec\_tensor.f90.

```
37   TYPE(coolist), DIMENSION(:), ALLOCATABLE, PUBLIC :: lxx !< Tensor holding the linear part of the
resolved tendencies involving the resolved variables
```

#### 8.4.3.13 type(coolist), dimension(:), allocatable, public dec\_tensor::lxy

Tensor holding the linear part of the resolved tendencies involving the unresolved variables.

Definition at line 38 of file dec\_tensor.f90.

```
38  TYPE(coolist), DIMENSION(:), ALLOCATABLE, PUBLIC :: lxy !< Tensor holding the linear part of the
   resolved tendencies involving the unresolved variables
```

#### 8.4.3.14 type(coolist), dimension(:), allocatable, public dec\_tensor::lyx

Tensor holding the linear part of the unresolved tendencies involving the resolved variables.

Definition at line 44 of file dec\_tensor.f90.

```
44  TYPE(coolist), DIMENSION(:), ALLOCATABLE, PUBLIC :: lyx !< Tensor holding the linear part of the
   unresolved tendencies involving the resolved variables
```

#### 8.4.3.15 type(coolist), dimension(:), allocatable, public dec\_tensor::lyy

Tensor holding the linear part of the unresolved tendencies involving the unresolved variables.

Definition at line 45 of file dec\_tensor.f90.

```
45  TYPE(coolist), DIMENSION(:), ALLOCATABLE, PUBLIC :: lyy !< Tensor holding the linear part of the
   unresolved tendencies involving the unresolved variables
```

#### 8.4.3.16 type(coolist), dimension(:), allocatable, public dec\_tensor::sf\_tensor

Tensor holding the part of the resolved tensor involving unresolved variables.

Definition at line 32 of file dec\_tensor.f90.

```
32  TYPE(coolist), DIMENSION(:), ALLOCATABLE, PUBLIC :: sf_tensor !< Tensor holding the part of the
   resolved tensor involving unresolved variables
```

#### 8.4.3.17 type(coolist), dimension(:), allocatable, public dec\_tensor::ss\_tensor

Tensor holding the part of the resolved tensor involving only resolved variables.

Definition at line 33 of file dec\_tensor.f90.

```
33  TYPE(coolist), DIMENSION(:), ALLOCATABLE, PUBLIC :: ss_tensor !< Tensor holding the part of the
   resolved tensor involving only resolved variables
```

#### 8.4.3.18 type(coolist), dimension(:), allocatable, public dec\_tensor::ss\_tl\_tensor

Tensor of the tangent linear model tendencies of the resolved component alone.

Definition at line 50 of file dec\_tensor.f90.

```
50  TYPE(coolist), DIMENSION(:), ALLOCATABLE, PUBLIC :: ss_tl_tensor !< Tensor of the tangent linear  
model tendencies of the resolved component alone
```

## 8.5 ic\_def Module Reference

Module to load the initial condition.

### Functions/Subroutines

- subroutine, public [load\\_ic](#)

*Subroutine to load the initial condition if IC.nml exists. If it does not, then write IC.nml with 0 as initial condition.*

### Variables

- logical [exists](#)  
*Boolean to test for file existence.*
- real(kind=8), dimension(:), allocatable, public [ic](#)  
*Initial condition vector.*

### 8.5.1 Detailed Description

Module to load the initial condition.

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### 8.5.2 Function/Subroutine Documentation

#### 8.5.2.1 subroutine, public ic\_def::load\_ic( )

Subroutine to load the initial condition if IC.nml exists. If it does not, then write IC.nml with 0 as initial condition.

Definition at line 32 of file ic\_def.f90.

```

32      INTEGER :: i,allocstat,j
33      CHARACTER(len=20) :: fm
34      REAL(KIND=8) :: size_of_random_noise
35      INTEGER, DIMENSION(:), ALLOCATABLE :: seed
36      CHARACTER(LEN=4) :: init_type
37      namelist /iclist/ ic
38      namelist /rand/ init_type,size_of_random_noise,seed
39
40      fm(1:6)='(F3.1)'
41
42      CALL random_seed(size=j)
43
44      IF (ndim == 0) stop "*** Number of dimensions is 0! ***"
45      ALLOCATE(ic(0:ndim),seed(j), stat=allocstat)
46      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
47
48      INQUIRE(file='./IC.nml',exist=exists)
49
50      IF (exists) THEN
51          OPEN(8, file="IC.nml", status='OLD', recl=80, delim='APOSTROPHE')
52          READ(8,nml=iclist)
53          READ(8,nml=rand)
54          CLOSE(8)
55          SELECT CASE (init_type)
56              CASE ('seed')
57                  CALL random_seed(put=seed)
58                  CALL random_number(ic)
59                  ic=2*(ic-0.5)
60                  ic=ic*size_of_random_noise*10.d0
61                  ic(0)=1.0d0
62                  WRITE(6,*) "*** IC.nml namelist written. Starting with 'seeded' random initial condition !***"
63              CASE ('rand')
64                  CALL init_random_seed()
65                  CALL random_seed(get=seed)
66                  CALL random_number(ic)
67                  ic=2*(ic-0.5)
68                  ic=ic*size_of_random_noise*10.d0
69                  ic(0)=1.0d0
70                  WRITE(6,*) "*** IC.nml namelist written. Starting with random initial condition !***"
71              CASE ('zero')
72                  CALL init_random_seed()
73                  CALL random_seed(get=seed)
74                  ic=0
75                  ic(0)=1.0d0
76                  WRITE(6,*) "*** IC.nml namelist written. Starting with initial condition in IC.nml !***"
77              CASE ('read')
78                  CALL init_random_seed()
79                  CALL random_seed(get=seed)
80                  ic(0)=1.0d0
81                  ! except IC(0), nothing has to be done IC has already the right values
82                  WRITE(6,*) "*** IC.nml namelist written. Starting with initial condition in IC.nml !***"
83          END SELECT
84
85      ELSE
86          CALL init_random_seed()
87          CALL random_seed(get=seed)
88          ic=0
89          ic(0)=1.0d0
90          init_type="zero"
91          size_of_random_noise=0.d0
92          WRITE(6,*) "*** IC.nml namelist written. Starting with 0 as initial condition !***"
93      END IF
94      OPEN(8, file="IC.nml", status='REPLACE')
95      WRITE(8,'(a)') "!-----"
96      WRITE(8,'(a)') "! Namelist file :"
97      WRITE(8,'(a)') "! Initial condition."
98      WRITE(8,'(a)') "!-----"
99      WRITE(8,*)
100     WRITE(8,'(a)') "&ICLIST"
101     WRITE(8,*)
102     DO i=1,natm
103         WRITE(8,*) " IC("//trim(str(i))//") = ",ic(i)," ! typ= "&
104             &/awavenum(i)%typ//", Nx= "//trim(rstr(awavenum(i)&
105             &%Nx,fm))//", Ny= "//trim(rstr(awavenum(i)%Ny,fm))
106     END DO
107     WRITE(8,*)
108     DO i=1,natm
109         WRITE(8,*) " IC("//trim(str(i+natm))//") = ",ic(i+natm)," ! typ= "&
110             &/awavenum(i)%typ//", Nx= "//trim(rstr(awavenum(i)&
111             &%Nx,fm))//", Ny= "//trim(rstr(awavenum(i)%Ny,fm))
112     END DO
113
114     WRITE(8,*)
115     DO i=1,noc
116         WRITE(8,*) " IC("//trim(str(i+2*natm))//") = ",ic(i+2*natm)," ! Nx&
117             &= "//trim(rstr(owavenum(i)%Nx,fm))//", Ny= "&
118             &/trim(rstr(owavenum(i)%Ny,fm))

```

```

119      END DO
120      WRITE(8,*)
121      DO i=1,noc
122        WRITE(8,*)
123        &! Nx= //trim(rstr(owavenum(i)%Nx,fm))//", Ny= "
124        &!/trim(rstr(owavenum(i)%Ny,fm))
125      END DO
126
127      WRITE(8,'(a)') "&END"
128      WRITE(8,*)
129      WRITE(8,'(a)') "!-----"
130      WRITE(8,'(a)') "! Initialisation type."
131      WRITE(8,'(a)') "!-----"
132      WRITE(8,'(a)') "! type = 'read': use IC above (will generate a new seed);"
133      WRITE(8,'(a)') "!           'rand': random state (will generate a new seed);"
134      WRITE(8,'(a)') "!           'zero': zero IC (will generate a new seed);"
135      WRITE(8,'(a)') "!           'seed': use the seed below (generate the same IC)"
136      WRITE(8,*)
137      WRITE(8,'(a)') "&RAND"
138      WRITE(8,'(a)') " init_type= '//init_type//'"
139      WRITE(8,'(a,d15.7)') " size_of_random_noise = ",size_of_random_noise
140      DO i=1,j
141        WRITE(8,*)
142      END DO
143      WRITE(8,'(a)') "&END"
144      WRITE(8,*)
145      CLOSE(8)
146

```

### 8.5.3 Variable Documentation

#### 8.5.3.1 logical ic\_def::exists [private]

Boolean to test for file existence.

Definition at line 21 of file ic\_def.f90.

```
21  LOGICAL :: exists !< Boolean to test for file existence.
```

#### 8.5.3.2 real(kind=8), dimension(:), allocatable, public ic\_def::ic

Initial condition vector.

Definition at line 23 of file ic\_def.f90.

```
23  REAL(KIND=8), DIMENSION(:), ALLOCATABLE, PUBLIC :: ic !< Initial condition vector
```

## 8.6 inprod\_analytic Module Reference

Inner products between the truncated set of basis functions for the ocean and atmosphere streamfunction fields. These are partly calculated using the analytical expressions from Cehelsky, P., & Tung, K. K. : Theories of multiple equilibria and weather regimes-A critical reexamination. Part II: Baroclinic two-layer models. Journal of the atmospheric sciences, 44(21), 3282-3303, 1987.

## Data Types

- type [atm\\_tensors](#)  
*Type holding the atmospheric inner products tensors.*
- type [atm\\_wavenum](#)  
*Atmospheric bloc specification type.*
- type [ocean\\_tensors](#)  
*Type holding the oceanic inner products tensors.*
- type [ocean\\_wavenum](#)  
*Oceanic bloc specification type.*

## Functions/Subroutines

- real(kind=8) function [b1](#) (Pi, Pj, Pk)  
*Cehelsky & Tung Helper functions.*
- real(kind=8) function [b2](#) (Pi, Pj, Pk)  
*Cehelsky & Tung Helper functions.*
- real(kind=8) function [delta](#) (r)  
*Integer Dirac delta function.*
- real(kind=8) function [flambda](#) (r)  
*"Odd or even" function*
- real(kind=8) function [s1](#) (Pj, Pk, Mj, Hk)  
*Cehelsky & Tung Helper functions.*
- real(kind=8) function [s2](#) (Pj, Pk, Mj, Hk)  
*Cehelsky & Tung Helper functions.*
- real(kind=8) function [s3](#) (Pj, Pk, Hj, Hk)  
*Cehelsky & Tung Helper functions.*
- real(kind=8) function [s4](#) (Pj, Pk, Hj, Hk)  
*Cehelsky & Tung Helper functions.*
- real(kind=8) function [calculate\\_a](#) (i, j)  
*Eigenvalues of the Laplacian (atmospheric)*
- real(kind=8) function [calculate\\_b](#) (i, j, k)  
*Streamfunction advection terms (atmospheric)*
- real(kind=8) function [calculate\\_c\\_atm](#) (i, j)  
*Beta term for the atmosphere.*
- real(kind=8) function [calculate\\_d](#) (i, j)  
*Forcing of the ocean on the atmosphere.*
- real(kind=8) function [calculate\\_g](#) (i, j, k)  
*Temperature advection terms (atmospheric)*
- real(kind=8) function [calculate\\_s](#) (i, j)  
*Forcing (thermal) of the ocean on the atmosphere.*
- real(kind=8) function [calculate\\_k](#) (i, j)  
*Forcing of the atmosphere on the ocean.*
- real(kind=8) function [calculate\\_m](#) (i, j)  
*Forcing of the ocean fields on the ocean.*
- real(kind=8) function [calculate\\_n](#) (i, j)  
*Beta term for the ocean.*
- real(kind=8) function [calculate\\_o](#) (i, j, k)  
*Temperature advection term (passive scalar)*
- real(kind=8) function [calculate\\_c\\_oc](#) (i, j, k)

- *Streamfunction advection terms (oceanic)*
- `real(kind=8) function calculate_w (i, j)`  
*Short-wave radiative forcing of the ocean.*
- subroutine, public `init_inprod`  
*Initialisation of the inner product.*

## Variables

- type(`atm_wavenum`), dimension(:), allocatable, public `awavenum`  
*Atmospheric blocs specification.*
- type(`ocean_wavenum`), dimension(:), allocatable, public `owavenum`  
*Oceanic blocs specification.*
- type(`atm_tensors`), public `atmos`  
*Atmospheric tensors.*
- type(`ocean_tensors`), public `ocean`  
*Oceanic tensors.*

### 8.6.1 Detailed Description

Inner products between the truncated set of basis functions for the ocean and atmosphere streamfunction fields. These are partly calculated using the analytical expressions from Cehelsky, P., & Tung, K. K. : Theories of multiple equilibria and weather regimes-A critical reexamination. Part II: Baroclinic two-layer models. Journal of the atmospheric sciences, 44(21), 3282-3303, 1987.

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#### Remarks

Generated Fortran90/95 code from inprod\_analytic.lua

### 8.6.2 Function/Subroutine Documentation

#### 8.6.2.1 `real(kind=8) function inprod_analytic::b1 ( integer Pi, integer Pj, integer Pk ) [private]`

Cehelsky & Tung Helper functions.

Definition at line 100 of file inprod\_analytic.f90.

```
100      INTEGER :: pi,pj,pk
101      b1 = (pk + pj) / REAL(pi)
```

#### 8.6.2.2 `real(kind=8) function inprod_analytic::b2 ( integer Pi, integer Pj, integer Pk ) [private]`

Cehelsky & Tung Helper functions.

Definition at line 106 of file inprod\_analytic.f90.

```
106      INTEGER :: pi,pj,pk
107      b2 = (pk - pj) / REAL(pi)
```

### 8.6.2.3 real(kind=8) function inprod\_analytic::calculate\_a ( integer, intent(in) i, integer, intent(in) j ) [private]

Eigenvalues of the Laplacian (atmospheric)

$$a_{i,j} = (F_i, \nabla^2 F_j).$$

Definition at line 164 of file inprod\_analytic.f90.

```
164      INTEGER, INTENT(IN) :: i,j
165      TYPE(atm_wavenum) :: ti
166
167      calculate_a = 0.d0
168      IF (i==j) THEN
169          ti = awavenum(i)
170          calculate_a = -(n**2) * ti%Nx**2 - ti%Ny**2
171      END IF
```

### 8.6.2.4 real(kind=8) function inprod\_analytic::calculate\_b ( integer, intent(in) i, integer, intent(in) j, integer, intent(in) k ) [private]

Streamfunction advection terms (atmospheric)

$$b_{i,j,k} = (F_i, J(F_j, \nabla^2 F_k)).$$

Definition at line 178 of file inprod\_analytic.f90.

```
178      INTEGER, INTENT(IN) :: i,j,k
179
180      calculate_b = calculate_a(k,k) * calculate_g(i,j,k)
181
```

### 8.6.2.5 real(kind=8) function inprod\_analytic::calculate\_c\_atm ( integer, intent(in) i, integer, intent(in) j ) [private]

Beta term for the atmosphere.

$$c_{i,j} = (F_i, \partial_x F_j).$$

Definition at line 188 of file inprod\_analytic.f90.

```
188      INTEGER, INTENT(IN) :: i,j
189      TYPE(atm_wavenum) :: ti, tj
190
191      ti = awavenum(i)
192      tj = awavenum(j)
193      calculate_c_atm = 0.d0
194      IF ((ti%typ == "K") .AND. (tj%typ == "L")) THEN
195          calculate_c_atm = n * ti%M * delta(ti%M - tj%H) * delta(ti%P - tj%P)
196      ELSE IF ((ti%typ == "L") .AND. (tj%typ == "K")) THEN
197          ti = awavenum(j)
198          tj = awavenum(i)
199          calculate_c_atm = - n * ti%M * delta(ti%M - tj%H) * delta(ti%P - tj%P)
200      END IF
```

---

8.6.2.6 real(kind=8) function inprod\_analytic::calculate\_c\_oc ( integer, intent(in) i, integer, intent(in) j, integer, intent(in) k )  
[private]

Streamfunction advection terms (oceanic)

$$C_{i,j,k} = (\eta_i, J(\eta_j, \nabla^2 \eta_k)) .$$

Definition at line 412 of file inprod\_analytic.f90.

```
412      INTEGER, INTENT(IN) :: i,j,k
413
414      calculate_c_oc = calculate_m(k,k) * calculate_o(i,j,k)
415
```

---

8.6.2.7 real(kind=8) function inprod\_analytic::calculate\_d ( integer, intent(in) i, integer, intent(in) j ) [private]

Forcing of the ocean on the atmosphere.

$$d_{i,j} = (F_i, \nabla^2 \eta_j) .$$

Definition at line 208 of file inprod\_analytic.f90.

```
208      INTEGER, INTENT(IN) :: i,j
209
210      calculate_d=calculate_s(i,j) * calculate_m(j,j)
211
```

---

8.6.2.8 real(kind=8) function inprod\_analytic::calculate\_g ( integer, intent(in) i, integer, intent(in) j, integer, intent(in) k )  
[private]

Temperature advection terms (atmospheric)

$$g_{i,j,k} = (F_i, J(F_j, F_k)) .$$

Definition at line 218 of file inprod\_analytic.f90.

```
218      INTEGER, INTENT(IN) :: i,j,k
219      TYPE(atm_wavenum) :: ti,tj,tk
220      REAL(KIND=8) :: val,vb1, vb2, vs1, vs2, vs3, vs4
221      INTEGER, DIMENSION(3) :: a,b
222      INTEGER, DIMENSION(3,3) :: w
223      CHARACTER, DIMENSION(3) :: s
224      INTEGER :: par
225
226      ti = awavenum(i)
227      tj = awavenum(j)
228      tk = awavenum(k)
229
230      a(1)=i
231      a(2)=j
232      a(3)=k
233
234      val=0.d0
235
236      IF ((ti%typ == "L") .AND. (tj%typ == "L") .AND. (tk%typ == "L")) THEN
237
238          CALL piksrt(3,a,par)
239
240          ti = awavenum(a(1))
241          tj = awavenum(a(2))
242          tk = awavenum(a(3))
243
```

```

244      vs3 = s3(tj%P,tk%P,tj%H,tk%H)
245      vs4 = s4(tj%P,tk%P,tj%H,tk%H)
246      val = vs3 * ((delta(tk%H - tj%H - ti%H) - delta(tk%H &
247          & - tj%H + ti%H)) * delta(tk%P + tj%P - ti%P) + &
248          & delta(tk%H + tj%H - ti%H) * (delta(tk%P - tj%P &
249          & + ti%P) - delta(tk%P - tj%P - ti%P)) + vs4 * &
250          & ((delta(tk%H + tj%H - ti%H) * delta(tk%P - tj%P &
251          & - ti%P)) + (delta(tk%H - tj%H + ti%H) - &
252          & delta(tk%H - tj%H - ti%H) * (delta(tk%P - tj%P &
253          & - ti%P) - delta(tk%P - tj%P + ti%P)))
254      ELSE
255
256      s(1)=ti%typ
257      s(2)=tj%typ
258      s(3)=tk%typ
259
260      w(1,:)=isin("A",s)
261      w(2,:)=isin("K",s)
262      w(3,:)=isin("L",s)
263
264      IF (any(w(1,:)/=0) .AND. any(w(2,:)/=0) .AND. any(w(3,:)/=0)) THEN
265          b=w(:,1)
266          ti = awavenum(a(b(1)))
267          tj = awavenum(a(b(2)))
268          tk = awavenum(a(b(3)))
269          call piksrt(3,b,par)
270          vb1 = b1(ti%P,tj%P,tk%P)
271          vb2 = b2(ti%P,tj%P,tk%P)
272          val = -2 * sqrt(2.) / pi * tj%M * delta(tj%M - tk%H) * flambda(ti%P + tj%P + tk%P)
273          IF (val /= 0.d0) val = val * (vb1**2 / (vb1**2 - 1) - vb2**2 / (vb2**2 - 1))
274      ELSEIF ((w(2,2)/=0) .AND. (w(2,3)==0) .AND. any(w(3,:)/=0)) THEN
275          ti = awavenum(a(w(2,1)))
276          tj = awavenum(a(w(2,2)))
277          tk = awavenum(a(w(3,1)))
278          b(1)=w(2,1)
279          b(2)=w(2,2)
280          b(3)=w(3,1)
281          call piksrt(3,b,par)
282          vs1 = s1(tj%P,tk%P,tj%M,tk%H)
283          vs2 = s2(tj%P,tk%P,tj%M,tk%H)
284          val = vs1 * (delta(ti%M - tk%H - tj%M) * delta(ti%P - &
285              & tk%P + tj%P) - delta(ti%M - tk%H - tj%M) * &
286              & delta(ti%P + tk%P - tj%P) + (delta(tk%H - tj%M &
287                  & + ti%M) + delta(tk%H - tj%M - ti%M)) * &
288                  & delta(tk%P + tj%P - ti%P)) + vs2 * (delta(ti%M &
289                      & - tk%H - tj%M) * delta(ti%P - tk%P - tj%P) + &
290                      & (delta(tk%H - tj%M - ti%M) + delta(ti%M + tk%H &
291                          & - tj%M)) * (delta(ti%P - tk%P + tj%P) - &
292                          & delta(tk%P - tj%P + ti%P)))
293      ENDIF
294  ENDIF
295  calculate_g=par*val*n
296

```

### 8.6.2.9 real(kind=8) function inprod\_analytic::calculate\_k ( integer, intent(in) i, integer, intent(in) j ) [private]

Forcing of the atmosphere on the ocean.

$$K_{i,j} = (\eta_i, \nabla^2 F_j).$$

Definition at line 336 of file inprod\_analytic.f90.

```

336      INTEGER, INTENT(IN) :: i,j
337
338      calculate_k = calculate_s(j,i) * calculate_a(j,j)

```

### 8.6.2.10 real(kind=8) function inprod\_analytic::calculate\_m ( integer, intent(in) i, integer, intent(in) j ) [private]

Forcing of the ocean fields on the ocean.

$$M_{i,j} = (eta_i, \nabla^2 \eta_j).$$

Definition at line 345 of file inprod\_analytic.f90.

```

345      INTEGER, INTENT(IN) :: i,j
346      TYPE(ocean_wavenum) :: di
347
348      calculate_m=0.d0
349      IF (i==j) THEN
350          di = owavenum(i)
351          calculate_m = -(n**2) * di%Nx**2 - di%Ny**2
352      END IF

```

### 8.6.2.11 real(kind=8) function inprod\_analytic::calculate\_n ( integer, intent(in) i, integer, intent(in) j ) [private]

Beta term for the ocean.

$$N_{i,j} = (\eta_i, \partial_x \eta_j).$$

Definition at line 359 of file inprod\_analytic.f90.

```

359      INTEGER, INTENT(IN) :: i,j
360      TYPE(ocean_wavenum) :: di,dj
361      REAL(KIND=8) :: val
362
363      di = owavenum(i)
364      dj = owavenum(j)
365      calculate_n = 0.d0
366      IF (dj%H==di%H) THEN
367          val = delta(di%P - dj%P) * flambda(di%H + dj%H)
368          calculate_n = val * (-2) * dj%H * di%H * n / ((dj%H**2 - di%H**2) * pi)
369      END IF
370

```

### 8.6.2.12 real(kind=8) function inprod\_analytic::calculate\_o ( integer, intent(in) i, integer, intent(in) j, integer, intent(in) k ) [private]

Temperature advection term (passive scalar)

$$O_{i,j,k} = (\eta_i, J(\eta_j, \eta_k)).$$

Definition at line 377 of file inprod\_analytic.f90.

```

377      INTEGER, INTENT(IN) :: i,j,k
378      TYPE(ocean_wavenum) :: di,dj,dk
379      REAL(KIND=8) :: vs3,vs4,val
380      INTEGER, DIMENSION(3) :: a
381      INTEGER :: par
382
383      val=0.d0
384
385      a(1)=i
386      a(2)=j
387      a(3)=k
388
389      CALL piksrt(3,a,par)
390
391      di = owavenum(a(1))
392      dj = owavenum(a(2))
393      dk = owavenum(a(3))
394
395      vs3 = s3(dj%P,dk%P,dj%H,dk%H)
396      vs4 = s4(dj%P,dk%,dj%H,dk%H)
397      val = vs3*((delta(dk%H - dj%H - di%H) - delta(dk%H - dj%
398          &%H + di%H)) * delta(dk%P + dj%P - di%P) + delta(dk%
399          &%H + dj%H - di%H) * (delta(dk%P - dj%P + di%P) -&
amp;400          & delta(dk%P - dj%P - di%P)) + vs4 * ((delta(dk%H - dj%P + di%P) + delta(dk%H - dj%P - di%P)) * delta(dk%P - dj%P - di%P) +&
401          & (delta(dk%H - dj%H + di%H) - delta(dk%H - dj%H - di%H)) * (delta(dk%P - dj%P - di%P) - delta(dk%P - dj%P + di%P)))
402          & (delta(dk%H - dj%H + di%H) - delta(dk%H - dj%H - di%H)) * (delta(dk%P - dj%P - di%P) - delta(dk%P - dj%P + di%P)))
403          & (delta(dk%P - dj%P - di%P) - delta(dk%P - dj%P + di%P)))
404      calculate_o = par * val * n / 2

```

### 8.6.2.13 real(kind=8) function inprod\_analytic::calculate\_s ( integer, intent(in) i, integer, intent(in) j ) [private]

Forcing (thermal) of the ocean on the atmosphere.

$$s_{i,j} = (F_i, \eta_j).$$

Definition at line 303 of file inprod\_analytic.f90.

```

303      INTEGER, INTENT(IN) :: i,j
304      TYPE(atm_wavenum) :: ti
305      TYPE(ocean_wavenum) :: dj
306      REAL(KIND=8) :: val
307
308      ti = awavenum(i)
309      dj = owavenum(j)
310      val=0.d0
311      IF (ti%typ == "A") THEN
312          val = flambda(dj%H) * flambda(dj%P + ti%P)
313          IF (val /= 0.d0) THEN
314              val = val*8*sqrt(2.)*dj%P/(pi**2 * (dj%P**2 - ti%P**2) * dj%H)
315          END IF
316      ELSEIF (ti%typ == "K") THEN
317          val = flambda(2 * ti%M + dj%H) * delta(dj%P - ti%P)
318          IF (val /= 0.d0) THEN
319              val = val*4*dj%H/(pi * (-4 * ti%M**2 + dj%H**2))
320          END IF
321      ELSEIF (ti%typ == "L") THEN
322          val = delta(dj%P - ti%P) * delta(2 * ti%H - dj%H)
323      END IF
324      calculate_s=val
325

```

### 8.6.2.14 real(kind=8) function inprod\_analytic::calculate\_w ( integer, intent(in) i, integer, intent(in) j ) [private]

Short-wave radiative forcing of the ocean.

$$W_{i,j} = (\eta_i, F_j).$$

Definition at line 422 of file inprod\_analytic.f90.

```

422      INTEGER, INTENT(IN) :: i,j
423
424      calculate_w = calculate_s(j,i)
425

```

### 8.6.2.15 real(kind=8) function inprod\_analytic::delta ( integer r ) [private]

Integer Dirac delta function.

Definition at line 112 of file inprod\_analytic.f90.

```

112      INTEGER :: r
113      IF (r==0) THEN
114          delta = 1.d0
115      ELSE
116          delta = 0.d0
117      ENDIF

```

## 8.6.2.16 real(kind=8) function inprod\_analytic::flambda( integer r ) [private]

"Odd or even" function

Definition at line 122 of file inprod\_analytic.f90.

```
122      INTEGER :: r
123      IF (mod(r,2)==0) THEN
124          flambda = 0.d0
125      ELSE
126          flambda = 1.d0
127      ENDIF
```

## 8.6.2.17 subroutine, public inprod\_analytic::init\_inprod( )

Initialisation of the inner product.

Definition at line 436 of file inprod\_analytic.f90.

```
436      INTEGER :: i,j
437      INTEGER :: allocstat
438
439      IF (natm == 0 ) THEN
440          stop "*** Problem : natm==0 ! ***"
441      ELSEIF (noc == 0) then
442          stop "*** Problem : noc==0 ! ***"
443      END IF
444
445
446      ! Definition of the types and wave numbers tables
447
448      ALLOCATE(owavenum(noc),awavenum(natm), stat=allocstat)
449      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
450
451      j=0
452      DO i=1,nbatm
453          IF (ams(i,1) ==1) THEN
454              awavenum(j+1)%typ='A'
455              awavenum(j+2)%typ='K'
456              awavenum(j+3)%typ='L'
457
458              awavenum(j+1)%P=ams(i,2)
459              awavenum(j+2)%M=ams(i,1)
460              awavenum(j+2)%P=ams(i,2)
461              awavenum(j+3)%H=ams(i,1)
462              awavenum(j+3)%P=ams(i,2)
463
464              awavenum(j+1)%Ny=REAL(ams(i,2))
465              awavenum(j+2)%Nx=REAL(ams(i,1))
466              awavenum(j+2)%Ny=REAL(ams(i,2))
467              awavenum(j+3)%Nx=REAL(ams(i,1))
468              awavenum(j+3)%Ny=REAL(ams(i,2))
469
470          j=j+3
471      ELSE
472          awavenum(j+1)%typ='K'
473          awavenum(j+2)%typ='L'
474
475          awavenum(j+1)%M=ams(i,1)
476          awavenum(j+1)%P=ams(i,2)
477          awavenum(j+2)%H=ams(i,1)
478          awavenum(j+2)%P=ams(i,2)
479
480          awavenum(j+1)%Nx=REAL(ams(i,1))
481          awavenum(j+1)%Ny=REAL(ams(i,2))
482          awavenum(j+2)%Nx=REAL(ams(i,1))
483          awavenum(j+2)%Ny=REAL(ams(i,2))
484
485          j=j+2
486
487      ENDIF
488      ENDDO
489
490      DO i=1,noc
491          owavenum(i)%H=oms(i,1)
```

```

492      owarenum(i)%P=oms(i,2)
493
494      owarenum(i)%Nx=oms(i,1)/2.d0
495      owarenum(i)%Ny=oms(i,2)
496
497      ENDDO
498
499      ! Pointing to the atmospheric inner products functions
500
501      atmos%a => calculate_a
502      atmos%g => calculate_g
503      atmos%s => calculate_s
504      atmos%b => calculate_b
505      atmos%d => calculate_d
506      atmos%c => calculate_c_atm
507
508      ! Pointing to the oceanic inner products functions
509
510      ocean%M => calculate_m
511      ocean%N => calculate_n
512      ocean%O => calculate_o
513      ocean%C => calculate_c_oc
514      ocean%W => calculate_w
515      ocean%K => calculate_k
516

```

#### 8.6.2.18 real(kind=8) function inprod\_analytic::s1 ( integer $P_j$ , integer $P_k$ , integer $M_j$ , integer $H_k$ ) [private]

Cehelsky & Tung Helper functions.

Definition at line 132 of file inprod\_analytic.f90.

```

132      INTEGER :: pk,pj,mj,hk
133      s1 = -(pk * mj + pj * hk) / 2.d0

```

#### 8.6.2.19 real(kind=8) function inprod\_analytic::s2 ( integer $P_j$ , integer $P_k$ , integer $M_j$ , integer $H_k$ ) [private]

Cehelsky & Tung Helper functions.

Definition at line 138 of file inprod\_analytic.f90.

```

138      INTEGER :: pk,pj,mj,hk
139      s2 = (pk * mj - pj * hk) / 2.d0

```

#### 8.6.2.20 real(kind=8) function inprod\_analytic::s3 ( integer $P_j$ , integer $P_k$ , integer $H_j$ , integer $H_k$ ) [private]

Cehelsky & Tung Helper functions.

Definition at line 144 of file inprod\_analytic.f90.

```

144      INTEGER :: pj,pk,hj,hk
145      s3 = (pk * hj + pj * hk) / 2.d0

```

8.6.2.21 `real(kind=8) function inprod_analytic::s4 ( integer Pj, integer Pk, integer Hj, integer Hk ) [private]`

Cehelsky & Tung Helper functions.

Definition at line 150 of file `inprod_analytic.f90`.

```
150      INTEGER :: pj,pk,hj,hk
151      s4 = (pk * hj - pj * hk) / 2.d0
```

### 8.6.3 Variable Documentation

8.6.3.1 `type(atm_tensors), public inprod_analytic::atmos`

Atmospheric tensors.

Definition at line 78 of file `inprod_analytic.f90`.

```
78      TYPE(atm_tensors), PUBLIC :: atmos
```

8.6.3.2 `type(atm_wavenum), dimension(:), allocatable, public inprod_analytic::awavenum`

Atmospheric blocs specification.

Definition at line 73 of file `inprod_analytic.f90`.

```
73      TYPE(atm_wavenum), DIMENSION(:), ALLOCATABLE, PUBLIC :: awavenum
```

8.6.3.3 `type(ocean_tensors), public inprod_analytic::ocean`

Oceanic tensors.

Definition at line 80 of file `inprod_analytic.f90`.

```
80      TYPE(ocean_tensors), PUBLIC :: ocean
```

8.6.3.4 `type(ocean_wavenum), dimension(:), allocatable, public inprod_analytic::owavenum`

Oceanic blocs specification.

Definition at line 75 of file `inprod_analytic.f90`.

```
75      TYPE(ocean_wavenum), DIMENSION(:), ALLOCATABLE, PUBLIC :: owavenum
```

## 8.7 int\_comp Module Reference

Utility module containing the routines to perform the integration of functions.

### Functions/Subroutines

- subroutine, public [integrate](#) (func, ss)
 

*Routine to compute integrals of function from O to #maxint.*
- subroutine [qromb](#) (func, a, b, ss)
 

*Romberg integration routine.*
- subroutine [qromo](#) (func, a, b, ss, choose)
 

*Romberg integration routine on an open interval.*
- subroutine [polint](#) (xa, ya, n, x, y, dy)
 

*Polynomial interpolation routine.*
- subroutine [trapzd](#) (func, a, b, s, n)
 

*Trapezoidal rule integration routine.*
- subroutine [midpnt](#) (func, a, b, s, n)
 

*Midpoint rule integration routine.*
- subroutine [midexp](#) (funk, aa, bb, s, n)
 

*Midpoint routine for bb infinite with funk decreasing infinitely rapidly at infinity.*

### 8.7.1 Detailed Description

Utility module containing the routines to perform the integration of functions.

#### Copyright

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#### Remarks

Most are taken from the Numerical Recipes

### 8.7.2 Function/Subroutine Documentation

#### 8.7.2.1 subroutine, public int\_comp::integrate ( external func, real(kind=8) ss )

Routine to compute integrals of function from O to #maxint.

#### Parameters

<i>func</i>	function to integrate
<i>ss</i>	result of the integration

Definition at line 30 of file int\_comp.f90.

```

30      REAL(KIND=8) :: ss,func,b
31      EXTERNAL func
32      b=maxint
33      ! CALL qromo(func,0.D0,1.D0,ss,midexp)
34      CALL qromb(func,0.d0,b,ss)

```

### 8.7.2.2 subroutine int\_comp::midexp ( external *func*, real(kind=8) *aa*, real(kind=8) *bb*, real(kind=8) *s*, integer *n* ) [private]

Midpoint routine for *bb* infinite with *func* decreasing infinitely rapidly at infinity.

#### Parameters

<i>func</i>	function to integrate
<i>aa</i>	lower limit of the integral
<i>bb</i>	higher limit of the integral
<i>s</i>	result of the integration
<i>n</i>	higher stage of the rule to be computed

Definition at line 200 of file int\_comp.f90.

```

200      INTEGER :: n
201      REAL(KIND=8) :: aa,bb,s,func
202      EXTERNAL func
203      INTEGER :: it,j
204      REAL(KIND=8) :: ddel,del,sum,tnm,x,func,a,b
205      func(x)=func(-log(x))/x
206      b=exp(-aa)
207      a=0.
208      if (n.eq.1) then
209          s=(b-a)*func(0.5*(a+b))
210      else
211          it=3** (n-2)
212          tnm=it
213          del=(b-a)/(3.*tnm)
214          ddel=del+del
215          x=a+0.5*del
216          sum=0.
217          do j=1,it
218              sum=sum+func(x)
219              x=x+ddel
220              sum=sum+func(x)
221              x=x+del
222          end do
223          s=(s+(b-a)*sum/tnm)/3.
224      endif
225      return

```

### 8.7.2.3 subroutine int\_comp::midpnt ( external *func*, real(kind=8) *a*, real(kind=8) *b*, real(kind=8) *s*, integer *n* ) [private]

Midpoint rule integration routine.

#### Parameters

<i>func</i>	function to integrate
<i>a</i>	lower limit of the integral
<i>b</i>	higher limit of the integral
<i>s</i>	result of the integration
<i>n</i>	higher stage of the rule to be computed

Definition at line 167 of file int\_comp.f90.

```

167      INTEGER :: n
168      REAL(KIND=8) :: a,b,s,func
169      EXTERNAL func
170      INTEGER :: it,j
171      REAL(KIND=8) :: ddel,del,sum,tnm,x
172      if (n.eq.1) then
173          s=(b-a)*func(0.5*(a+b))
174      else
175          it=3** (n-2)
176          tnm=it
177          del=(b-a)/(3.*tnm)
178          ddel=del+del
179          x=a+0.5*del
180          sum=0.
181          do j=1,it
182              sum=sum+func(x)
183              x=x+ddel
184              sum=sum+func(x)
185              x=x+del
186          end do
187          s=(s+(b-a)*sum/tnm)/3.
188      endif
189      return

```

**8.7.2.4 subroutine int\_comp::polint ( real(kind=8), dimension(n) xa, real(kind=8), dimension(n) ya, integer n, real(kind=8) x, real(kind=8) y, real(kind=8) dy ) [private]**

Polynomial interpolation routine.

Definition at line 91 of file int\_comp.f90.

```

91      INTEGER :: n,nmax
92      REAL(KIND=8) :: dy,x,y,xa(n),ya(n)
93      parameter(nmax=10)
94      INTEGER :: i,m,ns
95      REAL(KIND=8) :: den,dif,dift,ho,hp,w,c(nmax),d(nmax)
96      ns=1
97      dif=abs(x-xa(1))
98      do i=1,n
99          dift=abs(x-xa(i))
100         if (dift.lt.dif) then
101             ns=i
102             dif=dift
103         endif
104         c(i)=ya(i)
105         d(i)=ya(i)
106     end do
107     y=ya(ns)
108     ns=ns-1
109     do m=1,n-1
110         do i=1,n-m
111             ho=xa(i)-x
112             hp=xa(i+m)-x
113             w=c(i+1)-d(i)
114             den=ho-hp
115             if(den.eq.0.)stop 'failure in polint'
116             den=w/den
117             d(i)=hp*den
118             c(i)=ho*den
119         end do
120         if (2*ns.lt.n-m)then
121             dy=c(ns+1)
122         else
123             dy=d(ns)
124             ns=ns-1
125         endif
126         y=y+dy
127     end do
128     return

```

**8.7.2.5 subroutine int\_comp::qromb ( external func, real(kind=8) a, real(kind=8) b, real(kind=8) ss ) [private]**

Romberg integration routine.

### Parameters

<i>func</i>	function to integrate
<i>a</i>	lower limit of the integral
<i>b</i>	higher limit of the integral
<i>func</i>	function to integrate
<i>ss</i>	result of the integration

Definition at line 44 of file int\_comp.f90.

```

44      INTEGER :: jmax,jmaxp,k,km
45      REAL(KIND=8) :: a,b,func,ss,eps
46      EXTERNAL func
47      parameter(eps=1.d-6, jmax=20, jmaxp=jmax+1, k=5, km=k-1)
48      INTEGER j
49      REAL(KIND=8) :: dss,h(jmaxp),s(jmaxp)
50      h(1)=1.
51      DO j=1,jmax
52          CALL trapzd(func,a,b,s(j),j)
53          IF (j.ge.k) THEN
54              CALL point(h(j-km),s(j-km),k,0.d0,ss,dss)
55              IF (abs(dss).le.eps*abs(ss)) RETURN
56          ENDIF
57          s(j+1)=s(j)
58          h(j+1)=0.25*h(j)
59      ENDDO
60      stop 'too many steps in qromb'

```

### 8.7.2.6 subroutine int\_comp::qromo ( external *func*, real(kind=8) *a*, real(kind=8) *b*, real(kind=8) *ss*, external *choose* ) [private]

Romberg integration routine on an open interval.

### Parameters

<i>a</i>	lower limit of the integral
<i>b</i>	higher limit of the integral
<i>func</i>	function to integrate
<i>ss</i>	result of the integration
<i>choose</i>	routine to perform the integration

Definition at line 70 of file int\_comp.f90.

```

70      INTEGER :: jmax,jmaxp,k,km
71      REAL(KIND=8) :: a,b,func,ss,eps
72      EXTERNAL func,choose
73      parameter(eps=1.e-6, jmax=14, jmaxp=jmax+1, k=5, km=k-1)
74      INTEGER :: j
75      REAL(KIND=8) :: dss,h(jmaxp),s(jmaxp)
76      h(1)=1.
77      DO j=1,jmax
78          CALL choose(func,a,b,s(j),j)
79          IF (j.ge.k) THEN
80              call point(h(j-km),s(j-km),k,0.d0,ss,dss)
81              if (abs(dss).le.eps*abs(ss)) return
82          ENDIF
83          s(j+1)=s(j)
84          h(j+1)=h(j)/9.
85      ENDDO
86      stop 'too many steps in qromo'

```

### 8.7.2.7 subroutine int\_comp::trapzd ( external func, real(kind=8) a, real(kind=8) b, real(kind=8) s, integer n ) [private]

Trapezoidal rule integration routine.

#### Parameters

<i>func</i>	function to integrate
<i>a</i>	lower limit of the integral
<i>b</i>	higher limit of the integral
<i>s</i>	result of the integration
<i>n</i>	higher stage of the rule to be computed

Definition at line 138 of file int\_comp.f90.

```

138      INTEGER :: n
139      REAL(KIND=8) :: a,b,s,func
140      EXTERNAL func
141      INTEGER :: it,j
142      REAL(KIND=8) :: del,sum,tnm,x
143      if (n.eq.1) then
144          s=0.5*(b-a)*(func(a)+func(b))
145      else
146          it=2** (n-2)
147          tnm=it
148          del=(b-a)/tnm
149          x=a+0.5*del
150          sum=0.
151          do j=1,it
152              sum=sum+func(x)
153              x=x+del
154          end do
155          s=0.5*(s+(b-a)*sum/tnm)
156      endif
157      return

```

## 8.8 int\_corr Module Reference

Module to compute or load the integrals of the correlation matrices.

### Functions/Subroutines

- subroutine, public [init\\_corrint](#)

*Subroutine to initialise the integrated matrices and tensors.*

- real(kind=8) function [func\\_ij](#) (s)

*Function that returns the component oi and oj of the correlation matrix at time s.*

- real(kind=8) function [func\\_ijkl](#) (s)

*Function that returns the component oi,oj,ok and ol of the outer product of the correlation matrix with itself at time s.*

- subroutine, public [comp\\_corrint](#)

*Routine that actually compute or load the integrals.*

## Variables

- integer oi
  - integer oj
  - integer ok
  - integer ol
- Integers that specify the matrices and tensor component considered as a function of time.*
- real(kind=8), parameter **real\_eps** = 2.2204460492503131e-16
- Small epsilon constant to determine equality with zero.*
- real(kind=8), dimension(:, :, ), allocatable, public **corrint**
- Matrix holding the integral of the correlation matrix.*
- type(**coolist4**), dimension(:), allocatable, public **corr2int**
- Tensor holding the integral of the correlation outer product with itself.*

### 8.8.1 Detailed Description

Module to compute or load the integrals of the correlation matrices.

#### Copyright

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#### Remarks

### 8.8.2 Function/Subroutine Documentation

#### 8.8.2.1 subroutine, public int\_corr::comp\_corrint( )

Routine that actually compute or load the integrals.

Definition at line 75 of file int\_corr.f90.

```

75      IMPLICIT NONE
76      INTEGER :: i,j,k,l,n,allocstat
77      REAL(KIND=8) :: ss
78      LOGICAL :: ex
79
80      INQUIRE(file='corrint.def',exist=ex)
81      SELECT CASE (int_corr_mode)
82      CASE ('file')
83          IF (ex) THEN
84              OPEN(30,file='corrint.def',status='old')
85              READ(30,*) corrint
86              CLOSE(30)
87          ELSE
88              stop "*** File corrint.def not found ! ***"
89          END IF
90      CASE ('prog')
91          DO i = 1,n_unres
92              DO j= 1,n_unres
93                  oi=i
94                  oj=j
95                  CALL integrate(func_ij,ss)
96                  corrint(ind(i),ind(j))=ss
97              END DO
98          END DO
99
100         OPEN(30,file='corrint.def')
```

```

101      WRITE(30,*) corrint
102      CLOSE(30)
103  END SELECT
104
105
106  INQUIRE(file='corr2int.def',exist=ex)
107  SELECT CASE (int_corr_mode)
108  CASE ('file')
109    IF (ex) THEN
110      CALL load_tensor4_from_file("corr2int.def",corr2int)
111    ELSE
112      stop "*** File corr2int.def not found ! ***"
113    END IF
114  CASE ('prog')
115    DO i = 1,n_unres
116      n=0
117      DO j= 1,n_unres
118        DO k= 1,n_unres
119          DO l = 1,n_unres
120            oi=i
121            oj=j
122            ok=k
123            ol=l
124
125            CALL integrate(func_ijkl,ss)
126            IF (abs(ss)>real_eps) n=n+1
127          ENDDO
128        ENDDO
129      ENDDO
130      IF (n/=0) THEN
131        ALLOCATE(corr2int(ind(i))%elems(n), stat=allocstat)
132        IF (allocstat /= 0) stop "*** Not enough memory ! ***"
133
134      n=0
135      DO j= 1,n_unres
136        DO k= 1,n_unres
137          DO l = 1,n_unres
138            oi=i
139            oj=j
140            ok=k
141            ol=l
142
143            CALL integrate(func_ijkl,ss)
144            IF (abs(ss)>real_eps) THEN
145              n=n+1
146              corr2int(ind(i))%elems(n)%j=ind(j)
147              corr2int(ind(i))%elems(n)%k=ind(k)
148              corr2int(ind(i))%elems(n)%l=ind(l)
149              corr2int(ind(i))%elems(n)%v=ss
150            END IF
151          ENDDO
152        ENDDO
153      ENDDO
154      corr2int(ind(i))%nelems=n
155    END IF
156  ENDDO
157
158  CALL write_tensor4_to_file("corr2int.def",corr2int)
159 CASE DEFAULT
160   stop '*** INT_CORR_MODE variable not properly defined in corrmod.nml ***'
161 END SELECT
162

```

### 8.8.2.2 real(kind=8) function int\_corr::func\_ij( real(kind=8) s ) [private]

Function that returns the component oi and oj of the correlation matrix at time s.

#### Parameters

<b>s</b>	time at which the function is evaluated
----------	---

Definition at line 55 of file int\_corr.f90.

```

56      REAL(KIND=8) :: s, func_ij
57      CALL corrcomp(s)
58      func_ij=corr_ij(oi,oj)
59      RETURN

```

### 8.8.2.3 real(kind=8) function int\_corr::func\_ijkl ( real(kind=8) s ) [private]

Function that returns the component oi,oj,ok and ol of the outer product of the correlation matrix with itself at time s.

#### Parameters

<b>s</b>	time at which the function is evaluated
----------	---

Definition at line 66 of file int\_corr.f90.

```

66      IMPLICIT NONE
67      REAL(KIND=8) :: s, func_ijkl
68      CALL corrcomp(s)
69      func_ijkl=corr_ij(oi,oj)*corr_ij(ok,ol)
70      RETURN

```

### 8.8.2.4 subroutine, public int\_corr::init\_corrint ( )

Subroutine to initialise the integrated matrices and tensors.

Definition at line 38 of file int\_corr.f90.

```

38      INTEGER :: allocstat
39
40      ALLOCATE(corrint(ndim,ndim), stat=allocstat)
41      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
42
43      ALLOCATE(corr2int(ndim), stat=allocstat)
44      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
45
46      CALL init_corr ! Initialize the correlation matrix function
47
48      corrint=0.d0
49

```

## 8.8.3 Variable Documentation

### 8.8.3.1 type(coolist4), dimension(:), allocatable, public int\_corr::corr2int

Tensor holding the integral of the correlation outer product with itself.

Definition at line 30 of file int\_corr.f90.

```

30      TYPE(coolist4), DIMENSION(:), ALLOCATABLE, PUBLIC :: corr2int !< Tensor holding the integral of
                     the correlation outer product with itself

```

### 8.8.3.2 real(kind=8), dimension(:, :, ), allocatable, public int\_corr::corrint

Matrix holding the integral of the correlation matrix.

Definition at line 29 of file int\_corr.f90.

```
29    REAL(KIND=8), DIMENSION(:, :, ), ALLOCATABLE, PUBLIC :: corrint !< Matrix holding the integral of the
correlation matrix
```

### 8.8.3.3 integer int\_corr::oi [private]

Definition at line 26 of file int\_corr.f90.

```
26    INTEGER :: oi,oj,ok,ol !< Integers that specify the matrices and tensor component considered as a
function of time
```

### 8.8.3.4 integer int\_corr::oj [private]

Definition at line 26 of file int\_corr.f90.

### 8.8.3.5 integer int\_corr::ok [private]

Definition at line 26 of file int\_corr.f90.

### 8.8.3.6 integer int\_corr::ol [private]

Integers that specify the matrices and tensor component considered as a function of time.

Definition at line 26 of file int\_corr.f90.

### 8.8.3.7 real(kind=8), parameter int\_corr::real\_eps = 2.2204460492503131e-16 [private]

Small epsilon constant to determine equality with zero.

Definition at line 27 of file int\_corr.f90.

```
27    REAL(KIND=8), PARAMETER :: real_eps = 2.2204460492503131e-16 !< Small epsilon constant to determine
equality with zero
```

## 8.9 integrator Module Reference

Module with the integration routines.

## Functions/Subroutines

- subroutine, public `init_integrator`  
*Routine to initialise the integration buffers.*
- subroutine `tendencies` (`t, y, res`)  
*Routine computing the tendencies of the model.*
- subroutine, public `step` (`y, t, dt, res`)  
*Routine to perform an integration step (Heun algorithm). The incremented time is returned.*

## Variables

- real(kind=8), dimension(:), allocatable `buf_y1`  
*Buffer to hold the intermediate position (Heun algorithm)*
- real(kind=8), dimension(:), allocatable `buf_f0`  
*Buffer to hold tendencies at the initial position.*
- real(kind=8), dimension(:), allocatable `buf_f1`  
*Buffer to hold tendencies at the intermediate position.*
- real(kind=8), dimension(:), allocatable `buf_ka`  
*Buffer A to hold tendencies.*
- real(kind=8), dimension(:), allocatable `buf_kb`  
*Buffer B to hold tendencies.*

### 8.9.1 Detailed Description

Module with the integration routines.

Module with the RK4 integration routines.

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#### Remarks

This module actually contains the Heun algorithm routines. The user can modify it according to its preferred integration scheme. For higher-order schemes, additional buffers will probably have to be defined.

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#### Remarks

This module actually contains the RK4 algorithm routines. The user can modify it according to its preferred integration scheme. For higher-order schemes, additional buffers will probably have to be defined.

## 8.9.2 Function/Subroutine Documentation

### 8.9.2.1 subroutine public integrator::init\_integrator( )

Routine to initialise the integration buffers.

Definition at line 37 of file rk2\_integrator.f90.

```
37      INTEGER :: allocstat
38      ALLOCATE(buf_y1(0:ndim),buf_f0(0:ndim),buf_f1(0:ndim) ,stat=allocstat)
39      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
```

### 8.9.2.2 subroutine public integrator::step ( real(kind=8), dimension(0:ndim), intent(in) y, real(kind=8), intent(inout) t, real(kind=8), intent(in) dt, real(kind=8), dimension(0:ndim), intent(out) res )

Routine to perform an integration step (Heun algorithm). The incremented time is returned.

Routine to perform an integration step (RK4 algorithm). The incremented time is returned.

#### Parameters

<i>y</i>	Initial point.
<i>t</i>	Actual integration time
<i>dt</i>	Integration timestep.
<i>res</i>	Final point after the step.

Definition at line 61 of file rk2\_integrator.f90.

```
61      REAL(KIND=8), DIMENSION(0:ndim), INTENT(IN) :: y
62      REAL(KIND=8), INTENT(INOUT) :: t
63      REAL(KIND=8), INTENT(IN) :: dt
64      REAL(KIND=8), DIMENSION(0:ndim), INTENT(OUT) :: res
65
66      CALL tendencies(t,y,buf_f0)
67      buf_y1 = y+dt*buf_f0
68      CALL tendencies(t+dt,buf_y1,buf_f1)
69      res=y+0.5*(buf_f0+buf_f1)*dt
70      t=t+dt
```

### 8.9.2.3 subroutine integrator::tendencies ( real(kind=8), intent(in) t, real(kind=8), dimension(0:ndim), intent(in) y, real(kind=8), dimension(0:ndim), intent(out) res ) [private]

Routine computing the tendencies of the model.

#### Parameters

<i>t</i>	Time at which the tendencies have to be computed. Actually not needed for autonomous systems.
<i>y</i>	Point at which the tendencies have to be computed.
<i>res</i>	vector to store the result.

**Remarks**

Note that it is NOT safe to pass `y` as a result buffer, as this operation does multiple passes.

Definition at line 49 of file `rk2_integrator.f90`.

```
49      REAL(KIND=8), INTENT(IN) :: t
50      REAL(KIND=8), DIMENSION(0:ndim), INTENT(IN) :: y
51      REAL(KIND=8), DIMENSION(0:ndim), INTENT(OUT) :: res
52      CALL sparse_mul3(aotensor, y, y, res)
```

### 8.9.3 Variable Documentation

#### 8.9.3.1 real(kind=8), dimension(:), allocatable integrator::buf\_f0 [private]

Buffer to hold tendencies at the initial position.

Definition at line 28 of file `rk2_integrator.f90`.

```
28      REAL(KIND=8), DIMENSION(:, ALLOCATABLE :: buf_f0 !< Buffer to hold tendencies at the initial position
```

#### 8.9.3.2 real(kind=8), dimension(:), allocatable integrator::buf\_f1 [private]

Buffer to hold tendencies at the intermediate position.

Definition at line 29 of file `rk2_integrator.f90`.

```
29      REAL(KIND=8), DIMENSION(:, ALLOCATABLE :: buf_f1 !< Buffer to hold tendencies at the intermediate position
```

#### 8.9.3.3 real(kind=8), dimension(:), allocatable integrator::buf\_ka [private]

Buffer A to hold tendencies.

Definition at line 28 of file `rk4_integrator.f90`.

```
28      REAL(KIND=8), DIMENSION(:, ALLOCATABLE :: buf_ka !< Buffer A to hold tendencies
```

#### 8.9.3.4 real(kind=8), dimension(:), allocatable integrator::buf\_kb [private]

Buffer B to hold tendencies.

Definition at line 29 of file `rk4_integrator.f90`.

```
29      REAL(KIND=8), DIMENSION(:, ALLOCATABLE :: buf_kb !< Buffer B to hold tendencies
```

### 8.9.3.5 real(kind=8), dimension(:,), allocatable integrator::buf\_y1 [private]

Buffer to hold the intermediate position (Heun algorithm)

Definition at line 27 of file rk2\_integrator.f90.

```
27    REAL(KIND=8), DIMENSION(:,), ALLOCATABLE :: buf_y1 !< Buffer to hold the intermediate position (Heun
algorithm)
```

## 8.10 mar Module Reference

Multidimensional Autoregressive module to generate the correlation for the WL parameterization.

### Functions/Subroutines

- subroutine, public `init_mar`  
*Subroutine to initialise the MAR.*
- subroutine, public `mar_step (x)`  
*Routine to generate one step of the MAR.*
- subroutine, public `mar_step_red (xred)`  
*Routine to generate one step of the reduce MAR.*
- subroutine `stoch_vec (dW)`

### Variables

- real(kind=8), dimension(:, :, ), allocatable, public `q`  
*Square root of the noise covariance matrix.*
- real(kind=8), dimension(:, :, ), allocatable, public `qred`  
*Reduce version of Q.*
- real(kind=8), dimension(:, :, ), allocatable, public `rred`  
*Covariance matrix of the noise.*
- real(kind=8), dimension(:, :, :, ), allocatable, public `w`  
*W\_i matrix.*
- real(kind=8), dimension(:, :, :, ), allocatable, public `wred`  
*Reduce W\_i matrix.*
- real(kind=8), dimension(:, ), allocatable `buf_y`
- real(kind=8), dimension(:, ), allocatable `dw`
- integer, public `ms`  
*order of the MAR*

### 8.10.1 Detailed Description

Multidimensional Autoregressive module to generate the correlation for the WL parameterization.

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#### Remarks

Based on the equation  $y_n = \sum_{i=1}^m y_{n-i} \cdot W_i + Q \cdot \xi_n$  for an order

## 8.10.2 Function/Subroutine Documentation

### 8.10.2.1 subroutine, public mar::init\_mar( )

Subroutine to initialise the MAR.

Definition at line 45 of file MAR.f90.

```

45      INTEGER :: allocstat,nf,i,info,info2
46      INTEGER, DIMENSION(3) :: s
47
48      print*, 'Initializing the MAR integrator...'
49
50      print*, 'Loading the MAR config from files...'
51
52      OPEN(20,file='MAR_R_params.def',status='old')
53      READ(20,*) nf,ms
54      IF (nf /= n_unres) stop "*** Dimension in files MAR_R_params.def and sf.nml do not correspond ! ***"
55      ALLOCATE(qred(n_unres,n_unres),rred(n_unres,n_unres),wred(ms,n_unres,n_unres),
56      stat=allocstat)
57      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
58      ALLOCATE(q(ndim,ndim),w(ms,ndim,ndim), stat=allocstat)
59      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
60      ALLOCATE(buf_y(0:ndim), dw(ndim), stat=allocstat)
61      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
62      READ(20,*) rred
63      CLOSE(20)
64
65      OPEN(20,file='MAR_W_params.def',status='old')
66      READ(20,*) nf,ms
67      s=shape(wred)
68      IF (nf /= n_unres) stop "*** Dimension in files MAR_W_params.def and sf.nml do not correspond ! ***"
69      IF (s(1) /= ms) stop "*** MAR order in files MAR_R_params.def and MAR_W_params.def do not correspond !
70      ***"
71      DO i=1,ms
72          READ(20,*) wred(i,:,:)
73      ENDDO
74      CLOSE(20)
75
76      CALL init_sqrt
77      CALL sqrtm(rred,qred,info,info2)
78      CALL ireduce(q,qred,n_unres,ind,rind)
79
80      DO i=1,ms
81          CALL ireduce(w(i,:,:),wred(i,:,:),n_unres,ind,rind)
82      ENDDO
83
84      ! Kept for internal testing - Uncomment if not needed
85      ! DEALLOCATE(Wred,Rred,Qred, STAT=AllocStat)
86      ! IF (AllocStat /= 0) STOP "*** Deallocation problem ! ***"
87
88      print*, 'MAR of order',ms,'found!'
89

```

### 8.10.2.2 subroutine, public mar::mar\_step( real(kind=8), dimension(0:ndim,ms), intent(inout) x )

Routine to generate one step of the MAR.

#### Parameters

<i>x</i>	State vector of the MAR (store the $y_i$ )
----------	--

Definition at line 93 of file MAR.f90.

```

93      REAL(KIND=8), DIMENSION(0:ndim,ms), INTENT(INOUT) :: x
94      INTEGER :: j
95

```

```

96      CALL stoch_vec(dw)
97      buf_y=0.d0
98      buf_y(1:ndim)=matmul(q,dw)
99      DO j=1,ms
100         buf_y(1:ndim)=buf_y(1:ndim)+matmul(x(1:ndim,j),w(j,:,:))
101      ENDDO
102      x=eoshift(x,shift=-1,boundary=buf_y,dim=2)

```

### 8.10.2.3 subroutine, public mar::mar\_step\_red ( real(kind=8), dimension(0:ndim,ms), intent(inout) xred )

Routine to generate one step of the reduce MAR.

#### Parameters

xred	State vector of the MAR (store the $y_i$ )
------	--

#### Remarks

For debugging purpose only

Definition at line 110 of file MAR.f90.

```

110      REAL(KIND=8), DIMENSION(0:ndim,ms), INTENT(INOUT) :: xred
111      INTEGER :: j
112
113      CALL stoch_vec(dw)
114      buf_y=0.d0
115      buf_y(1:n_unres)=matmul(qred,dw(1:n_unres))
116      DO j=1,ms
117         buf_y(1:n_unres)=buf_y(1:n_unres)+matmul(xred(1:n_unres,j),wred(j,:,:))
118      ENDDO
119      xred=eoshift(xred,shift=-1,boundary=buf_y,dim=2)

```

### 8.10.2.4 subroutine mar::stoch\_vec ( real(kind=8), dimension(ndim), intent(inout) dw ) [private]

Definition at line 125 of file MAR.f90.

```

125      REAL(KIND=8), DIMENSION(ndim), INTENT(INOUT) :: dw
126      INTEGER :: i
127      DO i=1,ndim
128         dw(i)=gasdev()
129      ENDDO

```

## 8.10.3 Variable Documentation

### 8.10.3.1 real(kind=8), dimension(:), allocatable mar::buf\_y [private]

Definition at line 34 of file MAR.f90.

```

34      REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: buf_y,dw

```

**8.10.3.2 real(kind=8), dimension(:), allocatable mar::dw [private]**

Definition at line 34 of file MAR.f90.

**8.10.3.3 integer, public mar::ms**

order of the MAR

Definition at line 36 of file MAR.f90.

```
36    INTEGER :: ms !< order of the MAR
```

**8.10.3.4 real(kind=8), dimension(:, :, ), allocatable, public mar::q**

Square root of the noise covariance matrix.

Definition at line 29 of file MAR.f90.

```
29    REAL(KIND=8), DIMENSION(:, :, ), ALLOCATABLE :: q !< Square root of the noise covariance matrix
```

**8.10.3.5 real(kind=8), dimension(:, :, ), allocatable, public mar::qred**

Reduce version of Q.

Definition at line 30 of file MAR.f90.

```
30    REAL(KIND=8), DIMENSION(:, :, ), ALLOCATABLE :: qred !< Reduce version of Q
```

**8.10.3.6 real(kind=8), dimension(:, :, ), allocatable, public mar::rred**

Covariance matrix of the noise.

Definition at line 31 of file MAR.f90.

```
31    REAL(KIND=8), DIMENSION(:, :, ), ALLOCATABLE :: rred !< Covariance matrix of the noise
```

**8.10.3.7 real(kind=8), dimension(:, :, :, ), allocatable, public mar::w**

W\_i matrix.

Definition at line 32 of file MAR.f90.

```
32    REAL(KIND=8), DIMENSION(:, :, :, ), ALLOCATABLE :: w !< W_i matrix
```

### 8.10.3.8 real(kind=8), dimension(:,:,:), allocatable, public mar::wred

Reduce W\_i matrix.

Definition at line 33 of file MAR.f90.

```
33    REAL(KIND=8), DIMENSION(:,:,:), ALLOCATABLE :: wred !< Reduce W_i matrix
```

## 8.11 memory Module Reference

Module that compute the memory term  $M_3$  of the WL parameterization.

### Functions/Subroutines

- subroutine, public **init\_memory**  
*Subroutine to initialise the memory.*
- subroutine, public **compute\_m3** (y, dt, dtn, savey, save\_ev, evolve, inter, h\_int)  
*Compute the integrand of  $M_3$  at each time in the past and integrate to get the memory term.*
- subroutine, public **test\_m3** (y, dt, dtn, h\_int)  
*Routine to test the #compute\_M3 routine.*

### Variables

- real(kind=8), dimension(:, :, :), allocatable **x**  
*Array storing the previous state of the system.*
- real(kind=8), dimension(:, :, :), allocatable **xs**  
*Array storing the resolved time evolution of the previous state of the system.*
- real(kind=8), dimension(:, :, :), allocatable **zs**  
*Dummy array to replace Xs in case where the evolution is not stored.*
- real(kind=8), dimension(:, :), allocatable **buf\_m**  
*Dummy vector.*
- real(kind=8), dimension(:, :), allocatable **buf\_m3**  
*Dummy vector to store the  $M_3$  integrand.*
- integer **t\_index**  
*Integer storing the time index (current position in the arrays)*
- procedure(ss\_step), pointer **step**  
*Procedural pointer pointing on the resolved dynamics step routine.*

### 8.11.1 Detailed Description

Module that compute the memory term  $M_3$  of the WL parameterization.

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#### Remarks

## 8.11.2 Function/Subroutine Documentation

8.11.2.1 subroutine, public memory::compute\_m3 ( real(kind=8), dimension(0:ndim), intent(in) y, real(kind=8), intent(in) dt, real(kind=8), intent(in) dtn, logical, intent(in) savey, logical, intent(in) save\_ev, logical, intent(in) evolve, real(kind=8), intent(in) inter, real(kind=8), dimension(0:ndim), intent(out) h\_int )

Compute the integrand of  $M_3$  at each time in the past and integrate to get the memory term.

### Parameters

<i>y</i>	current state
<i>dt</i>	timestep
<i>dtn</i>	stochastic timestep
<i>savey</i>	set if the state is stored in X at the end
<i>save_ev</i>	set if the result of the resolved time evolution is stored in Xs at the end
<i>evolve</i>	set if the resolved time evolution is performed
<i>inter</i>	set over which time interval the resolved time evolution must be computed
<i>h_int</i>	result of the integration - give the memory term

Definition at line 86 of file memory.f90.

```

86      REAL(KIND=8), DIMENSION(0:ndim), INTENT(IN) :: y
87      REAL(KIND=8), INTENT(IN) :: dt,dtn
88      LOGICAL, INTENT(IN) :: savey,save_ev,evolve
89      REAL(KIND=8), INTENT(IN) :: inter
90      REAL(KIND=8), DIMENSION(0:ndim), INTENT(OUT) :: h_int
91      REAL(KIND=8) :: t
92      INTEGER :: i,j
93
94      x(:,t_index)=y
95      IF (b23def) THEN
96          xs(:,t_index)=y
97          zs(:,t_index)=y
98          DO i=1,mems-1
99              j=modulo(t_index+i-1,mems)+1
100             zs(:,j)=xs(:,j)
101             IF (evolve) THEN
102                 IF (dt.lt.inter) THEN
103                     t=0.d0
104                     DO WHILE (t+dt<inter)
105                         CALL step(zs(:,j),y,t,dt,dtn,zs(:,j))
106                     ENDDO
107                     CALL step(zs(:,j),y,t,inter-t,sqrt(inter-t),zs(:,j))
108                 ELSE
109                     CALL step(zs(:,j),y,t,inter,sqrt(inter),zs(:,j))
110                 ENDIF
111             ENDIF
112             IF (save_ev) xs(:,j)=zs(:,j)
113         ENDDO
114     ENDIF
115
116     ! Computing the integral
117     h_int=0.d0
118
119     DO i=1,mems
120         j=modulo(t_index+i-2,mems)+1
121         buf_m3=0.d0
122         IF (ldef) THEN
123             CALL sparse_mul3(ltot(:,i),y,x(:,j),buf_m)
124             buf_m3=buf_m3+buf_m
125         ENDIF
126         IF (b14def) THEN
127             CALL sparse_mul3(b14(:,i),x(:,j),x(:,j),buf_m)
128             buf_m3=buf_m3+buf_m
129         ENDIF
130         IF (b23def) THEN
131             CALL sparse_mul3(b23(:,i),x(:,j),zs(:,j),buf_m)
132             buf_m3=buf_m3+buf_m
133         ENDIF

```

```

135      IF (mdef) THEN
136          CALL sparse_mul4(mtot(:,i),x(:,j),x(:,j),zs(:,j),buf_m)
137          buf_m3=buf_m3+buf_m
138      ENDIF
139      IF ((i.eq.1).or.(i.eq.mems)) THEN
140          h_int=h_int+0.5*buf_m3
141      ELSE
142          h_int=h_int+buf_m3
143      ENDIF
144  ENDDO
145
146  h_int=muti*h_int
147  IF (savexy) THEN
148      t_index=t_index-1
149      IF (t_index.eq.0) t_index=mems
150  ENDIF

```

### 8.11.2.2 subroutine, public memory::init\_memory( )

Subroutine to initialise the memory.

Definition at line 45 of file memory.f90.

```

45      INTEGER :: allocstat
46
47      t_index=mems
48
49      ALLOCATE(x(0:ndim,mems), stat=allocstat)
50      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
51
52      x=0.d0
53
54      IF (b23def) THEN
55          ALLOCATE(xs(0:ndim,mems), zs(0:ndim,mems), stat=allocstat)
56          IF (allocstat /= 0) stop "*** Not enough memory ! ***"
57
58          xs=0.d0
59      ENDIF
60
61      ALLOCATE(buf_m3(0:ndim), buf_m(0:ndim), stat=allocstat)
62      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
63
64      SELECT CASE (x_int_mode)
65      CASE('reso')
66          step => ss_step
67      CASE('tang')
68          step => ss_t1_step
69      CASE DEFAULT
70          stop '*** X_INT_MODE variable not properly defined in stoch_params.nml ***'
71      END SELECT
72

```

### 8.11.2.3 subroutine, public memory::test\_m3 ( real(kind=8), dimension(0:ndim), intent(in) y, real(kind=8), intent(in) dt, real(kind=8), intent(in) dtn, real(kind=8), dimension(0:ndim), intent(out) h\_int )

Routine to test the #compute\_M3 routine.

#### Parameters

<i>y</i>	current state
<i>dt</i>	timestep
<i>dtn</i>	stochastic timestep
<i>h_int</i>	result of the integration - give the memory term

Definition at line 159 of file memory.f90.

```

159      REAL(KIND=8), DIMENSION(0:ndim), INTENT(IN) :: y
160      REAL(KIND=8), INTENT(IN) :: dt,dtn
161      REAL(KIND=8), DIMENSION(0:ndim), INTENT(OUT) :: h_int
162      INTEGER :: i,j
163
164      CALL compute_m3(y,dt,dtn,.true.,.true.,.true.,muti,h_int)
165      print*, t_index
166      print*, 'X'
167      DO i=1,mems
168          j=modulo(t_index+i-1,mems)+1
169          print*, i,j,x(1,j)
170      ENDDO
171
172      IF (b23def) THEN
173          print*, 'Xs'
174          DO i=1,mems
175              j=modulo(t_index+i-1,mems)+1
176              print*, i,j,xs(1,j)
177          ENDDO
178      ENDIF
179      print*, 'h_int',h_int

```

### 8.11.3 Variable Documentation

#### 8.11.3.1 real(kind=8), dimension(:), allocatable memory::buf\_m [private]

Dummy vector.

Definition at line 31 of file memory.f90.

```
31      REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: buf_m !< Dummy vector
```

#### 8.11.3.2 real(kind=8), dimension(:), allocatable memory::buf\_m3 [private]

Dummy vector to store the  $M_3$  integrand.

Definition at line 32 of file memory.f90.

```
32      REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: buf_m3 !< Dummy vector to store the \f$M_3\f$ integrand
```

#### 8.11.3.3 procedure(ss\_step), pointer memory::step [private]

Procedural pointer pointing on the resolved dynamics step routine.

Definition at line 36 of file memory.f90.

```
36      PROCEDURE(ss_step), POINTER :: step !< Procedural pointer pointing on the resolved dynamics step routine
```

#### 8.11.3.4 integer memory::t\_index [private]

Integer storing the time index (current position in the arrays)

Definition at line 34 of file memory.f90.

```
34      INTEGER :: t_index !< Integer storing the time index (current position in the arrays)
```

### 8.11.3.5 `real(kind=8), dimension(:, :, allocatable memory::x [private]`

Array storing the previous state of the system.

Definition at line 28 of file `memory.f90`.

```
28  REAL(KIND=8), DIMENSION(:, :, ALLOCATABLE :: x !< Array storing the previous state of the system
```

### 8.11.3.6 `real(kind=8), dimension(:, :, allocatable memory::xs [private]`

Array storing the resolved time evolution of the previous state of the system.

Definition at line 29 of file `memory.f90`.

```
29  REAL(KIND=8), DIMENSION(:, :, ALLOCATABLE :: xs !< Array storing the resolved time evolution of the
   previous state of the system
```

### 8.11.3.7 `real(kind=8), dimension(:, :, allocatable memory::zs [private]`

Dummy array to replace Xs in case where the evolution is not stored.

Definition at line 30 of file `memory.f90`.

```
30  REAL(KIND=8), DIMENSION(:, :, ALLOCATABLE :: zs !< Dummy array to replace Xs in case where the evolution
   is not stored
```

## 8.12 `mtv_int_tensor` Module Reference

The MTV tensors used to integrate the MTV model.

### Functions/Subroutines

- subroutine, public `init_mtv_int_tensor`

*Subroutine to initialise the MTV tensor.*

## Variables

- real(kind=8), dimension(:), allocatable, public **h1**

*First constant vector.*
- real(kind=8), dimension(:), allocatable, public **h2**

*Second constant vector.*
- real(kind=8), dimension(:), allocatable, public **h3**

*Third constant vector.*
- real(kind=8), dimension(:), allocatable, public **htot**

*Total constant vector.*
- type(**coolist**), dimension(:), allocatable, public **l1**

*First linear tensor.*
- type(**coolist**), dimension(:), allocatable, public **l2**

*Second linear tensor.*
- type(**coolist**), dimension(:), allocatable, public **l3**

*Third linear tensor.*
- type(**coolist**), dimension(:), allocatable, public **ltot**

*Total linear tensor.*
- type(**coolist**), dimension(:), allocatable, public **b1**

*First quadratic tensor.*
- type(**coolist**), dimension(:), allocatable, public **b2**

*Second quadratic tensor.*
- type(**coolist**), dimension(:), allocatable, public **btot**

*Total quadratic tensor.*
- type(**coolist4**), dimension(:), allocatable, public **mtot**

*Tensor for the cubic terms.*
- real(kind=8), dimension(:,:,), allocatable, public **q1**

*Constant terms for the state-dependent noise covariance matrix.*
- real(kind=8), dimension(:,:,), allocatable, public **q2**

*Constant terms for the state-independent noise covariance matrix.*
- type(**coolist**), dimension(:), allocatable, public **utot**

*Linear terms for the state-dependent noise covariance matrix.*
- type(**coolist4**), dimension(:), allocatable, public **vtot**

*Quadratic terms for the state-dependent noise covariance matrix.*
- real(kind=8), dimension(:), allocatable **dumb\_vec**

*Dummy vector.*
- real(kind=8), dimension(:,:,), allocatable **dumb\_mat1**

*Dummy matrix.*
- real(kind=8), dimension(:,:,), allocatable **dumb\_mat2**

*Dummy matrix.*
- real(kind=8), dimension(:,:,), allocatable **dumb\_mat3**

*Dummy matrix.*
- real(kind=8), dimension(:,:,), allocatable **dumb\_mat4**

*Dummy matrix.*

### 8.12.1 Detailed Description

The MTV tensors used to integrate the MTV model.

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#### Remarks

See : Franzke, C., Majda, A. J., & Vanden-Eijnden, E. (2005). Low-order stochastic mode reduction for a realistic barotropic model climate. *Journal of the atmospheric sciences*, 62(6), 1722-1745.

### 8.12.2 Function/Subroutine Documentation

#### 8.12.2.1 subroutine, public mtv\_int\_tensor::init\_mtv\_int\_tensor( )

Subroutine to initialise the MTV tensor.

Definition at line 89 of file MTV\_int\_tensor.f90.

```

89      INTEGER :: allocstat,i,j,k,l
90
91      print*, 'Initializing the decomposition tensors...'
92      CALL init_dec_tensor
93      print*, "Initializing the correlation matrices and tensors..."
94      CALL init_corrint
95      print*, "Computing the correlation integrated matrices and tensors..."
96      CALL comp_corrint
97
98      !H part
99      print*, "Computing the H term..."
100
101      ALLOCATE(h1(0:ndim), h2(0:ndim), h3(0:ndim), htot(0:ndim),
102      stat=allocstat)
103      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
104
105      ALLOCATE(dumb_mat1(ndim,ndim), dumb_mat2(ndim,ndim), stat=allocstat)
106      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
107
108      ALLOCATE(dumb_mat3(ndim,ndim), dumb_mat4(ndim,ndim), stat=allocstat)
109      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
110
111      !H1
112      CALL coo_to_mat_ik(lxy,dumb_mat1)
113      dumb_mat2=matmul(dumb_mat1,corrint)
114      CALL sparse_mul3_with_mat(bxxy,dumb_mat2,h1)
115
116      ! H2
117      h2=0.d0
118      IF (mode.ne.'ures') THEN
119          CALL coo_to_mat_ik(lyy,dumb_mat1)
120          dumb_mat1=matmul(inv_corr_i_full,dumb_mat1)
121
122          DO i=1,ndim
123              CALL coo_to_mat_i(i,bxxy,dumb_mat2)
124              CALL sparse_mul4_with_mat_jl(corr2int,dumb_mat2,dumb_mat3)
125              CALL sparse_mul4_with_mat_jl(corr2int,transpose(dumb_mat2),dumb_mat4)
126              dumb_mat3=dumb_mat3+dumb_mat4
127              h2(i)=mat_contract(dumb_mat1,dumb_mat3)
128          ENDDO
129      ENDIF
130
131      !H3
132      h3=0.d0
133      CALL sparse_mul3_with_mat(bxxy,corr_i_full,h3)
134
135      !Htot
136      htot=0.d0
137      htot=h1+h2+h3

```

```

137      print*, "Computing the L terms..."
138      ALLOCATE(l1(ndim), l2(ndim), l3(ndim), stat=allocstat)
139      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
140
141      !L1
142      CALL coo_to_mat_ik(lxy,dumb_mat1)
143      CALL coo_to_mat_ik(lxy,dumb_mat2)
144      dumb_mat3=matmul(inv_corr_i_full,corrint)
145      dumb_mat4=matmul(dumb_mat2,matmul(transpose(dumb_mat3),dumb_mat1))
146      CALL matc_to_coo(dumb_mat4,11)
147
148      !L2
149      dumb_mat4=0.d0
150      DO i=1,ndim
151          DO j=1,ndim
152              CALL coo_to_mat_i(i,bxxy,dumb_mat1)
153              CALL sparse_mul4_with_mat_jl(corr2int,dumb_mat1+transpose(dumb_mat1),dumb_mat2)
154
155              CALL coo_to_mat_j(j,byxy,dumb_mat1)
156              dumb_mat1=matmul(inv_corr_i_full,dumb_mat1)
157              dumb_mat4(i,j)=mat_contract(dumb_mat1,dumb_mat2)
158
159          END DO
160      END DO
161      CALL matc_to_coo(dumb_mat4,12)
162
163      !L3
164      dumb_mat4=0.d0
165      DO i=1,ndim
166          DO j=1,ndim
167              CALL coo_to_mat_j(j,bxxx,dumb_mat1)
168              CALL coo_to_mat_i(i,bxxx,dumb_mat2)
169              dumb_mat4(i,j)=mat_trace(matmul(dumb_mat1,matmul(corrint,transpose(dumb_mat2))))
170          ENDDO
171      END DO
172      CALL matc_to_coo(dumb_mat4,13)
173
174      !Ltot
175
176      ALLOCATE(ltot(ndim), stat=allocstat)
177      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
178
179      CALL add_to_tensor(l1,ltot)
180      CALL add_to_tensor(l2,ltot)
181      CALL add_to_tensor(l3,ltot)
182
183      print*, "Computing the B terms..."
184      ALLOCATE(b1(ndim), b2(ndim), btot(ndim), stat=allocstat)
185      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
186      ALLOCATE(dumb_vec(ndim), stat=allocstat)
187      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
188
189      ! B1
190      CALL coo_to_mat_ik(lxy,dumb_mat1)
191      dumb_mat2=matmul(inv_corr_i_full,corrint)
192
193      dumb_mat3=matmul(dumb_mat1,transpose(dumb_mat2))
194      DO j=1,ndim
195          DO k=1,ndim
196              CALL coo_to_vec_jk(j,k,byxx,dumb_vec)
197              dumb_vec=matmul(dumb_mat3,dumb_vec)
198              CALL add_vec_jk_to_tensor(j,k,dumb_vec,b1)
199          ENDDO
200      END DO
201
202      ! B2
203      CALL coo_to_mat_ik(lxy,dumb_mat3)
204      dumb_mat2=matmul(inv_corr_i_full,corrint)
205
206      dumb_mat4=matmul(transpose(dumb_mat2),dumb_mat3)
207      DO i=1,ndim
208          CALL coo_to_mat_i(i,bxxy,dumb_mat1)
209          dumb_mat2=matmul(dumb_mat1,dumb_mat4)
210          CALL add_matc_to_tensor(i,dumb_mat2,b2)
211      ENDDO
212
213      CALL add_to_tensor(b1,btot)
214      CALL add_to_tensor(b2,btot)
215
216      !M
217
218      print*, "Computing the M term..."
219
220      ALLOCATE(mtot(ndim), stat=allocstat)
221      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
222
223      dumb_mat2=matmul(inv_corr_i_full,corrint)

```

```

224
225    DO i=1,ndim
226        CALL coo_to_mat_i(i,bxxy,dumb_mat1)
227        dumb_mat3=matmul(dumb_mat1,transpose(dumb_mat2))
228        DO k=1,ndim
229            DO l=1,ndim
230                CALL coo_to_vec_jk(k,l,byxx,dumb_vec)
231                dumb_vec=matmul(dumb_mat3,dumb_vec)
232                CALL add_vec_ikl_to_tensor4(i,k,l,dumb_vec,mtot)
233            ENDDO
234        END DO
235    END DO
236
237    !Q
238
239    print*, "Computing the Q terms..."
240    ALLOCATE(q1(ndim,ndim), q2(ndim,ndim), stat=allocstat)
241    IF (allocstat /= 0) stop "*** Not enough memory ! ***"
242
243    !Q1
244
245    CALL coo_to_mat_ik(lxy,dumb_mat1)
246    q1=matmul(dumb_mat1,matmul(corrint,transpose(dumb_mat1)))
247
248    !Q2
249
250    DO i=1,ndim
251        DO j=1,ndim
252            CALL coo_to_mat_i(i,bxxy,dumb_mat1)
253            CALL coo_to_mat_i(j,bxxy,dumb_mat2)
254            CALL sparse_mul4_with_mat_jl(corr2int,dumb_mat2,dumb_mat3)
255            CALL sparse_mul4_with_mat_jl(corr2int,transpose(dumb_mat2),dumb_mat4)
256            dumb_mat2=dumb_mat3+dumb_mat4
257            q2(i,j)=mat_contract(dumb_mat1,dumb_mat2)
258        END DO
259    END DO
260
261    !U
262
263    ALLOCATE(utot(ndim), stat=allocstat)
264    IF (allocstat /= 0) stop "*** Not enough memory ! ***"
265
266    CALL coo_to_mat_ik(lxy,dumb_mat1)
267    DO i=1,ndim
268        CALL coo_to_mat_i(i,bxxy,dumb_mat2)
269        dumb_mat3=matmul(dumb_mat1,matmul(corrint,transpose(dumb_mat2)))
270        CALL add_matc_to_tensor(i,dumb_mat3,utot)
271    ENDDO
272
273    DO j=1,ndim
274        CALL coo_to_mat_i(j,bxxy,dumb_mat2)
275        dumb_mat3=matmul(dumb_mat1,matmul(corrint,transpose(dumb_mat2)))
276        DO k=1,ndim
277            CALL add_vec_jk_to_tensor(j,k,dumb_mat3(:,k),utot)
278        ENDDO
279    ENDDO
280
281    !V
282
283    ALLOCATE(vtot(ndim), stat=allocstat)
284    IF (allocstat /= 0) stop "*** Not enough memory ! ***"
285
286    DO i=1,ndim
287        DO j=1,ndim
288            CALL coo_to_mat_i(i,bxxy,dumb_mat1)
289            CALL coo_to_mat_i(j,bxxy,dumb_mat2)
290            dumb_mat3=matmul(dumb_mat1,matmul(corrint,transpose(dumb_mat2)))
291            CALL add_matc_to_tensor4(j,i,dumb_mat3,vtot)
292        ENDDO
293    ENDDO
294
295    DEALLOCATE(dumb_mat1, dumb_mat2, stat=allocstat)
296    IF (allocstat /= 0) stop "*** Problem to deallocate ! ***"
297
298    DEALLOCATE(dumb_mat3, dumb_mat4, stat=allocstat)
299    IF (allocstat /= 0) stop "*** Problem to deallocate ! ***"
300
301    DEALLOCATE(dumb_vec, stat=allocstat)
302    IF (allocstat /= 0) stop "*** Problem to deallocate ! ***"
303
304

```

### 8.12.3 Variable Documentation

**8.12.3.1 type(coolist), dimension(:), allocatable, public mtv\_int\_tensor::b1**

First quadratic tensor.

Definition at line 54 of file MTV\_int\_tensor.f90.

```
54    TYPE(coolist), DIMENSION(:), ALLOCATABLE, PUBLIC :: b1 !< First quadratic tensor
```

**8.12.3.2 type(coolist), dimension(:), allocatable, public mtv\_int\_tensor::b2**

Second quadratic tensor.

Definition at line 55 of file MTV\_int\_tensor.f90.

```
55    TYPE(coolist), DIMENSION(:), ALLOCATABLE, PUBLIC :: b2 !< Second quadratic tensor
```

**8.12.3.3 type(coolist), dimension(:), allocatable, public mtv\_int\_tensor::btot**

Total quadratic tensor.

Definition at line 56 of file MTV\_int\_tensor.f90.

```
56    TYPE(coolist), DIMENSION(:), ALLOCATABLE, PUBLIC :: btot !< Total quadratic tensor
```

**8.12.3.4 real(kind=8), dimension(:, :), allocatable mtv\_int\_tensor::dumb\_mat1 [private]**

Dummy matrix.

Definition at line 67 of file MTV\_int\_tensor.f90.

```
67    REAL(KIND=8), DIMENSION(:, :), ALLOCATABLE :: dumb_mat1 !< Dummy matrix
```

**8.12.3.5 real(kind=8), dimension(:, :), allocatable mtv\_int\_tensor::dumb\_mat2 [private]**

Dummy matrix.

Definition at line 68 of file MTV\_int\_tensor.f90.

```
68    REAL(KIND=8), DIMENSION(:, :), ALLOCATABLE :: dumb_mat2 !< Dummy matrix
```

**8.12.3.6 real(kind=8), dimension(:, :, ), allocatable mtv\_int\_tensor::dumb\_mat3 [private]**

Dummy matrix.

Definition at line 69 of file MTV\_int\_tensor.f90.

```
69    REAL(KIND=8), DIMENSION(:, :, ), ALLOCATABLE :: dumb_mat3 !< Dummy matrix
```

**8.12.3.7 real(kind=8), dimension(:, :, ), allocatable mtv\_int\_tensor::dumb\_mat4 [private]**

Dummy matrix.

Definition at line 70 of file MTV\_int\_tensor.f90.

```
70    REAL(KIND=8), DIMENSION(:, :, ), ALLOCATABLE :: dumb_mat4 !< Dummy matrix
```

**8.12.3.8 real(kind=8), dimension(:), allocatable mtv\_int\_tensor::dumb\_vec [private]**

Dummy vector.

Definition at line 66 of file MTV\_int\_tensor.f90.

```
66    REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: dumb_vec !< Dummy vector
```

**8.12.3.9 real(kind=8), dimension(:), allocatable, public mtv\_int\_tensor::h1**

First constant vector.

Definition at line 42 of file MTV\_int\_tensor.f90.

```
42    REAL(KIND=8), DIMENSION(:), ALLOCATABLE, PUBLIC :: h1 !< First constant vector
```

**8.12.3.10 real(kind=8), dimension(:), allocatable, public mtv\_int\_tensor::h2**

Second constant vector.

Definition at line 43 of file MTV\_int\_tensor.f90.

```
43    REAL(KIND=8), DIMENSION(:), ALLOCATABLE, PUBLIC :: h2 !< Second constant vector
```

**8.12.3.11 real(kind=8), dimension(:), allocatable, public mtv\_int\_tensor::h3**

Third constant vector.

Definition at line 44 of file MTV\_int\_tensor.f90.

```
44    REAL(KIND=8), DIMENSION(:), ALLOCATABLE, PUBLIC :: h3      !< Third constant vector
```

**8.12.3.12 real(kind=8), dimension(:), allocatable, public mtv\_int\_tensor::htot**

Total constant vector.

Definition at line 45 of file MTV\_int\_tensor.f90.

```
45    REAL(KIND=8), DIMENSION(:), ALLOCATABLE, PUBLIC :: htot !< Total constant vector
```

**8.12.3.13 type(coolist), dimension(:), allocatable, public mtv\_int\_tensor::l1**

First linear tensor.

Definition at line 48 of file MTV\_int\_tensor.f90.

```
48    TYPE(coolist), DIMENSION(:), ALLOCATABLE, PUBLIC :: l1      !< First linear tensor
```

**8.12.3.14 type(coolist), dimension(:), allocatable, public mtv\_int\_tensor::l2**

Second linear tensor.

Definition at line 49 of file MTV\_int\_tensor.f90.

```
49    TYPE(coolist), DIMENSION(:), ALLOCATABLE, PUBLIC :: l2      !< Second linear tensor
```

**8.12.3.15 type(coolist), dimension(:), allocatable, public mtv\_int\_tensor::l3**

Third linear tensor.

Definition at line 50 of file MTV\_int\_tensor.f90.

```
50    TYPE(coolist), DIMENSION(:), ALLOCATABLE, PUBLIC :: l3      !< Third linear tensor
```

### 8.12.3.16 type(coolist), dimension(:), allocatable, public mtv\_int\_tensor::ltot

Total linear tensor.

Definition at line 51 of file MTV\_int\_tensor.f90.

```
51  TYPE(coolist), DIMENSION(:), ALLOCATABLE, PUBLIC :: ltot !< Total linear tensor
```

### 8.12.3.17 type(coolist4), dimension(:), allocatable, public mtv\_int\_tensor::mtot

Tensor for the cubic terms.

Definition at line 58 of file MTV\_int\_tensor.f90.

```
58  TYPE(coolist4), DIMENSION(:), ALLOCATABLE, PUBLIC :: mtot !< Tensor for the cubic terms
```

### 8.12.3.18 real(kind=8), dimension(:,:,), allocatable, public mtv\_int\_tensor::q1

Constant terms for the state-dependent noise covariance matrix.

Definition at line 61 of file MTV\_int\_tensor.f90.

```
61  REAL(KIND=8), DIMENSION(:,:), ALLOCATABLE, PUBLIC :: q1 !< Constant terms for the state-dependent noise covariance matrix
```

### 8.12.3.19 real(kind=8), dimension(:,:,), allocatable, public mtv\_int\_tensor::q2

Constant terms for the state-independent noise covariance matrix.

Definition at line 62 of file MTV\_int\_tensor.f90.

```
62  REAL(KIND=8), DIMENSION(:,:), ALLOCATABLE, PUBLIC :: q2 !< Constant terms for the state-independent noise covariance matrix
```

### 8.12.3.20 type(coolist), dimension(:), allocatable, public mtv\_int\_tensor::utot

Linear terms for the state-dependent noise covariance matrix.

Definition at line 63 of file MTV\_int\_tensor.f90.

```
63  TYPE(coolist), DIMENSION(:), ALLOCATABLE, PUBLIC :: utot !< Linear terms for the state-dependent noise covariance matrix
```

### 8.12.3.21 type(coolist4), dimension(:), allocatable, public mtv\_int\_tensor::vtot

Quadratic terms for the state-dependent noise covariance matrix.

Definition at line 64 of file MTV\_int\_tensor.f90.

```
64   TYPE(coolist4), DIMENSION(:), ALLOCATABLE, PUBLIC :: vtot !< Quadratic terms for the
           state-dependent noise covariance matrix
```

## 8.13 params Module Reference

The model parameters module.

### Functions/Subroutines

- subroutine, private [init\\_nml](#)  
*Read the basic parameters and mode selection from the namelist.*
- subroutine [init\\_params](#)  
*Parameters initialisation routine.*

### Variables

- real(kind=8) [n](#)  
 $n = 2L_y/L_x$  - Aspect ratio
- real(kind=8) [phi0](#)  
*Latitude in radian.*
- real(kind=8) [rra](#)  
*Earth radius.*
- real(kind=8) [sig0](#)  
 $\sigma_0$  - Non-dimensional static stability of the atmosphere.
- real(kind=8) [k](#)  
*Bottom atmospheric friction coefficient.*
- real(kind=8) [kp](#)  
 $k'$  - Internal atmospheric friction coefficient.
- real(kind=8) [r](#)  
*Frictional coefficient at the bottom of the ocean.*
- real(kind=8) [d](#)  
*Mechanical coupling parameter between the ocean and the atmosphere.*
- real(kind=8) [f0](#)  
 $f_0$  - Coriolis parameter
- real(kind=8) [gp](#)  
 $g'$  Reduced gravity
- real(kind=8) [h](#)  
*Depth of the active water layer of the ocean.*
- real(kind=8) [phi0\\_npi](#)  
*Latitude expressed in fraction of pi.*
- real(kind=8) [lambda](#)  
 $\lambda$  - Sensible + turbulent heat exchange between the ocean and the atmosphere.

- real(kind=8) **co**  
 $C_a$  - Constant short-wave radiation of the ocean.
- real(kind=8) **go**  
 $\gamma_o$  - Specific heat capacity of the ocean.
- real(kind=8) **ca**  
 $C_a$  - Constant short-wave radiation of the atmosphere.
- real(kind=8) **to0**  
 $T_o^0$  - Stationary solution for the 0-th order ocean temperature.
- real(kind=8) **ta0**  
 $T_a^0$  - Stationary solution for the 0-th order atmospheric temperature.
- real(kind=8) **epsa**  
 $\epsilon_a$  - Emissivity coefficient for the grey-body atmosphere.
- real(kind=8) **ga**  
 $\gamma_a$  - Specific heat capacity of the atmosphere.
- real(kind=8) **rr**  
 $R$  - Gas constant of dry air
- real(kind=8) **scale**  
 $L_y = L \pi$  - The characteristic space scale.
- real(kind=8) **pi**  
 $\pi$
- real(kind=8) **lr**  
 $L_R$  - Rossby deformation radius
- real(kind=8) **g**  
 $\gamma$
- real(kind=8) **rp**  
 $r'$  - Frictional coefficient at the bottom of the ocean.
- real(kind=8) **dp**  
 $d'$  - Non-dimensional mechanical coupling parameter between the ocean and the atmosphere.
- real(kind=8) **kd**  
 $k_d$  - Non-dimensional bottom atmospheric friction coefficient.
- real(kind=8) **kdp**  
 $k'_d$  - Non-dimensional internal atmospheric friction coefficient.
- real(kind=8) **cpo**  
 $C'_a$  - Non-dimensional constant short-wave radiation of the ocean.
- real(kind=8) **lpo**  
 $\lambda'_o$  - Non-dimensional sensible + turbulent heat exchange from ocean to atmosphere.
- real(kind=8) **cpa**  
 $C'_a$  - Non-dimensional constant short-wave radiation of the atmosphere.
- real(kind=8) **ipa**  
 $\lambda'_a$  - Non-dimensional sensible + turbulent heat exchange from atmosphere to ocean.
- real(kind=8) **sbpo**  
 $\sigma'_{B,o}$  - Long wave radiation lost by ocean to atmosphere & space.
- real(kind=8) **sbpa**  
 $\sigma'_{B,a}$  - Long wave radiation from atmosphere absorbed by ocean.
- real(kind=8) **lsbpo**  
 $S'_{B,o}$  - Long wave radiation from ocean absorbed by atmosphere.
- real(kind=8) **lsbpa**  
 $S'_{B,a}$  - Long wave radiation lost by atmosphere to space & ocean.
- real(kind=8) **l**  
 $L$  - Domain length scale
- real(kind=8) **sc**

- real(kind=8) **sb**  
*Stefan–Boltzmann constant.*
- real(kind=8) **betp**  
 $\beta'$  - Non-dimensional beta parameter
- real(kind=8) **nua** =0.D0  
*Dissipation in the atmosphere.*
- real(kind=8) **nuo** =0.D0  
*Dissipation in the ocean.*
- real(kind=8) **nuap**  
*Non-dimensional dissipation in the atmosphere.*
- real(kind=8) **nuop**  
*Non-dimensional dissipation in the ocean.*
- real(kind=8) **t\_trans**  
*Transient time period.*
- real(kind=8) **t\_run**  
*Effective intergration time (length of the generated trajectory)*
- real(kind=8) **dt**  
*Integration time step.*
- real(kind=8) **tw**  
*Write all variables every tw time units.*
- logical **writeout**  
*Write to file boolean.*
- integer **nboc**  
*Number of atmospheric blocks.*
- integer **nbatm**  
*Number of oceanic blocks.*
- integer **natm** =0  
*Number of atmospheric basis functions.*
- integer **noc** =0  
*Number of oceanic basis functions.*
- integer **ndim**  
*Number of variables (dimension of the model)*
- integer, dimension(:, :), allocatable **oms**  
*Ocean mode selection array.*
- integer, dimension(:, :), allocatable **ams**  
*Atmospheric mode selection array.*

### 8.13.1 Detailed Description

The model parameters module.

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#### Remarks

Once the `init_params()` subroutine is called, the parameters are loaded globally in the main program and its subroutines and function

## 8.13.2 Function/Subroutine Documentation

### 8.13.2.1 subroutine, private params::init\_nml( ) [private]

Read the basic parameters and mode selection from the namelist.

Definition at line 97 of file params.f90.

```

97      INTEGER :: allocstat
98
99      namelist /aoscale/ scale,f0,n,rra,phi0_npi
100     namelist /oparams/ gp,r,h,d,nuo
101     namelist /aparams/ k,kp,sig0,nua
102     namelist /toparams/ go,co,to0
103     namelist /taparams/ ga,ca,epsa,ta0
104     namelist /otparams/ sc,lambda,rr,sb
105
106     namelist /modeselection/ oms,ams
107     namelist /numblocs/ nboc,nbatm
108
109     namelist /int_params/ t_trans,t_run,dt,tw,writeout
110
111     OPEN(8, file="params.nml", status='OLD', recl=80, delim='APOSTROPHE')
112
113     READ(8,nml=aoscale)
114     READ(8,nml=oparams)
115     READ(8,nml=aparams)
116     READ(8,nml=toparams)
117     READ(8,nml=taparams)
118     READ(8,nml=otparams)
119
120     CLOSE(8)
121
122     OPEN(8, file="modeselection.nml", status='OLD', recl=80, delim='APOSTROPHE')
123     READ(8,nml=numblocs)
124
125     ALLOCATE(oms(nboc,2),ams(nbatm,2), stat=allocstat)
126     IF (allocstat /= 0) stop "*** Not enough memory ! ***"
127
128     READ(8,nml=modeselection)
129     CLOSE(8)
130
131     OPEN(8, file="int_params.nml", status='OLD', recl=80, delim='APOSTROPHE')
132     READ(8,nml=int_params)
133

```

### 8.13.2.2 subroutine params::init\_params( )

Parameters initialisation routine.

Definition at line 138 of file params.f90.

```

138      INTEGER, DIMENSION(2) :: s
139      INTEGER :: i
140      CALL init_nml
141
142      !-----!
143      !
144      ! Computation of the dimension of the atmospheric
145      ! and oceanic components
146      !
147      !-----!
148
149      natm=0
150      DO i=1,nbatm
151          IF (ams(i,1)==1) THEN
152              natm=natm+3
153          ELSE
154              natm=natm+2
155          ENDIF
156      ENDDO
157      s=shape(oms)
158      noc=s(1)

```

```

159      ndim=2*natm+2*noc
160
161      !-----!
162      !
163      ! Some general parameters (Domain, beta, gamma, coupling) !
164      !
165      !-----!
166
167      pi=dacos(-1.d0)
168      l=scale/pi
169      phi0=phi0_npi*pi
170      lr=sqrt(gp*h)/f0
171      g=-l**2/lr**2
172      betp=l/rra*cos(phi0)/sin(phi0)
173      rp=r/f0
174      dp=d/f0
175      kd=k**2
176      kdp=kp
177
178      !-----!
179      !
180      ! DERIVED QUANTITIES
181      !
182      !
183      !-----!
184
185      cpo=co/(go*f0) * rr/(f0**2*l**2)
186      lpo=lambda/(go*f0)
187      cpa=ca/(ga*f0) * rr/(f0**2*l**2)/2 ! Cpa acts on psil-psi3, not on theta
188      lpa=lambda/(ga*f0)
189      sbpo=4*sb*t0**3/(go*f0) ! long wave radiation lost by ocean to atmosphere space
190      sbpa=8*epsa*sb*ta0**3/(go*f0) ! long wave radiation from atmosphere absorbed by ocean
191      lsbpo=2*epsa*sb*t0**3/(ga*f0) ! long wave radiation from ocean absorbed by atmosphere
192      lsbpa=8*epsa*sb*ta0**3/(ga*f0) ! long wave radiation lost by atmosphere to space & ocea
193      nuap=nua/(f0*l**2)
194      nuop=nuo/(f0*l**2)
195

```

### 8.13.3 Variable Documentation

#### 8.13.3.1 integer, dimension(:,:), allocatable params::ams

Atmospheric mode selection array.

Definition at line 87 of file params.f90.

```
87      INTEGER, DIMENSION(:,:,:), ALLOCATABLE :: ams      !< Atmospheric mode selection array
```

#### 8.13.3.2 real(kind=8) params::betp

$\beta'$  - Non-dimensional beta parameter

Definition at line 67 of file params.f90.

```
67      REAL(KIND=8) :: betp      !< \f$ \beta' \f$ - Non-dimensional beta parameter
```

#### 8.13.3.3 real(kind=8) params::ca

$C_a$  - Constant short-wave radiation of the atmosphere.

Definition at line 40 of file params.f90.

```
40      REAL(KIND=8) :: ca      !< \f$ C_a \f$ - Constant short-wave radiation of the atmosphere.
```

#### 8.13.3.4 real(kind=8) params::co

$C_a$  - Constant short-wave radiation of the ocean.

Definition at line 38 of file params.f90.

```
38    REAL(KIND=8) :: co      !< \f$C_a\f$ - Constant short-wave radiation of the ocean.
```

#### 8.13.3.5 real(kind=8) params::cpa

$C'_a$  - Non-dimensional constant short-wave radiation of the atmosphere.

##### Remarks

Cpa acts on psi1-psi3, not on theta.

Definition at line 58 of file params.f90.

```
58    REAL(KIND=8) :: cpa      !< \f$C'_a\f$ - Non-dimensional constant short-wave radiation of the atmosphere. @remark Cpa acts on psi1-psi3, not on theta.
```

#### 8.13.3.6 real(kind=8) params::cpo

$C'_a$  - Non-dimensional constant short-wave radiation of the ocean.

Definition at line 56 of file params.f90.

```
56    REAL(KIND=8) :: cpo      !< \f$C'_a\f$ - Non-dimensional constant short-wave radiation of the ocean.
```

#### 8.13.3.7 real(kind=8) params::d

Mechanical coupling parameter between the ocean and the atmosphere.

Definition at line 31 of file params.f90.

```
31    REAL(KIND=8) :: d      !< Mechanical coupling parameter between the ocean and the atmosphere.
```

#### 8.13.3.8 real(kind=8) params::dp

$d'$  - Non-dimensional mechanical coupling parameter between the ocean and the atmosphere.

Definition at line 52 of file params.f90.

```
52    REAL(KIND=8) :: dp      !< \f$d'\f$ - Non-dimensional mechanical coupling parameter between the ocean and the atmosphere.
```

**8.13.3.9 real(kind=8) params::dt**

Integration time step.

Definition at line 77 of file params.f90.

```
77    REAL(KIND=8) :: dt           !< Integration time step
```

**8.13.3.10 real(kind=8) params::epsa**

$\epsilon_a$  - Emissivity coefficient for the grey-body atmosphere.

Definition at line 43 of file params.f90.

```
43    REAL(KIND=8) :: epsa        !< \f$ \epsilon_a \f$ - Emissivity coefficient for the grey-body atmosphere.
```

**8.13.3.11 real(kind=8) params::f0**

$f_0$  - Coriolis parameter

Definition at line 32 of file params.f90.

```
32    REAL(KIND=8) :: f0          !< \f$ f_0 \f$ - Coriolis parameter
```

**8.13.3.12 real(kind=8) params::g**

$\gamma$

Definition at line 50 of file params.f90.

```
50    REAL(KIND=8) :: g           !< \f$ \gamma \f$
```

**8.13.3.13 real(kind=8) params::ga**

$\gamma_a$  - Specific heat capacity of the atmosphere.

Definition at line 44 of file params.f90.

```
44    REAL(KIND=8) :: ga         !< \f$ \gamma_a \f$ - Specific heat capacity of the atmosphere.
```

### 8.13.3.14 real(kind=8) params::go

$\gamma_o$  - Specific heat capacity of the ocean.

Definition at line 39 of file params.f90.

```
39    REAL(KIND=8) :: go           !< \f$gamma_o\f$ - Specific heat capacity of the ocean.
```

### 8.13.3.15 real(kind=8) params::gp

$g'$ Reduced gravity

Definition at line 33 of file params.f90.

```
33    REAL(KIND=8) :: gp          !< \f$g'\f$Reduced gravity
```

### 8.13.3.16 real(kind=8) params::h

Depth of the active water layer of the ocean.

Definition at line 34 of file params.f90.

```
34    REAL(KIND=8) :: h           !< Depth of the active water layer of the ocean.
```

### 8.13.3.17 real(kind=8) params::k

Bottom atmospheric friction coefficient.

Definition at line 28 of file params.f90.

```
28    REAL(KIND=8) :: k           !< Bottom atmospheric friction coefficient.
```

### 8.13.3.18 real(kind=8) params::kd

$k_d$  - Non-dimensional bottom atmospheric friction coefficient.

Definition at line 53 of file params.f90.

```
53    REAL(KIND=8) :: kd          !< \f$kd\f$ - Non-dimensional bottom atmospheric friction coefficient.
```

## 8.13.3.19 real(kind=8) params::kdp

$k'_d$  - Non-dimensional internal atmospheric friction coefficient.

Definition at line 54 of file params.f90.

```
54    REAL(KIND=8) :: kdp      !< \f$ k'_d\f$ - Non-dimensional internal atmospheric friction coefficient.
```

## 8.13.3.20 real(kind=8) params::kp

$k'$  - Internal atmospheric friction coefficient.

Definition at line 29 of file params.f90.

```
29    REAL(KIND=8) :: kp      !< \f$ k'\f$ - Internal atmospheric friction coefficient.
```

## 8.13.3.21 real(kind=8) params::l

$L$  - Domain length scale

Definition at line 64 of file params.f90.

```
64    REAL(KIND=8) :: l      !< \f$ L\f$ - Domain length scale
```

## 8.13.3.22 real(kind=8) params::lambda

$\lambda$  - Sensible + turbulent heat exchange between the ocean and the atmosphere.

Definition at line 37 of file params.f90.

```
37    REAL(KIND=8) :: lambda   !< \f$ \lambda\f$ - Sensible + turbulent heat exchange between the ocean and the atmosphere.
```

## 8.13.3.23 real(kind=8) params::lpa

$\lambda'_a$  - Non-dimensional sensible + turbulent heat exchange from atmosphere to ocean.

Definition at line 59 of file params.f90.

```
59    REAL(KIND=8) :: lpa     !< \f$ \lambda'_a\f$ - Non-dimensional sensible + turbulent heat exchange from atmosphere to ocean.
```

### 8.13.3.24 real(kind=8) params::lpo

$\lambda'_o$  - Non-dimensional sensible + turbulent heat exchange from ocean to atmosphere.

Definition at line 57 of file params.f90.

```
57    REAL(KIND=8) :: lpo      !< \f$ \lambda'_o \f$ - Non-dimensional sensible + turbulent heat exchange from
     ocean to atmosphere.
```

### 8.13.3.25 real(kind=8) params::lr

$L_R$  - Rossby deformation radius

Definition at line 49 of file params.f90.

```
49    REAL(KIND=8) :: lr      !< \f$ L_R \f$ - Rossby deformation radius
```

### 8.13.3.26 real(kind=8) params::lsbpa

$S'_{B,a}$  - Long wave radiation lost by atmosphere to space & ocean.

Definition at line 63 of file params.f90.

```
63    REAL(KIND=8) :: lsbpa   !< \f$ S'_{B,a} \f$ - Long wave radiation lost by atmosphere to space & ocean.
```

### 8.13.3.27 real(kind=8) params::lsbpo

$S'_{B,o}$  - Long wave radiation from ocean absorbed by atmosphere.

Definition at line 62 of file params.f90.

```
62    REAL(KIND=8) :: lsbpo   !< \f$ S'_{B,o} \f$ - Long wave radiation from ocean absorbed by atmosphere.
```

### 8.13.3.28 real(kind=8) params::n

$n = 2L_y/L_x$  - Aspect ratio

Definition at line 24 of file params.f90.

```
24    REAL(KIND=8) :: n      !< \f$ n = 2 L_y / L_x \f$ - Aspect ratio
```

**8.13.3.29 integer params::natm =0**

Number of atmospheric basis functions.

Definition at line 83 of file params.f90.

```
83    INTEGER :: natm=0 !< Number of atmospheric basis functions
```

**8.13.3.30 integer params::nbatm**

Number of oceanic blocks.

Definition at line 82 of file params.f90.

```
82    INTEGER :: nbatm !< Number of oceanic blocks
```

**8.13.3.31 integer params::nboc**

Number of atmospheric blocks.

Definition at line 81 of file params.f90.

```
81    INTEGER :: nboc !< Number of atmospheric blocks
```

**8.13.3.32 integer params::ndim**

Number of variables (dimension of the model)

Definition at line 85 of file params.f90.

```
85    INTEGER :: ndim !< Number of variables (dimension of the model)
```

**8.13.3.33 integer params::noc =0**

Number of oceanic basis functions.

Definition at line 84 of file params.f90.

```
84    INTEGER :: noc=0 !< Number of oceanic basis functions
```

**8.13.3.34 real(kind=8) params::nua =0.D0**

Dissipation in the atmosphere.

Definition at line 69 of file params.f90.

```
69    REAL(KIND=8) :: nua=0.d0 !< Dissipation in the atmosphere
```

**8.13.3.35 real(kind=8) params::nuap**

Non-dimensional dissipation in the atmosphere.

Definition at line 72 of file params.f90.

```
72    REAL(KIND=8) :: nuap      !< Non-dimensional dissipation in the atmosphere
```

**8.13.3.36 real(kind=8) params::nuo =0.D0**

Dissipation in the ocean.

Definition at line 70 of file params.f90.

```
70    REAL(KIND=8) :: nuo=0.d0 !< Dissipation in the ocean
```

**8.13.3.37 real(kind=8) params::nuop**

Non-dimensional dissipation in the ocean.

Definition at line 73 of file params.f90.

```
73    REAL(KIND=8) :: nuop      !< Non-dimensional dissipation in the ocean
```

**8.13.3.38 integer, dimension(:, :), allocatable params::oms**

Ocean mode selection array.

Definition at line 86 of file params.f90.

```
86    INTEGER, DIMENSION(:, :), ALLOCATABLE :: oms !< Ocean mode selection array
```

**8.13.3.39 real(kind=8) params::phi0**

Latitude in radian.

Definition at line 25 of file params.f90.

```
25    REAL(KIND=8) :: phi0      !< Latitude in radian
```

**8.13.3.40 real(kind=8) params::phi0\_npi**

Latitude exprimed in fraction of pi.

Definition at line 35 of file params.f90.

```
35    REAL(KIND=8) :: phi0_npi !< Latitude exprimed in fraction of pi.
```

**8.13.3.41 real(kind=8) params::pi**

$\pi$

Definition at line 48 of file params.f90.

```
48    REAL(KIND=8) :: pi      !< \f$ \pi \f$
```

**8.13.3.42 real(kind=8) params::r**

Frictional coefficient at the bottom of the ocean.

Definition at line 30 of file params.f90.

```
30    REAL(KIND=8) :: r      !< Frictional coefficient at the bottom of the ocean.
```

**8.13.3.43 real(kind=8) params::rp**

$r'$  - Frictional coefficient at the bottom of the ocean.

Definition at line 51 of file params.f90.

```
51    REAL(KIND=8) :: rp      !< \f$ r' \f$ - Frictional coefficient at the bottom of the ocean.
```

#### 8.13.3.44 real(kind=8) params::rr

$R$  - Gas constant of dry air

Definition at line 45 of file params.f90.

```
45    REAL(KIND=8) :: rr          !< \f$R\f$ - Gas constant of dry air
```

#### 8.13.3.45 real(kind=8) params::rra

Earth radius.

Definition at line 26 of file params.f90.

```
26    REAL(KIND=8) :: rra        !< Earth radius
```

#### 8.13.3.46 real(kind=8) params::sb

Stefan–Boltzmann constant.

Definition at line 66 of file params.f90.

```
66    REAL(KIND=8) :: sb          !< Stefan-Boltzmann constant
```

#### 8.13.3.47 real(kind=8) params::sbpa

$\sigma'_{B,a}$  - Long wave radiation from atmosphere absorbed by ocean.

Definition at line 61 of file params.f90.

```
61    REAL(KIND=8) :: sbpa       !< \f$\sigma'_{\{B,a\}}\f$ - Long wave radiation from atmosphere absorbed by ocean.
```

#### 8.13.3.48 real(kind=8) params::sbpo

$\sigma'_{B,o}$  - Long wave radiation lost by ocean to atmosphere & space.

Definition at line 60 of file params.f90.

```
60    REAL(KIND=8) :: sbpo       !< \f$\sigma'_{\{B,o\}}\f$ - Long wave radiation lost by ocean to atmosphere & space.
```

**8.13.3.49 real(kind=8) params::sc**

Ratio of surface to atmosphere temperature.

Definition at line 65 of file params.f90.

```
65    REAL(KIND=8) :: sc          !< Ratio of surface to atmosphere temperature.
```

**8.13.3.50 real(kind=8) params::scale**

$L_y = L \pi$  - The characteristic space scale.

Definition at line 47 of file params.f90.

```
47    REAL(KIND=8) :: scale      !< \f$L_y = L \, , \pi\f$ - The characteristic space scale.
```

**8.13.3.51 real(kind=8) params::sig0**

$\sigma_0$  - Non-dimensional static stability of the atmosphere.

Definition at line 27 of file params.f90.

```
27    REAL(KIND=8) :: sig0      !< \f$\sigma_0\f$ - Non-dimensional static stability of the atmosphere.
```

**8.13.3.52 real(kind=8) params::t\_run**

Effective intergration time (length of the generated trajectory)

Definition at line 76 of file params.f90.

```
76    REAL(KIND=8) :: t_run     !< Effective intergration time (length of the generated trajectory)
```

**8.13.3.53 real(kind=8) params::t\_trans**

Transient time period.

Definition at line 75 of file params.f90.

```
75    REAL(KIND=8) :: t_trans   !< Transient time period
```

#### 8.13.3.54 real(kind=8) params::ta0

$T_a^0$  - Stationary solution for the 0-th order atmospheric temperature.

Definition at line 42 of file params.f90.

```
42    REAL(KIND=8) :: ta0      !< \f$T_a^0\f$ - Stationary solution for the 0-th order atmospheric
   temperature.
```

#### 8.13.3.55 real(kind=8) params::to0

$T_o^0$  - Stationary solution for the 0-th order ocean temperature.

Definition at line 41 of file params.f90.

```
41    REAL(KIND=8) :: to0      !< \f$T_o^0\f$ - Stationary solution for the 0-th order ocean temperature.
```

#### 8.13.3.56 real(kind=8) params::tw

Write all variables every tw time units.

Definition at line 78 of file params.f90.

```
78    REAL(KIND=8) :: tw      !< Write all variables every tw time units
```

#### 8.13.3.57 logical params::writeout

Write to file boolean.

Definition at line 79 of file params.f90.

```
79    LOGICAL :: writeout     !< Write to file boolean
```

## 8.14 rk2\_mtv\_integrator Module Reference

Module with the MTV rk2 integration routines.

## Functions/Subroutines

- subroutine, public **init\_integrator**  
*Subroutine to initialize the MTV rk2 integrator.*
- subroutine **init\_noise**  
*Routine to initialize the noise vectors and buffers.*
- subroutine **init\_g**  
*Routine to initialize the G term.*
- subroutine **compg (y)**  
*Routine to actualize the G term based on the state y of the MTV system.*
- subroutine, public **step (y, t, dt, dtn, res, tend)**  
*Routine to perform an integration step (Heun algorithm) of the MTV system. The incremented time is returned.*
- subroutine, public **full\_step (y, t, dt, dtn, res)**  
*Routine to perform an integration step (Heun algorithm) of the full stochastic system. The incremented time is returned.*

## Variables

- real(kind=8), dimension(:), allocatable **buf\_y1**
- real(kind=8), dimension(:), allocatable **buf\_f0**
- real(kind=8), dimension(:), allocatable **buf\_f1**  
*Integration buffers.*
- real(kind=8), dimension(:), allocatable **dw**
- real(kind=8), dimension(:), allocatable **dwmult**  
*Standard gaussian noise buffers.*
- real(kind=8), dimension(:), allocatable **dwar**
- real(kind=8), dimension(:), allocatable **dwau**
- real(kind=8), dimension(:), allocatable **dwor**
- real(kind=8), dimension(:), allocatable **dwou**  
*Standard gaussian noise buffers.*
- real(kind=8), dimension(:), allocatable **anoise**
- real(kind=8), dimension(:), allocatable **noise**  
*Additive noise term.*
- real(kind=8), dimension(:), allocatable **noisemult**  
*Multiplicative noise term.*
- real(kind=8), dimension(:), allocatable **g**  
*G term of the MTV tendencies.*
- real(kind=8), dimension(:), allocatable **buf\_g**  
*Buffer for the G term computation.*
- logical **mult**  
*Logical indicating if the sigma1 matrix must be computed for every state change.*
- logical **q1fill**  
*Logical indicating if the matrix Q1 is non-zero.*
- logical **compute\_mult**  
*Logical indicating if the Gaussian noise for the multiplicative noise must be computed.*
- real(kind=8), parameter **sq2 = sqrt(2.D0)**  
*Hard coded square root of 2.*

### 8.14.1 Detailed Description

Module with the MTV rk2 integration routines.

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#### Remarks

This module actually contains the Heun algorithm routines.

### 8.14.2 Function/Subroutine Documentation

#### 8.14.2.1 subroutine rk2\_mtv\_integrator::compg ( real(kind=8), dimension(0:ndim), intent(in) y ) [private]

Routine to actualize the G term based on the state y of the MTV system.

##### Parameters

<i>y</i>	State of the MTV system
----------	-------------------------

Definition at line 105 of file rk2\_MTV\_integrator.f90.

```

105      REAL(KIND=8), DIMENSION(0:ndim), INTENT(IN) :: y
106
107      g=htot
108      CALL sparse_mul2_k(ltot,y,buf_g)
109      g=g+buf_g
110      CALL sparse_mul3(btot,y,y,buf_g)
111      g=g+buf_g
112      CALL sparse_mul4(mtot,y,y,y,buf_g)
113      g=g+buf_g

```

#### 8.14.2.2 subroutine, public rk2\_mtv\_integrator::full\_step ( real(kind=8), dimension(0:ndim), intent(in) y, real(kind=8), intent(inout) t, real(kind=8), intent(in) dt, real(kind=8), intent(in) dtn, real(kind=8), dimension(0:ndim), intent(out) res )

Routine to perform an integration step (Heun algorithm) of the full stochastic system. The incremented time is returned.

##### Parameters

<i>y</i>	Initial point.
<i>t</i>	Actual integration time
<i>dt</i>	Integration timestep.
<i>dtn</i>	Stochastic integration timestep (normally square-root of dt).
<i>res</i>	Final point after the step.

Definition at line 170 of file rk2\_MTV\_integrator.f90.

```

170      REAL (KIND=8), DIMENSION(0:ndim), INTENT(IN) :: y
171      REAL (KIND=8), INTENT(INOUT) :: t
172      REAL (KIND=8), INTENT(IN) :: dt,dtn
173      REAL (KIND=8), DIMENSION(0:ndim), INTENT(OUT) :: res
174      CALL stoch_atm_res_vec(dwar)
175      CALL stoch_atm_unres_vec(dwau)
176      CALL stoch_oc_res_vec(dwor)
177      CALL stoch_oc_unres_vec(dwou)
178      anoise=(q_ar*dwar+q_au*dwau+q_or*dwor+q_ou*dwou)*dtn
179      CALL sparse_mul3(aotensor,y,y,buf_f0)
180      buf_y1 = y+dt*buf_f0+anoise
181      CALL sparse_mul3(aotensor,buf_y1,buf_y1,buf_f1)
182      res=y+0.5*(buf_f0+buf_f1)*dt+anoise
183      t=t+dt

```

#### 8.14.2.3 subroutine rk2\_mtv\_integrator::init\_g( ) [private]

Routine to initialize the G term.

Definition at line 97 of file rk2\_MTV\_integrator.f90.

```

97      INTEGER :: allocstat
98      ALLOCATE(g(0:ndim), buf_g(0:ndim), stat=allocstat)
99      IF (allocstat /= 0) stop "*** Not enough memory ! ***"

```

#### 8.14.2.4 subroutine, public rk2\_mtv\_integrator::init\_integrator( )

Subroutine to initialize the MTV rk2 integrator.

Definition at line 50 of file rk2\_MTV\_integrator.f90.

```

50      INTEGER :: allocstat
51
52      CALL init_ss_integrator ! Initialize the uncoupled resolved dynamics
53
54      ALLOCATE(buf_y1(0:ndim),buf_f0(0:ndim),buf_f1(0:ndim),stat=allocstat)
55      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
56
57      buf_y1=0.d0
58      buf_f1=0.d0
59      buf_f0=0.d0
60
61      print*, 'Initializing the integrator ...'
62      CALL init_sigma(mult,q1fill)
63      CALL init_noise
64      CALL init_g

```

#### 8.14.2.5 subroutine rk2\_mtv\_integrator::init\_noise( ) [private]

Routine to initialize the noise vectors and buffers.

Definition at line 69 of file rk2\_MTV\_integrator.f90.

```

69      INTEGER :: allocstat
70      ALLOCATE(dw(0:ndim), dwmult(0:ndim), stat=allocstat)
71      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
72
73      ALLOCATE(dwar(0:ndim), dwau(0:ndim), dwor(0:ndim), dwou(0:ndim),
74      stat=allocstat)
75      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
76
77      ALLOCATE(anoise(0:ndim), noise(0:ndim), noisemult(0:ndim), stat=allocstat)
78      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
79
80      dw=0.d0
81      dwmult=0.d0
82
83      dwar=0.d0
84      dwor=0.d0
85      dwau=0.d0
86      dwou=0.d0
87
88      anoise=0.d0
89      noise=0.d0
90      noisemult=0.d0
91
92      compute_mult=((qlfill).OR.(mult))

```

### 8.14.2.6 subroutine, public rk2\_mtv\_integrator::step ( real(kind=8), dimension(0:ndim), intent(in) y, real(kind=8), intent(inout) t, real(kind=8), intent(in) dt, real(kind=8), intent(in) dtn, real(kind=8), dimension(0:ndim), intent(out) res, real(kind=8), dimension(0:ndim), intent(out) tend )

Routine to perform an integration step (Heun algorithm) of the MTV system. The incremented time is returned.

#### Parameters

<i>y</i>	Initial point.
<i>t</i>	Actual integration time
<i>dt</i>	Integration timestep.
<i>dtn</i>	Stochastic integration timestep (normally square-root of dt).
<i>res</i>	Final point after the step.
<i>tend</i>	Partial or full tendencies used to perform the step (used for debugging).

Definition at line 124 of file rk2\_MTV\_integrator.f90.

```

124      REAL(KIND=8), DIMENSION(0:ndim), INTENT(IN) :: y
125      REAL(KIND=8), INTENT(INOUT) :: t
126      REAL(KIND=8), INTENT(IN) :: dt,dtn
127      REAL(KIND=8), DIMENSION(0:ndim), INTENT(OUT) :: res,tend
128
129      CALL compg(y)
130
131      CALL stoch_atm_res_vec(dwar)
132      CALL stoch_oc_res_vec(dwor)
133      anoise=q_ar*dwar+q_or*dwor
134      CALL stoch_vec(dw)
135      IF (compute_mult) CALL stoch_vec(dwmult)
136      noise(1:ndim)=matmul(sig2,dw(1:ndim))
137      IF ((mult).and.(mod(t,mnuni)<dt)) CALL compute_mult_sigma(y)
138      IF (compute_mult) noisemult(1:ndim)=matmul(sig1,dwmult(1:ndim))
139
140      CALL tendencies(t,y,buf_f0)
141      buf_y1 = y+dt*(buf_f0+g)+(anoise+sq2*(noise+noisemult))*dtn
142
143      buf_f1=g
144      CALL compg(buf_y1)
145      g=0.5*(g+buf_f1)
146
147      IF ((mult).and.(mod(t,mnuni)<dt)) CALL compute_mult_sigma(buf_y1)
148      IF (compute_mult) THEN
149          buf_f1(1:ndim)=matmul(sig1,dwmult(1:ndim))

```

```

150      noisemult(1:ndim)=0.5*(noisemult(1:ndim)+buf_f1(1:ndim))
151  ENDIF
152
153
154  CALL tendencies(t,buf_y1,buf_f1)
155  buf_f0=0.5*(buf_f0+buf_f1)
156  res=y+dt*(buf_f0+g)+(anoise+sq2*(noise+noisemult))*dtn
157  ! tend=G+sq2*(noise+noisemult)/dtn
158  tend=sq2*noisemult/dtn
159  t=t+dt
160

```

### 8.14.3 Variable Documentation

#### 8.14.3.1 real(kind=8), dimension(:), allocatable rk2\_mtv\_integrator::anoise [private]

Definition at line 33 of file rk2\_MTV\_integrator.f90.

```
33  REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: anoise,noise           !< Additive noise term
```

#### 8.14.3.2 real(kind=8), dimension(:), allocatable rk2\_mtv\_integrator::buf\_f0 [private]

Definition at line 30 of file rk2\_MTV\_integrator.f90.

#### 8.14.3.3 real(kind=8), dimension(:), allocatable rk2\_mtv\_integrator::buf\_f1 [private]

Integration buffers.

Definition at line 30 of file rk2\_MTV\_integrator.f90.

#### 8.14.3.4 real(kind=8), dimension(:), allocatable rk2\_mtv\_integrator::buf\_g [private]

Buffer for the G term computation.

Definition at line 36 of file rk2\_MTV\_integrator.f90.

```
36  REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: buf_g                  !< Buffer for the G term computation
```

#### 8.14.3.5 real(kind=8), dimension(:), allocatable rk2\_mtv\_integrator::buf\_y1 [private]

Definition at line 30 of file rk2\_MTV\_integrator.f90.

```
30  REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: buf_y1,buf_f0,buf_f1 !< Integration buffers
```

#### 8.14.3.6 logical rk2\_mtv\_integrator::compute\_mult [private]

Logical indicating if the Gaussian noise for the multiplicative noise must be computed.

Definition at line 40 of file rk2\_MTV\_integrator.f90.

```
40  LOGICAL :: compute_mult
      noise for the multiplicative noise must be computed !< Logical indicating if the Gaussian
```

#### 8.14.3.7 real(kind=8), dimension(:), allocatable rk2\_mtv\_integrator::dw [private]

Definition at line 31 of file rk2\_MTV\_integrator.f90.

```
31  REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: dw, dwmult !< Standard gaussian noise buffers
```

#### 8.14.3.8 real(kind=8), dimension(:), allocatable rk2\_mtv\_integrator::dwar [private]

Definition at line 32 of file rk2\_MTV\_integrator.f90.

```
32  REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: dwar,dwau,dwor,dwou !< Standard gaussian noise buffers
```

#### 8.14.3.9 real(kind=8), dimension(:), allocatable rk2\_mtv\_integrator::dwau [private]

Definition at line 32 of file rk2\_MTV\_integrator.f90.

#### 8.14.3.10 real(kind=8), dimension(:), allocatable rk2\_mtv\_integrator::dwmult [private]

Standard gaussian noise buffers.

Definition at line 31 of file rk2\_MTV\_integrator.f90.

#### 8.14.3.11 real(kind=8), dimension(:), allocatable rk2\_mtv\_integrator::dwor [private]

Definition at line 32 of file rk2\_MTV\_integrator.f90.

#### 8.14.3.12 real(kind=8), dimension(:), allocatable rk2\_mtv\_integrator::dwou [private]

Standard gaussian noise buffers.

Definition at line 32 of file rk2\_MTV\_integrator.f90.

## 8.14.3.13 real(kind=8), dimension(:), allocatable rk2\_mtv\_integrator::g [private]

G term of the MTV tendencies.

Definition at line 35 of file rk2\_MTV\_integrator.f90.

```
35  REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: g           !< G term of the MTV tendencies
```

## 8.14.3.14 logical rk2\_mtv\_integrator::mult [private]

Logical indicating if the sigma1 matrix must be computed for every state change.

Definition at line 38 of file rk2\_MTV\_integrator.f90.

```
38  LOGICAL :: mult                                !< Logical indicating if the sigma1  
      matrix must be computed for every state change
```

## 8.14.3.15 real(kind=8), dimension(:), allocatable rk2\_mtv\_integrator::noise [private]

Additive noise term.

Definition at line 33 of file rk2\_MTV\_integrator.f90.

## 8.14.3.16 real(kind=8), dimension(:), allocatable rk2\_mtv\_integrator::noisemult [private]

Multiplicative noise term.

Definition at line 34 of file rk2\_MTV\_integrator.f90.

```
34  REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: noisemult      !< Multiplicative noise term
```

## 8.14.3.17 logical rk2\_mtv\_integrator::q1fill [private]

Logical indicating if the matrix Q1 is non-zero.

Definition at line 39 of file rk2\_MTV\_integrator.f90.

```
39  LOGICAL :: q1fill                               !< Logical indicating if the matrix Q1 is  
      non-zero
```

## 8.14.3.18 real(kind=8), parameter rk2\_mtv\_integrator::sq2 = sqrt(2.D0) [private]

Hard coded square root of 2.

Definition at line 42 of file rk2\_MTV\_integrator.f90.

```
42  REAL(KIND=8), PARAMETER :: sq2 = sqrt(2.d0)          !< Hard coded square root of 2
```

## 8.15 rk2\_ss\_integrator Module Reference

Module with the stochastic uncoupled resolved nonlinear and tangent linear rk2 dynamics integration routines.

### Functions/Subroutines

- subroutine, public [init\\_ss\\_integrator](#)  
*Subroutine to initialize the uncoupled resolved rk2 integrator.*
- subroutine, public [tendencies](#) (t, y, res)  
*Routine computing the tendencies of the uncoupled resolved model.*
- subroutine, public [tl\\_tendencies](#) (t, y, ys, res)  
*Tendencies for the tangent linear model of the uncoupled resolved dynamics in point ystar for perturbation delty.*
- subroutine, public [ss\\_step](#) (y, ys, t, dt, dtn, res)  
*Routine to perform a stochastic integration step of the unresolved uncoupled dynamics (Heun algorithm). The incremented time is returned.*
- subroutine, public [ss\\_tl\\_step](#) (y, ys, t, dt, dtn, res)  
*Routine to perform a stochastic integration step of the unresolved uncoupled tangent linear dynamics (Heun algorithm). The incremented time is returned.*

### Variables

- real(kind=8), dimension(:), allocatable [dwar](#)
- real(kind=8), dimension(:), allocatable [dwor](#)  
*Standard gaussian noise buffers.*
- real(kind=8), dimension(:), allocatable [anoise](#)  
*Additive noise term.*
- real(kind=8), dimension(:), allocatable [buf\\_y1](#)
- real(kind=8), dimension(:), allocatable [buf\\_f0](#)
- real(kind=8), dimension(:), allocatable [buf\\_f1](#)  
*Integration buffers.*

### 8.15.1 Detailed Description

Module with the stochastic uncoupled resolved nonlinear and tangent linear rk2 dynamics integration routines.

#### Copyright

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#### Remarks

This module actually contains the Heun algorithm routines.

## 8.15.2 Function/Subroutine Documentation

### 8.15.2.1 subroutine, public rk2\_ss\_integrator::init\_ss\_integrator( )

Subroutine to initialize the uncoupled resolved rk2 integrator.

Definition at line 40 of file rk2\_ss\_integrator.f90.

```

40      INTEGER :: allocstat
41
42      ALLOCATE(buf_y1(0:ndim),buf_f0(0:ndim),buf_f1(0:ndim),stat=allocstat)
43      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
44
45      ALLOCATE(anoise(0:ndim),stat=allocstat)
46      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
47
48      ALLOCATE(dwar(0:ndim),dwor(0:ndim),stat=allocstat)
49      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
50
51      dwar=0.d0
52      dwor=0.d0
53

```

### 8.15.2.2 subroutine, public rk2\_ss\_integrator::ss\_step ( real(kind=8), dimension(0:ndim), intent(in) y, real(kind=8), dimension(0:ndim), intent(in) ys, real(kind=8), intent(inout) t, real(kind=8), intent(in) dt, real(kind=8), intent(in) dtn, real(kind=8), dimension(0:ndim), intent(out) res )

Routine to perform a stochastic integration step of the unresolved uncoupled dynamics (Heun algorithm). The incremented time is returned.

#### Parameters

<i>y</i>	Initial point.
<i>ys</i>	Dummy argument for compatibility.
<i>t</i>	Actual integration time
<i>dt</i>	Integration timestep.
<i>dtn</i>	Stochastic integration timestep (normally square-root of dt).
<i>res</i>	Final point after the step.

Definition at line 92 of file rk2\_ss\_integrator.f90.

```

92      REAL(KIND=8), DIMENSION(0:ndim), INTENT(IN) :: y,ys
93      REAL(KIND=8), INTENT(INOUT) :: t
94      REAL(KIND=8), INTENT(IN) :: dt,dtn
95      REAL(KIND=8), DIMENSION(0:ndim), INTENT(OUT) :: res
96
97      CALL stoch_atm_res_vec(dwar)
98      CALL stoch_oc_res_vec(dwor)
99      anoise=(q_ar*dwar+q_or*dwor)*dtn
100     CALL tendencies(t,y,buf_f0)
101     buf_y1 = y+dt*buf_f0+anoise
102     CALL tendencies(t,buf_y1,buf_f1)
103     res=y+0.5*(buf_f0+buf_f1)*dt+anoise
104     t=t+dt

```

---

8.15.2.3 subroutine, public rk2\_ss\_integrator::ss\_tl\_step ( real(kind=8), dimension(0:ndim), intent(in) *y*, real(kind=8), dimension(0:ndim), intent(in) *ys*, real(kind=8), intent(inout) *t*, real(kind=8), intent(in) *dt*, real(kind=8), intent(in) *dtn*, real(kind=8), dimension(0:ndim), intent(out) *res* )

Routine to perform a stochastic integration step of the unresolved uncoupled tangent linear dynamics (Heun algorithm). The incremented time is returned.

#### Parameters

<i>y</i>	Initial point.
<i>ys</i>	point in trajectory to which the tangent space belongs.
<i>t</i>	Actual integration time
<i>dt</i>	Integration timestep.
<i>dtn</i>	Stochastic integration timestep (normally square-root of dt).
<i>res</i>	Final point after the step.

Definition at line 117 of file rk2\_ss\_integrator.f90.

```

117  REAL (KIND=8), DIMENSION(0:ndim), INTENT(IN) :: y,ys
118  REAL (KIND=8), INTENT(INOUT) :: t
119  REAL (KIND=8), INTENT(IN) :: dt,dtn
120  REAL (KIND=8), DIMENSION(0:ndim), INTENT(OUT) :: res
121
122  CALL stoch_atm_res_vec(dwar)
123  CALL stoch_oc_res_vec(dwor)
124  anoise=(q_ar*dwar+q_or*dwor)*dtn
125  CALL tl_tendencies(t,y,ys,buf_f0)
126  buf_y1 = y+dt*buf_f0+anoise
127  CALL tl_tendencies(t,buf_y1,ys,buf_f1)
128  res=y+0.5*(buf_f0+buf_f1)*dt+anoise
129  t=t+dt

```

---

8.15.2.4 subroutine, public rk2\_ss\_integrator::tendencies ( real(kind=8), intent(in) *t*, real(kind=8), dimension(0:ndim), intent(in) *y*, real(kind=8), dimension(0:ndim), intent(out) *res* )

Routine computing the tendencies of the uncoupled resolved model.

#### Parameters

<i>t</i>	Time at which the tendencies have to be computed. Actually not needed for autonomous systems.
<i>y</i>	Point at which the tendencies have to be computed.
<i>res</i>	vector to store the result.

#### Remarks

Note that it is NOT safe to pass *y* as a result buffer, as this operation does multiple passes.

Definition at line 63 of file rk2\_ss\_integrator.f90.

```

63      REAL (KIND=8), INTENT(IN) :: t
64      REAL (KIND=8), DIMENSION(0:ndim), INTENT(IN) :: y
65      REAL (KIND=8), DIMENSION(0:ndim), INTENT(OUT) :: res
66      CALL sparse_mul3(ss_tensor, y, y, res)

```

8.15.2.5 subroutine, public rk2\_ss\_integrator::tl\_tendencies ( real(kind=8), intent(in)  $t$ , real(kind=8), dimension(0:ndim), intent(in)  $y$ , real(kind=8), dimension(0:ndim), intent(in)  $ys$ , real(kind=8), dimension(0:ndim), intent(out)  $res$  )

Tendencies for the tangent linear model of the uncoupled resolved dynamics in point  $ystar$  for perturbation  $deltay$ .

#### Parameters

$t$	time
$y$	point of the tangent space at which the tendencies have to be computed.
$ys$	point in trajectory to which the tangent space belongs.
$res$	vector to store the result.

Definition at line 76 of file rk2\_ss\_integrator.f90.

```
76      REAL(KIND=8), INTENT(IN) :: t
77      REAL(KIND=8), DIMENSION(0:ndim), INTENT(IN) :: y,ys
78      REAL(KIND=8), DIMENSION(0:ndim), INTENT(OUT) :: res
79      CALL sparse_mul3(ss_tl_tensor, y, ys, res)
```

### 8.15.3 Variable Documentation

8.15.3.1 real(kind=8), dimension(:), allocatable rk2\_ss\_integrator::anoise [private]

Additive noise term.

Definition at line 30 of file rk2\_ss\_integrator.f90.

```
30      REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: anoise !< Additive noise term
```

8.15.3.2 real(kind=8), dimension(:), allocatable rk2\_ss\_integrator::buf\_f0 [private]

Definition at line 32 of file rk2\_ss\_integrator.f90.

8.15.3.3 real(kind=8), dimension(:), allocatable rk2\_ss\_integrator::buf\_f1 [private]

Integration buffers.

Definition at line 32 of file rk2\_ss\_integrator.f90.

8.15.3.4 real(kind=8), dimension(:), allocatable rk2\_ss\_integrator::buf\_y1 [private]

Definition at line 32 of file rk2\_ss\_integrator.f90.

```
32      REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: buf_y1,buf_f0,buf_f1 !< Integration buffers
```

### 8.15.3.5 `real(kind=8), dimension(:), allocatable rk2_ss_integrator::dwar` [private]

Definition at line 28 of file `rk2_ss_integrator.f90`.

```
28    REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: dwar,dwau !< Standard gaussian noise buffers
```

### 8.15.3.6 `real(kind=8), dimension(:), allocatable rk2_ss_integrator::dwor` [private]

Standard gaussian noise buffers.

Definition at line 28 of file `rk2_ss_integrator.f90`.

## 8.16 rk2\_stoch\_integrator Module Reference

Module with the stochastic rk2 integration routines.

### Functions/Subroutines

- subroutine, public `init_integrator` (force)
 

*Subroutine to initialize the integrator.*
- subroutine `tendencies` (t, y, res)
 

*Routine computing the tendencies of the selected model.*
- subroutine, public `step` (y, t, dt, dtn, res, tend)
 

*Routine to perform a stochastic step of the selected dynamics (Heun algorithm). The incremented time is returned.*

### Variables

- `real(kind=8), dimension(:), allocatable dwar`
- `real(kind=8), dimension(:), allocatable dwau`
- `real(kind=8), dimension(:), allocatable dwor`
- `real(kind=8), dimension(:), allocatable dwou`

*Standard gaussian noise buffers.*
- `real(kind=8), dimension(:), allocatable buf_y1`
- `real(kind=8), dimension(:), allocatable buf_f0`
- `real(kind=8), dimension(:), allocatable buf_f1`

*Integration buffers.*
- `real(kind=8), dimension(:), allocatable anoise`

*Additive noise term.*
- type(`coolist`), dimension(:), allocatable `int_tensor`

*Dummy tensor that will hold the tendencies tensor.*

### 8.16.1 Detailed Description

Module with the stochastic rk2 integration routines.

#### Copyright

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#### Remarks

This module actually contains the Heun algorithm routines. There are four modes for this integrator:

- full: use the full dynamics
- ures: use the intrinsic unresolved dynamics
- qfst: use the quadratic terms of the unresolved tendencies
- reso: use the resolved dynamics alone

### 8.16.2 Function/Subroutine Documentation

#### 8.16.2.1 subroutine, public rk2\_stoch\_integrator::init\_integrator ( character\*4, intent(in), optional force )

Subroutine to initialize the integrator.

##### Parameters

<i>force</i>	Parameter to force the mode of the integrator
--------------	---

Definition at line 48 of file rk2\_stoch\_integrator.f90.

```

48      INTEGER :: allocstat
49      CHARACTER*4, INTENT(IN), OPTIONAL :: force
50      CHARACTER*4 :: test
51
52      ALLOCATE(buf_y1(0:ndim),buf_f0(0:ndim),buf_f1(0:ndim),stat=allocstat)
53      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
54
55      ALLOCATE(anoise(0:ndim),stat=allocstat)
56      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
57
58      ALLOCATE(dwar(0:ndim),dwau(0:ndim),dwor(0:ndim),dwou(0:ndim),
59      stat=allocstat)
59      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
60
61      ALLOCATE(int_tensor(ndim),stat=allocstat)
62      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
63
64      dwar=0.d0
65      dwor=0.d0
66      dwau=0.d0
67      dwou=0.d0
68
69      IF (PRESENT(force)) THEN
70          test=force
71      ELSE
72          test=mode
73      ENDIF
74
75      SELECT CASE (test)
76      CASE('full')
77          CALL copy_coo(aotensor,int_tensor)
78      CASE('ures')
79          CALL copy_coo(ff_tensor,int_tensor)

```

```

80      CASE('qfst')
81          CALL copy_coo(byyy,int_tensor)
82      CASE('reso')
83          CALL copy_coo(ss_tensor,int_tensor)
84      CASE DEFAULT
85          stop '*** MODE variable not properly defined ***'
86      END SELECT
87

```

### 8.16.2.2 subroutine, public rk2\_stoch\_integrator::step ( real(kind=8), dimension(0:ndim), intent(in) y, real(kind=8), intent(inout) t, real(kind=8), intent(in) dt, real(kind=8), intent(in) dtn, real(kind=8), dimension(0:ndim), intent(out) res, real(kind=8), dimension(0:ndim), intent(out) tend )

Routine to perform a stochastic step of the selected dynamics (Heun algorithm). The incremented time is returned.

#### Parameters

<i>y</i>	Initial point.
<i>t</i>	Actual integration time
<i>dt</i>	Integration timestep.
<i>dtn</i>	Stochastic integration timestep (normally square-root of dt).
<i>res</i>	Final point after the step.
<i>tend</i>	Partial or full tendencies used to perform the step (used for debugging).

Definition at line 112 of file rk2\_stoch\_integrator.f90.

```

112      REAL(KIND=8), DIMENSION(0:ndim), INTENT(IN) :: y
113      REAL(KIND=8), INTENT(INOUT) :: t
114      REAL(KIND=8), INTENT(IN) :: dt,dtn
115      REAL(KIND=8), DIMENSION(0:ndim), INTENT(OUT) :: res,tend
116
117      CALL stoch_atm_res_vec(dwar)
118      CALL stoch_atm_unres_vec(dwau)
119      CALL stoch_oc_res_vec(dwor)
120      CALL stoch_oc_unres_vec(dwou)
121      anoise=(q_ar*dwar+q_au*dwau+q_or*dwor+q_ou*dwou)*dtn
122      CALL tendencies(t,y,buf_f0)
123      CALL sparse_mul3(int_tensor,y,y,tend)
124      buf_y1 = y+dt*buf_f0+anoise
125      CALL sparse_mul3(int_tensor,buf_y1,buf_y1,buf_f1)
126      tend=0.5*(tend+buf_f1)
127      CALL tendencies(t,buf_y1,buf_f1)
128      res=y+0.5*(buf_f0+buf_f1)*dt+anoise
129      t=t+dt

```

### 8.16.2.3 subroutine rk2\_stoch\_integrator::tendencies ( real(kind=8), intent(in) t, real(kind=8), dimension(0:ndim), intent(in) y, real(kind=8), dimension(0:ndim), intent(out) res ) [private]

Routine computing the tendencies of the selected model.

#### Parameters

<i>t</i>	Time at which the tendencies have to be computed. Actually not needed for autonomous systems.
<i>y</i>	Point at which the tendencies have to be computed.
<i>res</i>	vector to store the result.

**Remarks**

Note that it is NOT safe to pass `y` as a result buffer, as this operation does multiple passes.

Definition at line 97 of file `rk2_stoch_integrator.f90`.

```
97      REAL(KIND=8), INTENT(IN) :: t
98      REAL(KIND=8), DIMENSION(0:ndim), INTENT(IN) :: y
99      REAL(KIND=8), DIMENSION(0:ndim), INTENT(OUT) :: res
100     CALL sparse_mul3(int_tensor, y, y, res)
```

### 8.16.3 Variable Documentation

**8.16.3.1 real(kind=8), dimension(:), allocatable rk2\_stoch\_integrator::anoise [private]**

Additive noise term.

Definition at line 37 of file `rk2_stoch_integrator.f90`.

```
37      REAL(KIND=8), DIMENSION(:, ), ALLOCATABLE :: anoise !< Additive noise term
```

**8.16.3.2 real(kind=8), dimension(:), allocatable rk2\_stoch\_integrator::buf\_f0 [private]**

Definition at line 35 of file `rk2_stoch_integrator.f90`.

**8.16.3.3 real(kind=8), dimension(:), allocatable rk2\_stoch\_integrator::buf\_f1 [private]**

Integration buffers.

Definition at line 35 of file `rk2_stoch_integrator.f90`.

**8.16.3.4 real(kind=8), dimension(:), allocatable rk2\_stoch\_integrator::buf\_y1 [private]**

Definition at line 35 of file `rk2_stoch_integrator.f90`.

```
35      REAL(KIND=8), DIMENSION(:, ), ALLOCATABLE :: buf_y1,buf_f0,buf_f1 !< Integration buffers
```

**8.16.3.5 real(kind=8), dimension(:), allocatable rk2\_stoch\_integrator::dwar [private]**

Definition at line 33 of file `rk2_stoch_integrator.f90`.

```
33      REAL(KIND=8), DIMENSION(:, ), ALLOCATABLE :: dwar,dwau,dwor,dwou !< Standard gaussian noise buffers
```

8.16.3.6 `real(kind=8), dimension(:), allocatable rk2_stoch_integrator::dwau` [private]

Definition at line 33 of file rk2\_stoch\_integrator.f90.

8.16.3.7 `real(kind=8), dimension(:), allocatable rk2_stoch_integrator::dwor` [private]

Definition at line 33 of file rk2\_stoch\_integrator.f90.

8.16.3.8 `real(kind=8), dimension(:), allocatable rk2_stoch_integrator::dwou` [private]

Standard gaussian noise buffers.

Definition at line 33 of file rk2\_stoch\_integrator.f90.

8.16.3.9 `type(coolist), dimension(:), allocatable rk2_stoch_integrator::int_tensor` [private]

Dummy tensor that will hold the tendencies tensor.

Definition at line 39 of file rk2\_stoch\_integrator.f90.

```
39   TYPE(coolist), DIMENSION(:), ALLOCATABLE :: int_tensor !< Dummy tensor that will hold the
           tendencies tensor
```

## 8.17 rk2\_wl\_integrator Module Reference

Module with the WL rk2 integration routines.

### Functions/Subroutines

- subroutine, public `init_integrator`

*Subroutine that initialize the MARs, the memory unit and the integration buffers.*

- subroutine `compute_m1 (y)`

*Routine to compute the  $M_1$  term.*

- subroutine `compute_m2 (y)`

*Routine to compute the  $M_2$  term.*

- subroutine, public `step (y, t, dt, dtn, res, tend)`

*Routine to perform an integration step (Heun algorithm) of the WL system. The incremented time is returned.*

- subroutine, public `full_step (y, t, dt, dtn, res)`

*Routine to perform an integration step (Heun algorithm) of the full stochastic system. The incremented time is returned.*

## Variables

- real(kind=8), dimension(:), allocatable `buf_y1`
- real(kind=8), dimension(:), allocatable `buf_f0`
- real(kind=8), dimension(:), allocatable `buf_f1`
  - Integration buffers.*
- real(kind=8), dimension(:), allocatable `buf_m2`
- real(kind=8), dimension(:), allocatable `buf_m1`
- real(kind=8), dimension(:), allocatable `buf_m3`
- real(kind=8), dimension(:), allocatable `buf_m`
- real(kind=8), dimension(:), allocatable `buf_m3s`
  - Dummy buffers holding the terms  $f_i M_i$ .*
- real(kind=8), dimension(:), allocatable `anoise`
  - Additive noise term.*
- real(kind=8), dimension(:), allocatable `dwar`
- real(kind=8), dimension(:), allocatable `dwau`
- real(kind=8), dimension(:), allocatable `dwor`
- real(kind=8), dimension(:), allocatable `dwou`
  - Standard gaussian noise buffers.*
- real(kind=8), dimension(:, :), allocatable `x1`
  - Buffer holding the subsequent states of the first MAR.*
- real(kind=8), dimension(:, :), allocatable `x2`
  - Buffer holding the subsequent states of the second MAR.*

### 8.17.1 Detailed Description

Module with the WL rk2 integration routines.

#### Copyright

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#### Remarks

This module actually contains the Heun algorithm routines.

### 8.17.2 Function/Subroutine Documentation

#### 8.17.2.1 subroutine rk2\_wl\_integrator::compute\_m1 ( real(kind=8), dimension(0:ndim), intent(in) y ) [private]

Routine to compute the  $M_1$  term.

#### Parameters

<code>y</code>	Present state of the WL system
----------------	--------------------------------

Definition at line 106 of file rk2\_WL\_integrator.f90.

```
106      REAL (KIND=8), DIMENSION(0:ndim), INTENT(IN) :: y
```

```

107     buf_m1=0.d0
108     IF (m12def) CALL sparse_mul2_k(m12, y, buf_m1)
109     buf_m1=buf_m1+mltot

```

### 8.17.2.2 subroutine rk2\_wl\_integrator::compute\_m2 ( real(kind=8), dimension(0:ndim), intent(in) y ) [private]

Routine to compute the  $M_2$  term.

#### Parameters

<i>y</i>	Present state of the WL system
----------	--------------------------------

Definition at line 115 of file rk2\_WL\_integrator.f90.

```

115     REAL (KIND=8), DIMENSION(0:ndim), INTENT(IN) :: y
116     buf_m=0.d0
117     buf_m2=0.d0
118     IF (m21def) CALL sparse_mul3(m21, y, x1(0:ndim,1), buf_m)
119     IF (m22def) CALL sparse_mul3(m22, x2(0:ndim,1), x2(0:ndim,1), buf_m2)
120     buf_m2=buf_m2+buf_m

```

### 8.17.2.3 subroutine, public rk2\_wl\_integrator::full\_step ( real(kind=8), dimension(0:ndim), intent(in) y, real(kind=8), intent(inout) t, real(kind=8), intent(in) dt, real(kind=8), intent(in) dtn, real(kind=8), dimension(0:ndim), intent(out) res )

Routine to perform an integration step (Heun algorithm) of the full stochastic system. The incremented time is returned.

#### Parameters

<i>y</i>	Initial point.
<i>t</i>	Actual integration time
<i>dt</i>	Integration timestep.
<i>dtn</i>	Stochastoc integration timestep (normally square-root of dt).
<i>res</i>	Final point after the step.

Definition at line 185 of file rk2\_WL\_integrator.f90.

```

185     REAL (KIND=8), DIMENSION(0:ndim), INTENT(IN) :: y
186     REAL (KIND=8), INTENT(INOUT) :: t
187     REAL (KIND=8), INTENT(IN) :: dt,dtn
188     REAL (KIND=8), DIMENSION(0:ndim), INTENT(OUT) :: res
189     CALL stoch_atm_res_vec(dwar)
190     CALL stoch_atm_unres_vec(dwau)
191     CALL stoch_oc_res_vec(dwor)
192     CALL stoch_oc_unres_vec(dwou)
193     anoise=(q_ar*dwar+q_au*dwau+q_or*dwor+q_ou*dwou)*dtn
194     CALL sparse_mul3(aotensor,y,y,buf_f0)
195     buf_y1 = y+dt*buf_f0+anoise
196     CALL sparse_mul3(aotensor,buf_y1,buf_y1,buf_f1)
197     res=y+0.5*(buf_f0+buf_f1)*dt+anoise
198     t=t+dt

```

## 8.17.2.4 subroutine, public rk2\_wl\_integrator::init\_integrator( )

Subroutine that initialize the MARs, the memory unit and the integration buffers.

Definition at line 44 of file rk2\_WL\_integrator.f90.

```

44      INTEGER :: allocstat,i
45
46      CALL init_ss_integrator
47
48      print*, 'Initializing the integrator ...'
49
50      IF (mode.ne.'ures') THEN
51          print*, '*** Mode set to ',mode,' in stoch_params.nml ***'
52          print*, '*** WL configuration only support unresolved mode ***'
53          stop "*** Please change to 'ures' and perform the configuration again ! ***"
54      ENDIF
55
56      ALLOCATE(buf_y1(0:ndim),buf_f0(0:ndim),buf_f1(0:ndim),stat=allocstat)
57      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
58
59      ALLOCATE(buf_m1(0:ndim), buf_m2(0:ndim), buf_m3(0:ndim), buf_m(0:
60      ndim), buf_m3s(0:ndim), stat=allocstat)
61      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
62
63      ALLOCATE(dwar(0:ndim),dwau(0:ndim),dwor(0:ndim),dwou(0:ndim),
64      stat=allocstat)
65      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
66
67      ALLOCATE(anoise(0:ndim), stat=allocstat)
68      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
69
70      buf_y1=0.d0
71      buf_f1=0.d0
72      buf_f0=0.d0
73
74      dwar=0.d0
75      dwor=0.d0
76      dwau=0.d0
77      dwou=0.d0
78
79      buf_m1=0.d0
80      buf_m2=0.d0
81      buf_m3=0.d0
82      buf_m3s=0.d0
83      buf_m=0.d0
84
85      print*, 'Initializing the MARs ...'
86
87      CALL init_mar
88
89      ALLOCATE(x1(0:ndim,ms), x2(0:ndim,ms), stat=allocstat)
90
91      x1=0.d0
92      DO i=1,50000
93          CALL mar_step(x1)
94      ENDDO
95
96      x2=0.d0
97      DO i=1,50000
98          CALL mar_step(x2)
99      ENDDO
100
101      CALL init_memory

```

## 8.17.2.5 subroutine, public rk2\_wl\_integrator::step( real(kind=8), dimension(0:ndim), intent(in) y, real(kind=8), intent(inout) t, real(kind=8), intent(in) dt, real(kind=8), intent(in) dtn, real(kind=8), dimension(0:ndim), intent(out) res, real(kind=8), dimension(0:ndim), intent(out) tend )

Routine to perform an integration step (Heun algorithm) of the WL system. The incremented time is returned.

### Parameters

<i>y</i>	Initial point.
<i>t</i>	Actual integration time
<i>dt</i>	Integration timestep.
<i>dtn</i>	Stochastic integration timestep (normally square-root of dt).
<i>res</i>	Final point after the step.
<i>tend</i>	Partial or full tendencies used to perform the step (used for debugging).

Definition at line 132 of file rk2\_WL\_integrator.f90.

```

132      REAL(KIND=8), DIMENSION(0:ndim), INTENT(IN) :: y
133      REAL(KIND=8), INTENT(INOUT) :: t
134      REAL(KIND=8), INTENT(IN) :: dt,dtn
135      REAL(KIND=8), DIMENSION(0:ndim), INTENT(OUT) :: res,tend
136      INTEGER :: i
137
138      IF (mod(t,muti)<dt) THEN
139          CALL compute_m3(y,dts,dtsn,.true.,.true.,.true.,muti/2,buf_m3s)
140          buf_m3=buf_m3s
141          DO i=1,1
142              CALL compute_m3(y,dts,dtsn,.false.,.true.,.true.,.true.,muti/2,buf_m3s)
143              buf_m3=buf_m3+buf_m3s
144          ENDDO
145          !DO i=1,2
146              ! CALL compute_M3(y,dts,dtsn,.false.,.true.,.true.,.true.,muti/2,buf_M3s)
147              ! buf_M3=buf_M3+buf_M3s
148          !ENDDO
149          buf_m3=buf_m3/2
150      ENDIF
151
152
153      CALL stoch_atm_res_vec(dwar)
154      CALL stoch_oc_res_vec(dwor)
155      anoise=(q_ar*dwar+q_or*dwor)*dtn
156
157      CALL tendencies(t,y,buf_f0)
158      CALL mar_step(x1)
159      CALL mar_step(x2)
160      CALL compute_m1(y)
161      CALL compute_m2(y)
162      buf_f0= buf_f0+buf_m1+buf_m2+buf_m3
163      buf_y1 = y+dt*buf_f0+anoise
164
165      CALL tendencies(t+dt,buf_y1,buf_f1)
166      CALL compute_m1(buf_y1)
167      CALL compute_m2(buf_y1)
168      !IF (mod(t,muti)<dt) CALL compute_M3(buf_y1,dts,dtsn,.false.,.true.,buf_M3)
169
170      buf_f0=0.5*(buf_f0+buf_f1+buf_m1+buf_m2+buf_m3)
171      res=y+dt*buf_f0+anoise
172
173      tend=buf_m3
174      t=t+dt
175

```

### 8.17.3 Variable Documentation

#### 8.17.3.1 real(kind=8), dimension(:), allocatable rk2\_wl\_integrator::anoise [private]

Additive noise term.

Definition at line 33 of file rk2\_WL\_integrator.f90.

```
33      REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: anoise           !< Additive noise term
```

8.17.3.2 `real(kind=8), dimension(:), allocatable rk2_wl_integrator::buf_f0 [private]`

Definition at line 31 of file rk2\_WL\_integrator.f90.

8.17.3.3 `real(kind=8), dimension(:), allocatable rk2_wl_integrator::buf_f1 [private]`

Integration buffers.

Definition at line 31 of file rk2\_WL\_integrator.f90.

8.17.3.4 `real(kind=8), dimension(:), allocatable rk2_wl_integrator::buf_m [private]`

Definition at line 32 of file rk2\_WL\_integrator.f90.

8.17.3.5 `real(kind=8), dimension(:), allocatable rk2_wl_integrator::buf_m1 [private]`

Definition at line 32 of file rk2\_WL\_integrator.f90.

8.17.3.6 `real(kind=8), dimension(:), allocatable rk2_wl_integrator::buf_m2 [private]`

Definition at line 32 of file rk2\_WL\_integrator.f90.

```
32  REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: buf_m2,buf_m1,buf_m3,buf_m,buf_m3s !< Dummy buffers holding  
      the terms /f$M_i\f$ of the parameterization
```

8.17.3.7 `real(kind=8), dimension(:), allocatable rk2_wl_integrator::buf_m3 [private]`

Definition at line 32 of file rk2\_WL\_integrator.f90.

8.17.3.8 `real(kind=8), dimension(:), allocatable rk2_wl_integrator::buf_m3s [private]`

Dummy buffers holding the terms /f\$M\_i.

Definition at line 32 of file rk2\_WL\_integrator.f90.

8.17.3.9 `real(kind=8), dimension(:), allocatable rk2_wl_integrator::buf_y1 [private]`

Definition at line 31 of file rk2\_WL\_integrator.f90.

```
31  REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: buf_y1,buf_f0,buf_f1 !< Integration buffers
```

8.17.3.10 `real(kind=8), dimension(:), allocatable rk2_wl_integrator::dwar [private]`

Definition at line 34 of file `rk2_WL_integrator.f90`.

```
34    REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: dwar,dwau,dwor,dwou !< Standard gaussian noise buffers
```

8.17.3.11 `real(kind=8), dimension(:), allocatable rk2_wl_integrator::dwau [private]`

Definition at line 34 of file `rk2_WL_integrator.f90`.

8.17.3.12 `real(kind=8), dimension(:), allocatable rk2_wl_integrator::dwor [private]`

Definition at line 34 of file `rk2_WL_integrator.f90`.

8.17.3.13 `real(kind=8), dimension(:), allocatable rk2_wl_integrator::dwou [private]`

Standard gaussian noise buffers.

Definition at line 34 of file `rk2_WL_integrator.f90`.

8.17.3.14 `real(kind=8), dimension(:,:,), allocatable rk2_wl_integrator::x1 [private]`

Buffer holding the subsequent states of the first MAR.

Definition at line 36 of file `rk2_WL_integrator.f90`.

```
36    REAL(KIND=8), DIMENSION(:,:,), ALLOCATABLE :: x1 !< Buffer holding the subsequent states of the first MAR
```

8.17.3.15 `real(kind=8), dimension(:,:,), allocatable rk2_wl_integrator::x2 [private]`

Buffer holding the subsequent states of the second MAR.

Definition at line 37 of file `rk2_WL_integrator.f90`.

```
37    REAL(KIND=8), DIMENSION(:,:,), ALLOCATABLE :: x2 !< Buffer holding the subsequent states of the second MAR
```

## 8.18 sf\_def Module Reference

Module to select the resolved-unresolved components.

## Functions/Subroutines

- subroutine, public `load_sf`

*Subroutine to load the unresolved variable defintion vector SF from SF.nml if it exists. If it does not, then write SF.nml with no unresolved variables specified (null vector).*

## Variables

- logical `exists`

*Boolean to test for file existence.*

- integer, dimension(:), allocatable, public `sf`

*Unresolved variable definition vector.*

- integer, dimension(:), allocatable, public `ind`

- integer, dimension(:), allocatable, public `rind`

*Unresolved reduction indices.*

- integer, dimension(:), allocatable, public `sl_ind`

- integer, dimension(:), allocatable, public `sl_rind`

*Resolved reduction indices.*

- integer, public `n_unres`

*Number of unresolved variables.*

- integer, public `n_res`

*Number of resolved variables.*

- integer, dimension(:, :, ), allocatable, public `bar`

- integer, dimension(:, :, ), allocatable, public `bau`

- integer, dimension(:, :, ), allocatable, public `bor`

- integer, dimension(:, :, ), allocatable, public `bou`

*Filter matrices.*

### 8.18.1 Detailed Description

Module to select the resolved-unresolved components.

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### 8.18.2 Function/Subroutine Documentation

#### 8.18.2.1 subroutine, public sf\_def::load\_sf( )

Subroutine to load the unresolved variable defintion vector SF from SF.nml if it exists. If it does not, then write SF.nml with no unresolved variables specified (null vector).

Definition at line 37 of file sf\_def.f90.

```

37      INTEGER :: i,allocstat,n,ns
38      CHARACTER(len=20) :: fm
39
40      namelist /sflist/ sf
41
42      fm(1:6)='(F3.1)'
43
44      IF (ndim == 0) stop "*** Number of dimensions is 0! ***"
45      ALLOCATE(sf(0:ndim), stat=allocstat)
46      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
47
48      INQUIRE(file='./SF.nml', exist=exists)
49
50      IF (exists) THEN
51          OPEN(8, file="SF.nml", status='OLD', recl=80, delim='APOSTROPHE')
52          READ(8,nml=sflist)
53          CLOSE(8)
54          n_unres=0
55          DO i=1,ndim ! Computing the number of unresolved variables
56              IF (sf(i)==1) n_unres=n_unres+1
57          ENDDO
58          IF (n_unres==0) stop "*** No unresolved variable specified! ***"
59          n_res=ndim-n_unres
60          ALLOCATE(ind(n_unres), rind(0:ndim), sl_ind(n_res), sl_rind(0:ndim),
61 stat=allocstat)
62          IF (allocstat /= 0) stop "*** Not enough memory ! ***"
63          ALLOCATE(bar(0:ndim,0:ndim), bau(0:ndim,0:ndim), bor(0:
64 ndim,0:ndim), bou(0:ndim,0:ndim), stat=allocstat)
65          IF (allocstat /= 0) stop "*** Not enough memory ! ***"
66          rind=0
67          n=1
68          ns=1
69          DO i=1,ndim
70              IF (sf(i)==1) THEN
71                  ind(i)=i
72                  rind(i)=n
73                  n=n+1
74              ELSE
75                  sl_ind(ns)=i
76                  sl_rind(i)=ns
77                  ns=ns+1
78              ENDIF
79          ENDDO
80          bar=0
81          bau=0
82          bor=0
83          bou=0
84          DO i=1,2*natm
85              IF (sf(i)==1) THEN
86                  bau(i,i)=1
87              ELSE
88                  bar(i,i)=1
89              ENDIF
90          ENDDO
91          DO i=2*natm+1,ndim
92              IF (sf(i)==1) THEN
93                  bou(i,i)=1
94              ELSE
95                  bor(i,i)=1
96              ENDIF
97          ELSE
98              OPEN(8, file="SF.nml", status='NEW')
99              WRITE(8,'(a)') "!-----!""
100             WRITE(8,'(a)') "! Namelist file : "
101             WRITE(8,'(a)') "! Unresolved variables specification (1 -> unresolved, 0 -> resolved)
102             WRITE(8,'(a)') "!"
103             WRITE(8,*)
104             WRITE(8,'(a)') "&SFLIST"
105             WRITE(8,*)
106             DO i=1,natm
107                 WRITE(8,*)
108                     WRITE(8,"//trim(str(i))//") = 0 //"
109                     & //awavenum(i)%typ//", Nx= "//trim(rstr(awavenum(i)&
110                     &%Nx,fm))//", Ny= "//trim(rstr(awavenum(i)%Ny,fm))
111             END DO
112             WRITE(8,*)
113             DO i=1,natm
114                 WRITE(8,*)
115                     WRITE(8,"//trim(str(i+natm))//") = 0 //"
116                     & //awavenum(i)%typ//", Nx= "//trim(rstr(awavenum(i)&
117                     &%Nx,fm))//", Ny= "//trim(rstr(awavenum(i)%Ny,fm))
118             END DO
119             WRITE(8,*)
120             DO i=1,noc
121                 WRITE(8,*)
122                     WRITE(8,"//trim(str(i+2*natm))//") = 0 //"
123                     & //trim(rstr(owavenum(i)%Nx,fm))//", Ny= "&
```

```

121      &//trim(rstr(owavenum(i)%Ny, fm))
122      END DO
123      WRITE(8,*)
124      DO i=1,noc
125        WRITE(8,*)
126        &! Nx= //trim(rstr(owavenum(i)%Nx, fm)) //", Ny= "&
127        &//trim(rstr(owavenum(i)%Ny, fm))
128      END DO
129
130      WRITE(8,'(a)')  "&END"
131      WRITE(8,*)
132      CLOSE(8)
133      stop "*** SF.nml namelist written. Fill in the file and rerun !***"
134  ENDIF

```

### 8.18.3 Variable Documentation

#### 8.18.3.1 integer, dimension(:, :, ), allocatable, public sf\_def::bar

Definition at line 28 of file sf\_def.f90.

```
28  INTEGER, DIMENSION(:, :, ), ALLOCATABLE, PUBLIC :: bar,bau,bor,bou !< Filter matrices
```

#### 8.18.3.2 integer, dimension(:, :, ), allocatable, public sf\_def::bau

Definition at line 28 of file sf\_def.f90.

#### 8.18.3.3 integer, dimension(:, :, ), allocatable, public sf\_def::bor

Definition at line 28 of file sf\_def.f90.

#### 8.18.3.4 integer, dimension(:, :, ), allocatable, public sf\_def::bou

Filter matrices.

Definition at line 28 of file sf\_def.f90.

#### 8.18.3.5 logical sf\_def::exists [private]

Boolean to test for file existence.

Definition at line 21 of file sf\_def.f90.

```
21  LOGICAL :: exists !< Boolean to test for file existence.
```

#### 8.18.3.6 integer, dimension(:, ), allocatable, public sf\_def::ind

Definition at line 24 of file sf\_def.f90.

```
24  INTEGER, DIMENSION(:, ), ALLOCATABLE, PUBLIC :: ind,rind !< Unresolved reduction indices
```

**8.18.3.7 integer, public sf\_def::n\_res**

Number of resolved variables.

Definition at line 27 of file sf\_def.f90.

```
27    INTEGER, PUBLIC :: n_res !< Number of resolved variables
```

**8.18.3.8 integer, public sf\_def::n\_unres**

Number of unresolved variables.

Definition at line 26 of file sf\_def.f90.

```
26    INTEGER, PUBLIC :: n_unres !< Number of unresolved variables
```

**8.18.3.9 integer, dimension(:), allocatable, public sf\_def::rind**

Unresolved reduction indices.

Definition at line 24 of file sf\_def.f90.

**8.18.3.10 integer, dimension(:), allocatable, public sf\_def::sf**

Unresolved variable definition vector.

Definition at line 23 of file sf\_def.f90.

```
23    INTEGER, DIMENSION(:), ALLOCATABLE, PUBLIC :: sf           !< Unresolved variable definition vector
```

**8.18.3.11 integer, dimension(:), allocatable, public sf\_def::sl\_ind**

Definition at line 25 of file sf\_def.f90.

```
25    INTEGER, DIMENSION(:), ALLOCATABLE, PUBLIC :: sl_ind, sl_rind !< Resolved reduction indices
```

**8.18.3.12 integer, dimension(:), allocatable, public sf\_def::sl\_rind**

Resolved reduction indices.

Definition at line 25 of file sf\_def.f90.

## 8.19 sigma Module Reference

The MTV noise sigma matrices used to integrate the MTV model.

### Functions/Subroutines

- subroutine, public `init_sigma` (mult, Q1fill)
 

*Subroutine to initialize the sigma matices.*
- subroutine, public `compute_mult_sigma` (y)
 

*Routine to actualize the matrix  $\sigma_1$  based on the state y of the MTV system.*

### Variables

- real(kind=8), dimension(:, :, :), allocatable, public `sig1`

$\sigma_1(X)$  state-dependent noise matrix
- real(kind=8), dimension(:, :, :), allocatable, public `sig2`

$\sigma_2$  state-independent noise matrix
- real(kind=8), dimension(:, :, :), allocatable, public `sig1r`

Reduced  $\sigma_1(X)$  state-dependent noise matrix.
- real(kind=8), dimension(:, :, :), allocatable `dumb_mat1`

Dummy matrix.
- real(kind=8), dimension(:, :, :), allocatable `dumb_mat2`

Dummy matrix.
- real(kind=8), dimension(:, :, :), allocatable `dumb_mat3`

Dummy matrix.
- real(kind=8), dimension(:, :, :), allocatable `dumb_mat4`

Dummy matrix.
- integer, dimension(:, :), allocatable `ind1`
- integer, dimension(:, :), allocatable `rind1`
- integer, dimension(:, :), allocatable `ind2`
- integer, dimension(:, :), allocatable `rind2`

Reduction indices.
- integer `n1`
- integer `n2`

#### 8.19.1 Detailed Description

The MTV noise sigma matrices used to integrate the MTV model.

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#### Remarks

See : Franzke, C., Majda, A. J., & Vanden-Eijnden, E. (2005). Low-order stochastic mode reduction for a realistic barotropic model climate. *Journal of the atmospheric sciences*, 62(6), 1722-1745.

#### 8.19.2 Function/Subroutine Documentation

##### 8.19.2.1 subroutine, public `sigma::compute_mult_sigma` ( real(kind=8), dimension(0:ndim), intent(in) y )

Routine to actualize the matrix  $\sigma_1$  based on the state y of the MTV system.

## Parameters

<i>y</i>	State of the MTV system
----------	-------------------------

Definition at line 93 of file MTV\_sigma\_tensor.f90.

```

93      REAL(KIND=8), DIMENSION(0:ndim), INTENT(IN) :: y
94      INTEGER :: info,info2
95      CALL sparse_mul3_mat(utot,y,dumb_mat1)
96      CALL sparse_mul4_mat(vtot,y,y,dumb_mat2)
97      dumb_mat3=dumb_mat1+dumb_mat2+q1
98      CALL reduce(dumb_mat3,dumb_mat1,n1,ind1,rind1)
99      IF (n1 /= 0) THEN
100         CALL sqrtm_svd(dumb_mat1(1:n1,1:n1),dumb_mat2(1:n1,1:n1),info,info2,min(max(n1/2,2),64))
101         ! dumb_mat2=0.D0
102         ! CALL chol(0.5*(dumb_mat1(1:n1,1:n1)+transpose(dumb_mat1(1:n1,1:n1))),dumb_mat2(1:n1,1:n1),info)
103         IF ((.not.any(isnan(dumb_mat2))).and.(info.eq.0).and.(.not.any(dumb_mat2>huge(0.d0)))) THEN
104             CALL ireduce(sig1,dumb_mat2,n1,ind1,rind1)
105         ELSE
106             sig1=sig1r
107         ENDIF
108     ELSE
109         sig1=sig1r
110     ENDIF

```

### 8.19.2.2 subroutine, public sigma::init\_sigma ( logical, intent(out) mult, logical, intent(out) Q1fill )

Subroutine to initialize the sigma matices.

Definition at line 48 of file MTV\_sigma\_tensor.f90.

```

48      LOGICAL, INTENT(OUT) :: mult,q1fill
49      INTEGER :: allocstat,inf0l,info2
50
51      CALL init_sqrt
52
53      ALLOCATE(sig1(ndim,ndim), sig2(ndim,ndim), sig1r(ndim,
54      ndim),stat=allocstat)
54      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
55
56      ALLOCATE(ind1(ndim), rind1(ndim), ind2(ndim), rind2(ndim),
57      stat=allocstat)
57      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
58
59      ALLOCATE(dumb_mat1(ndim,ndim), dumb_mat2(ndim,ndim), stat=allocstat)
60      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
61
62      ALLOCATE(dumb_mat3(ndim,ndim), dumb_mat4(ndim,ndim), stat=allocstat)
63      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
64
65      print*, "Initializing the sigma matrices"
66
67      CALL reduce(q2,dumb_mat1,n2,ind2,rind2)
68      IF (n2 /= 0) THEN
69          CALL sqrtm_svd(dumb_mat1(1:n2,1:n2),dumb_mat2(1:n2,1:n2),inf0l,info2,min(max(n2/2,2),64))
70          CALL ireduce(sig2,dumb_mat2,n2,ind2,rind2)
71      ELSE
72          sig2=0.d0
73      ENDIF
74
75      mult=(.not.((tensor_empty(utot)).and.(tensor4_empty(vtot))))
76      q1fill=.true.
77      CALL reduce(q1,dumb_mat1,n1,ind1,rind1)
78      IF (n1 /= 0) THEN
79
80          CALL sqrtm_svd(dumb_mat1(1:n1,1:n1),dumb_mat2(1:n1,1:n1),inf0l,info2,min(max(n1/2,2),64))
81          CALL ireduce(sig1,dumb_mat2,n1,ind1,rind1)
82      ELSE
83          q1fill=.false.
84          sig1=0.d0
85      ENDIF
86      sig1r=sig1
87

```

### 8.19.3 Variable Documentation

#### 8.19.3.1 real(kind=8), dimension(:, :), allocatable sigma::dumb\_mat1 [private]

Dummy matrix.

Definition at line 35 of file MTV\_sigma\_tensor.f90.

```
35    REAL(KIND=8), DIMENSION(:, :, :), ALLOCATABLE :: dumb_mat1 !< Dummy matrix
```

#### 8.19.3.2 real(kind=8), dimension(:, :), allocatable sigma::dumb\_mat2 [private]

Dummy matrix.

Definition at line 36 of file MTV\_sigma\_tensor.f90.

```
36    REAL(KIND=8), DIMENSION(:, :, :), ALLOCATABLE :: dumb_mat2 !< Dummy matrix
```

#### 8.19.3.3 real(kind=8), dimension(:, :), allocatable sigma::dumb\_mat3 [private]

Dummy matrix.

Definition at line 37 of file MTV\_sigma\_tensor.f90.

```
37    REAL(KIND=8), DIMENSION(:, :, :), ALLOCATABLE :: dumb_mat3 !< Dummy matrix
```

#### 8.19.3.4 real(kind=8), dimension(:, :), allocatable sigma::dumb\_mat4 [private]

Dummy matrix.

Definition at line 38 of file MTV\_sigma\_tensor.f90.

```
38    REAL(KIND=8), DIMENSION(:, :, :), ALLOCATABLE :: dumb_mat4 !< Dummy matrix
```

#### 8.19.3.5 integer, dimension(:), allocatable sigma::ind1 [private]

Definition at line 39 of file MTV\_sigma\_tensor.f90.

```
39    INTEGER, DIMENSION(:), ALLOCATABLE :: ind1, rind1, ind2, rind2 !< Reduction indices
```

#### 8.19.3.6 integer, dimension(:), allocatable sigma::ind2 [private]

Definition at line 39 of file MTV\_sigma\_tensor.f90.

### 8.19.3.7 integer sigma::n1 [private]

Definition at line 41 of file MTV\_sigma\_tensor.f90.

```
41    INTEGER :: n1,n2
```

### 8.19.3.8 integer sigma::n2 [private]

Definition at line 41 of file MTV\_sigma\_tensor.f90.

### 8.19.3.9 integer, dimension(:), allocatable sigma::rind1 [private]

Definition at line 39 of file MTV\_sigma\_tensor.f90.

### 8.19.3.10 integer, dimension(:), allocatable sigma::rind2 [private]

Reduction indices.

Definition at line 39 of file MTV\_sigma\_tensor.f90.

### 8.19.3.11 real(kind=8), dimension(:, :), allocatable, public sigma::sig1

$\sigma_1(X)$  state-dependent noise matrix

Definition at line 31 of file MTV\_sigma\_tensor.f90.

```
31    REAL(KIND=8), DIMENSION(:, :), ALLOCATABLE, PUBLIC :: sig1 !< \f$\\sigma_1(X)\\f$ state-dependent noise
matrix
```

### 8.19.3.12 real(kind=8), dimension(:, :), allocatable, public sigma::sig1r

Reduced  $\sigma_1(X)$  state-dependent noise matrix.

Definition at line 33 of file MTV\_sigma\_tensor.f90.

```
33    REAL(KIND=8), DIMENSION(:, :), ALLOCATABLE, PUBLIC :: sig1r !< Reduced \f$\\sigma_1(X)\\f$ state-dependent
noise matrix
```

### 8.19.3.13 real(kind=8), dimension(:, :), allocatable, public sigma::sig2

$\sigma_2$  state-independent noise matrix

Definition at line 32 of file MTV\_sigma\_tensor.f90.

```
32    REAL(KIND=8), DIMENSION(:, :), ALLOCATABLE, PUBLIC :: sig2 !< \f$\\sigma_2\\f$ state-independent noise
matrix
```

## 8.20 sqrt\_mod Module Reference

Utility module with various routine to compute matrix square root.

### Functions/Subroutines

- subroutine, public `init_sqrt`
- subroutine, public `sqrtm` (A, sqA, info, info\_triu, bs)  
*Routine to compute a real square-root of a matrix.*
- logical function `selectev` (a, b)
- subroutine `sqrtm_triu` (A, sqA, info, bs)
- subroutine `csqrtm_triu` (A, sqA, info, bs)
- subroutine `rsf2csf` (T, Z, Tz, Zz)  
*Routine to perform a Cholesky decomposition.*
- subroutine, public `chol` (A, sqA, info)
- subroutine, public `sqrtm_svd` (A, sqA, info, info\_triu, bs)  
*Routine to compute a real square-root of a matrix via a SVD decomposition.*

### Variables

- real(kind=8), dimension(:), allocatable `work`
- integer `lwork`
- real(kind=8), parameter `real_eps` = 2.2204460492503131e-16

#### 8.20.1 Detailed Description

Utility module with various routine to compute matrix square root.

#### Copyright

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#### Remarks

Mainly based on the numerical recipes and from: Edvin Deadman, Nicholas J. Higham, Rui Ralha (2013) "← Blocked Schur Algorithms for Computing the Matrix Square Root", Lecture Notes in Computer Science, 7782. pp. 171-182.

#### 8.20.2 Function/Subroutine Documentation

##### 8.20.2.1 subroutine, public `sqrt_mod::chol` ( `real(kind=8), dimension(:, :), intent(in) A`, `real(kind=8), dimension(:, :), intent(out) sqA`, `integer, intent(out) info` )

Routine to perform a Cholesky decomposition.

## Parameters

<b>A</b>	Matrix whose decomposition is evaluated.
<b>sqA</b>	Cholesky decomposition of A.
<b>info</b>	Information code returned by the Lapack routines.

Definition at line 386 of file sqrt\_mod.f90.

```
386    REAL(KIND=8), DIMENSION(:,:), INTENT(IN) :: a
387    REAL(KIND=8), DIMENSION(:,:), INTENT(OUT) :: sqa
388    INTEGER, INTENT(OUT) :: info
389
390    sqa=a
391    CALL dpotrf('L',SIZE(sqa,1),sqa,SIZE(sqa,1),info)
```

### 8.20.2.2 subroutine sqrt\_mod::csqrmtm\_triu ( complex(kind=16), dimension(:,:), intent(in) A, complex(kind=16), dimension(:,:), intent(out) sqA, integer, intent(out) info, integer, intent(in), optional bs ) [private]

Definition at line 235 of file sqrt\_mod.f90.

```
235    COMPLEX(KIND=16), DIMENSION(:,:), INTENT(IN) :: a
236    INTEGER, INTENT(IN), OPTIONAL :: bs
237    COMPLEX(KIND=16), DIMENSION(:,:), INTENT(OUT) :: sqa
238    INTEGER, INTENT(OUT) :: info
239    COMPLEX(KIND=16), DIMENSION(SIZE(a,1)) :: a_diag
240    COMPLEX(KIND=16), DIMENSION(SIZE(a,1),SIZE(a,1)) :: r,sm,rii,rjj
241    INTEGER, DIMENSION(2*SIZE(a,1),2) :: start_stop_pairs
242    COMPLEX(KIND=16) :: s,denom, scale
243    INTEGER :: i,j,k,start,n,sstop,m
244    INTEGER :: istart,istop,jstart,jstop
245    INTEGER :: nblocks,blocksize
246    INTEGER :: bsmall,blarge,nlarge,nsmall
247
248    blocksize=64
249    IF (PRESENT(bs)) blocksize=bs
250    n=SIZE(a,1)
251    ! print*, blocksize
252
253    CALL cdiag(a,a_diag)
254    r=0.d0
255    DO i=1,n
256      r(i,i)=sqrt(a_diag(i))
257    ENDDO
258
259    nblocks=max(floordiv(n,blocksize),1)
260    bsmall=floordiv(n,nblocks)
261    nlarge=mod(n,nblocks)
262    blarge=bsmall+1
263    nsmall=nblocks-nlarge
264    IF (nsmall*bsmall + nlarge*blarge /= n) stop 'Sqrmtm: Internal inconsistency'
265
266    ! print*, nblocks,bsmall,nsmall,blarge,nlarge
267
268    start=1
269    DO i=1,nsmall
270      start_stop_pairs(i,1)=start
271      start_stop_pairs(i,2)=start+bsmall-1
272      start=start+bsmall
273    ENDDO
274    DO i=nsmall+1,nsmall+nlarge
275      start_stop_pairs(i,1)=start
276      start_stop_pairs(i,2)=start+blarge-1
277      start=start+blarge
278    ENDDO
279
280    ! DO i=1,SIZE(start_stop_pairs,1)
281    !   print*, i
282    !   print*, start_stop_pairs(i,1),start_stop_pairs(i,2)
283    ! END DO
284
285    DO k=1,nsmall+nlarge
```

```

287     start=start_stop_pairs(k,1)
288     sstop=start_stop_pairs(k,2)
289     DO j=start,sstop
290       DO i=j-1,start,-1
291         s=0.d0
292         IF (j-i>1) s= dot_product(r(i,i+1:j-1),r(i+1:j-1,j))
293         denom= r(i,i)+r(j,j)
294         IF (denom==0.d0) stop 'Sqrtn: Failed to find the matrix square root'
295         r(i,j)=(a(i,j)-s)/denom
296       END DO
297     END DO
298   END DO
299
300 ! print*, 'R'
301 ! CALL printmat(R)
302
303 DO j=1,nblocks
304   jstart=start_stop_pairs(j,1)
305   jstop=start_stop_pairs(j,2)
306   DO i=j-1,1,-1
307     istart=start_stop_pairs(i,1)
308     istop=start_stop_pairs(i,2)
309     sm=0.d0
310     sm(istart:istop,jstart:jstop)=a(istart:istop,jstart:jstop)
311     IF (j-i>1) sm(istart:istop,jstart:jstop) = sm(istart:istop&
312       &,jstart:jstop) - matmul(r(istart:istop,istop:jstart)&
313       &,r(istop:jstart,jstart:jstop))
314     rii=0.d0
315     rii = r(istart:istop, istart:istop)
316     rjj=0.d0
317     rjj = r(jstart:jstop, jstart:jstop)
318     m=istop-istart+1
319     n=jstop-jstart+1
320     k=1
321     ! print*, m,n
322     ! print*, istart,istop
323     ! print*, jstart,jstop
324
325     ! print*, 'Rii',Rii(istart:istop, istart:istop)
326     ! print*, 'Rjj',Rjj(jstart:jstop, jstart:jstop)
327     ! print*, 'Sm',Sm(istart:istop, jstart:jstop)
328
329     CALL ztrsvl('N','N',k,m,n,rii(istart:istop, istart:istop),m&
330       &,rjj(jstart:jstop, jstart:jstop),n,sm(istart:istop&
331       &,jstart:jstop),m, scale,info)
332     r(istart:istop,jstart:jstop)=sm(istart:istop,jstart:jstop)*scale
333   ENDDO
334 ENDDO
335 sqar=r

```

### 8.20.2.3 subroutine, public sqrt\_mod::init\_sqrt( )

Definition at line 39 of file sqrt\_mod.f90.

```

39      INTEGER :: allocstat
40      lwork=10
41      lwork=ndim*lwork
42
43      ! print*, lwork
44
45      IF (ALLOCATED(work)) THEN
46        DEALLOCATE (work, stat=allocstat)
47        IF (allocstat /= 0) stop "*** Deallocation problem ! ***"
48      ENDIF
49      ALLOCATE (work(lwork), stat=allocstat)
50      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
51
52      ! ALLOCATE (zwork(lwork), STAT=AllocStat)
53      ! IF (AllocStat /= 0) STOP "*** Not enough memory ! ***"

```

### 8.20.2.4 subroutine sqrt\_mod::rsf2csf ( real(kind=8), dimension(:,:), intent(in) T, real(kind=8), dimension(:, :), intent(in) Z, complex(kind=16), dimension(:, :), intent(out) Tz, complex(kind=16), dimension(:, :), intent(out) Zz ) [private]

Definition at line 339 of file sqrt\_mod.f90.

```

339      REAL(KIND=8), DIMENSION(:,:), INTENT(IN) :: t,z
340      COMPLEX(KIND=16), DIMENSION(:,:), INTENT(OUT) :: tz,zz
341      INTEGER, PARAMETER :: nb=2
342      COMPLEX(KIND=16), DIMENSION(nb) :: w
343      !COMPLEX(KIND=16), DIMENSION(nb,nb) :: vl,vr
344      COMPLEX(KIND=16) :: r,c,s,mu
345      COMPLEX(KIND=16), DIMENSION(nb,nb) :: g,gc
346      INTEGER :: n,m!,info
347      !REAL(KIND=8), DIMENSION(2*nb) :: rwork
348      !REAL(KIND=8), DIMENSION(2*nb) :: ztwork
349
350      ! print*, lwork
351      tz=cmplx(t,kind=16)
352      zz=cmplx(z,kind=16)
353      n=SIZE(t,1)
354      DO m=n,2,-1
355          IF (abs(tz(m,m-1)) > real_eps*(abs(tz(m-1,m-1)) + abs(tz(m,m)))) THEN
356              g=tz(m-1:m,m-1:m)
357              ! CALL printmat(dble(G))
358              ! CALL zgeev('N','N',nb,G,nb,w,vl,vr,nb,ztwork,2*nb,rwork,info)
359              ! CALL cprintmat(G)
360              ! print*, m,w,info
361              s=g(1,1)+g(2,2)
362              c=g(1,1)*g(2,2)-g(1,2)*g(2,1)
363              w(1)=s/2+sqrt(s**2/4-c)
364              mu=w(1)-tz(m,m)
365              r=sqrt(mu*conjg(mu)+tz(m,m-1)*conjg(tz(m,m-1)))
366              c=mu/r
367              s=tz(m,m-1)/r
368              g(1,1)=conjg(c)
369              g(1,2)=s
370              g(2,1)=-s
371              g(2,2)=c
372              gc=conjg(transpose(g))
373              tz(m-1:m,m-1:n)=matmul(g,tz(m-1:m,m-1:n))
374              tz(1:m,m-1:m)=matmul(tz(1:m,m-1:m),gc)
375              zz(:,m-1:m)=matmul(zz(:,m-1:m),gc)
376      END IF
377      tz(m,m-1)=cmplx(0.d0,kind=16)
378  END DO

```

#### 8.20.2.5 logical function sqrt\_mod::selectev ( real(kind=8) a, real(kind=8) b ) [private]

Definition at line 122 of file sqrt\_mod.f90.

```

122      REAL(KIND=8) :: a,b
123      LOGICAL selectev
124      selectev=.false.
125      ! IF (a>b) selectev=.true.
126      RETURN

```

#### 8.20.2.6 subroutine, public sqrt\_mod::sqrtm ( real(kind=8), dimension(:,:), intent(in) A, real(kind=8), dimension(:,:), intent(out) sqA, integer, intent(out) info, integer, intent(out) info\_triu, integer, intent(in), optional bs )

Routine to compute a real square-root of a matrix.

##### Parameters

<i>A</i>	Matrix whose square root to evaluate.
<i>sqA</i>	Square root of <i>A</i> .
<i>info</i>	Information code returned by the Lapack routines.
<i>info_triu</i>	Information code returned by the triangular matrix Lapack routines.
<i>bs</i>	Optional blocksize specification variable.

Definition at line 63 of file sqrt\_mod.f90.

```

63      REAL(KIND=8), DIMENSION(:,:), INTENT(IN) :: a
64      REAL(KIND=8), DIMENSION(:,:), INTENT(OUT) :: sqa
65      INTEGER, INTENT(IN), OPTIONAL :: bs
66      INTEGER, INTENT(OUT) :: info,info_triu
67      REAL(KIND=8), DIMENSION(SIZE(A,1),SIZE(A,1)) :: t,z,r
68      COMPLEX(KIND=16), DIMENSION(SIZE(A,1),SIZE(A,1)) :: tz,zz,rz
69      REAL(KIND=8), DIMENSION(SIZE(A,1)) :: wr,wi
70      LOGICAL, DIMENSION(SIZE(A,1)) :: bwork
71      LOGICAL :: selectev
72      INTEGER :: n
73      INTEGER :: sdim=0
74      n=SIZE(a,1)
75      t=a
76      ! print*, n, size(work,1)
77      CALL dgees('V','N',selectev,n,t,n,sdim,wr,wi,z,n,work,lwork,bwork,info)
78      ! print*, 'Z'
79      ! CALL printmat(Z)
80      ! print*, 'T'
81      ! CALL printmat(T)
82      ! CALL DGEES('V','N',SIZE(T,1),T,SIZE(T,1),0,wr,wi,Z,SIZE(Z,1),work,lwork,info)
83      ! print*, info
84      CALL triu(t,r)
85      IF (any(t /= r)) THEN
86          ! print*, 'T'
87          ! CALL printmat(T)
88          ! print*, 'Z'
89          ! CALL printmat(Z)
90          CALL rsf2csf(t,z,tz,zz)
91          ! print*, 'Tz'
92          ! CALL printmat(dble(Tz))
93          ! print*, 'iTz'
94          ! CALL printmat(dble(aimag(Tz)))
95          ! print*, 'Zz'
96          ! CALL printmat(dble(Zz))
97          ! print*, 'iZz'
98          ! CALL printmat(dble(aimag(Zz)))
99      IF (PRESENT(bs)) THEN
100          CALL csqrmtm_triu(tz,rz,info_triu,bs)
101      ELSE
102          CALL csqrmtm_triu(tz,rz,info_triu)
103      END IF
104      rz=matmul(zz,matmul(rz,conjg(transpose(z))))
105      ! print*, 'sqAz'
106      ! CALL printmat(dble(Rz))
107      ! print*, 'isqAz'
108      ! CALL printmat(dble(aimag(Rz)))
109      sqa=dble(rz)
110  ELSE
111      IF (PRESENT(bs)) THEN
112          CALL sqrmtm_triu(t,r,info_triu,bs)
113      ELSE
114          CALL sqrmtm_triu(t,r,info_triu)
115      END IF
116      sqa=matmul(z,matmul(r,transpose(z)))
117  ENDIF
118

```

### 8.20.2.7 subroutine, public sqrt\_mod::sqrmtm\_svd ( real(kind=8), dimension(:, :), intent(in) A, real(kind=8), dimension(:, :), intent(out) sqA, integer, intent(out) info, integer, intent(out) info\_triu, integer, intent(in), optional bs )

Routine to compute a real square-root of a matrix via a SVD decomposition.

#### Parameters

<i>A</i>	Matrix whose square root to evaluate.
<i>sqA</i>	Square root of <i>A</i> .
<i>info</i>	Information code returned by the Lapack routines.
<i>info_triu</i>	Not used (present for compatibility).
<i>bs</i>	Not used (present for compatibility).

Definition at line 401 of file sqrt\_mod.f90.

```

401    REAL(KIND=8), DIMENSION(:,:), INTENT(IN) :: a
402    REAL(KIND=8), DIMENSION(:,:), INTENT(OUT) :: sqa
403    INTEGER, INTENT(IN), OPTIONAL :: bs
404    INTEGER, INTENT(OUT) :: info,info_triu
405    REAL(KIND=8), DIMENSION(SIZE(A,1)) :: s
406    REAL(KIND=8), DIMENSION(SIZE(A,1),SIZE(A,1)) :: sq,u,vt
407    INTEGER :: i,n
408
409    sqa=a
410    n=SIZE(sqa,1)
411    CALL dgesvd('A','A',n,n,sqa,n,s,u,n,vt,n,work,lwork,info)
412    sq=0.d0
413    DO i=1,n
414      sq(i,i)=sqrt(s(i))
415    ENDDO
416    sqa=matmul(u,matmul(sq,vt))

```

### 8.20.2.8 subroutine sqrt\_mod::sqrtm\_triu ( real(kind=8), dimension(:,:), intent(in) A, real(kind=8), dimension(:,:), intent(out) sqA, integer, intent(out) info, integer, intent(in), optional bs ) [private]

Definition at line 131 of file sqrt\_mod.f90.

```

131    REAL(KIND=8), DIMENSION(:,:), INTENT(IN) :: a
132    INTEGER, INTENT(IN), OPTIONAL :: bs
133    REAL(KIND=8), DIMENSION(:,:), INTENT(OUT) :: sqa
134    INTEGER, INTENT(OUT) :: info
135    REAL(KIND=8), DIMENSION(SIZE(A,1)) :: a_diag
136    REAL(KIND=8), DIMENSION(SIZE(A,1),SIZE(A,1)) :: r,sm,rii,rjj
137    INTEGER, DIMENSION(2*SIZE(A,1),2) :: start_stop_pairs
138    REAL(KIND=8) :: s,denom, scale
139    INTEGER :: i,j,k,start,nstop,m
140    INTEGER :: istart,istop,jstart,jstop
141    INTEGER :: nblocks,blocksize
142    INTEGER :: bsmall,blarge,nlarge,nsmall
143
144    blocksize=64
145    IF (PRESENT(bs)) blocksize=bs
146    n=SIZE(a,1)
147    ! print*, blocksize
148
149    CALL diag(a,a_diag)
150    r=0.d0
151    DO i=1,n
152      r(i,i)=sqrt(a_diag(i))
153    ENDDO
154
155
156    nblocks=max(floordiv(n,blocksize),1)
157    bsmall=floordiv(n,nblocks)
158    nlarge=mod(n,nblocks)
159    blarge=bsmall+1
160    nsmall=nblocks-nlarge
161    IF (nsmall*bsmall + nlarge*blarge /= n) stop 'Sqrtm: Internal inconsistency'
162
163    ! print*, nblocks,bsmall,nsmall,blarge,nlarge
164
165    start=1
166    DO i=1,nsmall
167      start_stop_pairs(i,1)=start
168      start_stop_pairs(i,2)=start+bsmall-1
169      start=start+bsmall
170    ENDDO
171    DO i=nsmall+1,nsmall+nlarge
172      start_stop_pairs(i,1)=start
173      start_stop_pairs(i,2)=start+blarge-1
174      start=start+blarge
175    ENDDO
176
177    ! DO i=1,SIZE(start_stop_pairs,1)
178    !   print*, i
179    !   print*, start_stop_pairs(i,1),start_stop_pairs(i,2)
180    ! END DO
181
182    DO k=1,nsmall+nlarge
183      start=start_stop_pairs(k,1)
184      sstop=start_stop_pairs(k,2)
185      DO j=start,sstop
186        DO i=j-1,start,-1
187          s=0.d0

```

```

188      IF (j-i>1) s= dot_product(r(i,i+1:j-1),r(i+1:j-1,j))
189      denom= r(i,i)+r(j,j)
190      IF (denom==0.d0) stop 'Sqrtn: Failed to find the matrix square root'
191      r(i,j)=(a(i,j)-s)/denom
192      END DO
193   END DO
194 END DO
195
196 ! print*, 'R'
197 ! CALL printmat(R)
198
199 DO j=1,nblocks
200   jstart=start_stop_pairs(j,1)
201   jstop=start_stop_pairs(j,2)
202   DO i=j-1,1,-1
203     istart=start_stop_pairs(i,1)
204     istop=start_stop_pairs(i,2)
205     sm=0.d0
206     sm(istart:istop,jstart:jstop)=a(istart:istop,jstart:jstop)
207     IF (j-i>1) sm(istart:istop,jstart:jstop) = sm(istart:istop&
208           &,jstart:jstop) - matmul(r(istart:istop,istop:jstart)&
209           &,r(istop:jstart,jstart:jstop))
210     rii=0.d0
211     rii = r(istart:istop, istart:istop)
212     rjj=0.d0
213     rjj = r(jstart:jstop, jstart:jstop)
214     m=istop-istart+1
215     n=jstop-jstart+1
216     k=1
217     ! print*, m,n
218     ! print*, istart,istop
219     ! print*, jstart,jstop
220
221     ! print*, 'Rii',Rii(istart:istop, istart:istop)
222     ! print*, 'Rjj',Rjj(jstart:jstop, jstart:jstop)
223     ! print*, 'Sm',Sm(istart:istop,jstart:jstop)
224
225     CALL dtrsyl('N','N',k,m,n,rii(istart:istop, istart:istop),m&
226           &,rjj(jstart:jstop, jstart:jstop),n,sm(istart:istop&
227           &,jstart:jstop),m,scale,info)
228     r(istart:istop,jstart:jstop)=sm(istart:istop,jstart:jstop)*scale
229   ENDDO
230 ENDDO
231 sqa=r

```

### 8.20.3 Variable Documentation

#### 8.20.3.1 integer sqrt\_mod::lwork [private]

Definition at line 30 of file sqrt\_mod.f90.

```
30  INTEGER :: lwork
```

#### 8.20.3.2 real(kind=8), parameter sqrt\_mod::real\_eps = 2.2204460492503131e-16 [private]

Definition at line 32 of file sqrt\_mod.f90.

```
32  REAL(KIND=8), PARAMETER :: real_eps = 2.2204460492503131e-16
```

#### 8.20.3.3 real(kind=8), dimension(:), allocatable sqrt\_mod::work [private]

Definition at line 27 of file sqrt\_mod.f90.

```
27  REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: work
```

## 8.21 stat Module Reference

Statistics accumulators.

### Functions/Subroutines

- subroutine, public `init_stat`  
*Initialise the accumulators.*
- subroutine, public `acc` (`x`)  
*Accumulate one state.*
- real(kind=8) function, dimension(0:ndim), public `mean` ()  
*Function returning the mean.*
- real(kind=8) function, dimension(0:ndim), public `var` ()  
*Function returning the variance.*
- integer function, public `iter` ()  
*Function returning the number of data accumulated.*
- subroutine, public `reset`  
*Routine resetting the accumulators.*

### Variables

- integer `i` =0  
*Number of stats accumulated.*
- real(kind=8), dimension(:), allocatable `m`  
*Vector storing the inline mean.*
- real(kind=8), dimension(:), allocatable `mprev`  
*Previous mean vector.*
- real(kind=8), dimension(:), allocatable `v`  
*Vector storing the inline variance.*
- real(kind=8), dimension(:), allocatable `mtmp`

#### 8.21.1 Detailed Description

Statistics accumulators.

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#### 8.21.2 Function/Subroutine Documentation

##### 8.21.2.1 subroutine, public `stat::acc` ( `real(kind=8), dimension(0:ndim), intent(in) x` )

Accumulate one state.

Definition at line 48 of file `stat.f90`.

```

48      IMPLICIT NONE
49      REAL(KIND=8), DIMENSION(0:ndim), INTENT(IN) :: x
50      i=i+1
51      mprev=m+(x-m)/i
52      mtmp=mprev
53      mprev=m
54      m=mtmp
55      v=v+(x-mprev)*(x-m)

```

### 8.21.2.2 subroutine, public stat::init\_stat( )

Initialise the accumulators.

Definition at line 35 of file stat.f90.

```

35      INTEGER :: allocstat
36
37      ALLOCATE(m(0:ndim),mprev(0:ndim),v(0:ndim),mtmp(0:ndim),
38      stat=allocstat)
39      IF (allocstat /= 0) stop '*** Not enough memory ***'
40      m=0.d0
41      mprev=0.d0
42      v=0.d0
43      mtmp=0.d0

```

### 8.21.2.3 integer function, public stat::iter( )

Function returning the number of data accumulated.

Definition at line 72 of file stat.f90.

```

72      INTEGER :: iter
73      iter=i

```

### 8.21.2.4 real(kind=8) function, dimension(0:ndim), public stat::mean( )

Function returning the mean.

Definition at line 60 of file stat.f90.

```

60      REAL(KIND=8), DIMENSION(0:ndim) :: mean
61      mean=m

```

### 8.21.2.5 subroutine, public stat::reset( )

Routine resetting the accumulators.

Definition at line 78 of file stat.f90.

```

78      m=0.d0
79      mprev=0.d0
80      v=0.d0
81      i=0

```

### 8.21.2.6 real(kind=8) function, dimension(0:ndim), public stat::var( )

Function returning the variance.

Definition at line 66 of file stat.f90.

```

66      REAL(KIND=8), DIMENSION(0:ndim) :: var
67      var=v/(i-1)

```

### 8.21.3 Variable Documentation

#### 8.21.3.1 integer stat::i =0 [private]

Number of stats accumulated.

Definition at line 20 of file stat.f90.

```
20    INTEGER :: i=0 !< Number of stats accumulated
```

#### 8.21.3.2 real(kind=8), dimension(:), allocatable stat::m [private]

Vector storing the inline mean.

Definition at line 23 of file stat.f90.

```
23    REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: m           !< Vector storing the inline mean
```

#### 8.21.3.3 real(kind=8), dimension(:), allocatable stat::mprev [private]

Previous mean vector.

Definition at line 24 of file stat.f90.

```
24    REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: mprev      !< Previous mean vector
```

#### 8.21.3.4 real(kind=8), dimension(:), allocatable stat::mtmp [private]

Definition at line 26 of file stat.f90.

```
26    REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: mttmp
```

#### 8.21.3.5 real(kind=8), dimension(:), allocatable stat::v [private]

Vector storing the inline variance.

Definition at line 25 of file stat.f90.

```
25    REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: v           !< Vector storing the inline variance
```

## 8.22 stoch\_mod Module Reference

Utility module containing the stochastic related routines.

## Functions/Subroutines

- real(kind=8) function, public **gasdev** ()
  - subroutine, public **stoch\_vec** (dW)
 

*Routine to fill a vector with standard Gaussian noise process values.*
  - subroutine, public **stoch\_atm\_vec** (dW)
 

*routine to fill the atmospheric component of a vector with standard gaussian noise process values*
  - subroutine, public **stoch\_atm\_res\_vec** (dW)
 

*routine to fill the resolved atmospheric component of a vector with standard gaussian noise process values*
  - subroutine, public **stoch\_atm\_unres\_vec** (dW)
 

*routine to fill the unresolved atmospheric component of a vector with standard gaussian noise process values*
  - subroutine, public **stoch\_oc\_vec** (dW)
 

*routine to fill the oceanic component of a vector with standard gaussian noise process values*
  - subroutine, public **stoch\_oc\_res\_vec** (dW)
 

*routine to fill the resolved oceanic component of a vector with standard gaussian noise process values*
  - subroutine, public **stoch\_oc\_unres\_vec** (dW)
 

*routine to fill the unresolved oceanic component of a vector with standard gaussian noise process values*

## Variables

- integer **iset** =0
- real(kind=8) **gset**

### 8.22.1 Detailed Description

Utility module containing the stochastic related routines.

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#### Remarks

### 8.22.2 Function/Subroutine Documentation

#### 8.22.2.1 real(kind=8) function, public stoch\_mod::gasdev ( )

Definition at line 32 of file stoch\_mod.f90.

```

32      REAL(KIND=8) :: gasdev
33      REAL(KIND=8) :: fac,rsq,v1,v2,r
34      IF (iset.eq.0) THEN
35          DO
36              CALL random_number(r)
37              v1=2.d0*r-1.
38              CALL random_number(r)
39              v2=2.d0*r-1.
40              rsq=v1**2+v2**2
41              IF (rsq.lt.1.d0.and.rsq.ne.0.d0) EXIT
42          ENDDO
43          fac=sqrt(-2.*log(rsq)/rsq)
44          gset=v1*fac
45          gasdev=v2*fac
46          iset=1
47      ELSE
48          gasdev=gset
49          iset=0
50      ENDIF
51      RETURN

```

### 8.22.2.2 subroutine, public stoch\_mod::stoch\_atm\_res\_vec ( real(kind=8), dimension(0:ndim), intent(inout) dW )

routine to fill the resolved atmospheric component of a vector with standard gaussian noise process values

#### Parameters

<i>dW</i>	vector to fill
-----------	----------------

Definition at line 77 of file stoch\_mod.f90.

```
77      real(kind=8), dimension(0:ndim), intent(inout) :: dw
78      integer :: i
79      dw=0.d0
80      do i=1,2*natm
81        IF (sf(i)==0) dw(i)=gasdev()
82      enddo
```

### 8.22.2.3 subroutine, public stoch\_mod::stoch\_atm\_unres\_vec ( real(kind=8), dimension(0:ndim), intent(inout) dW )

routine to fill the unresolved atmospheric component of a vector with standard gaussian noise process values

#### Parameters

<i>dW</i>	vector to fill
-----------	----------------

Definition at line 88 of file stoch\_mod.f90.

```
88      real(kind=8), dimension(0:ndim), intent(inout) :: dw
89      integer :: i
90      dw=0.d0
91      do i=1,2*natm
92        IF (sf(i)==1) dw(i)=gasdev()
93      enddo
```

### 8.22.2.4 subroutine, public stoch\_mod::stoch\_atm\_vec ( real(kind=8), dimension(0:ndim), intent(inout) dW )

routine to fill the atmospheric component of a vector with standard gaussian noise process values

#### Parameters

<i>dW</i>	vector to fill
-----------	----------------

Definition at line 67 of file stoch\_mod.f90.

```
67      real(kind=8), dimension(0:ndim), intent(inout) :: dw
68      integer :: i
69      do i=1,2*natm
70        dw(i)=gasdev()
71      enddo
```

### 8.22.2.5 subroutine, public stoch\_mod::stoch\_oc\_res\_vec ( real(kind=8), dimension(0:ndim), intent(inout) dW )

routine to fill the resolved oceanic component of a vector with standard gaussian noise process values

#### Parameters

<i>dW</i>	vector to fill
-----------	----------------

Definition at line 109 of file stoch\_mod.f90.

```
109      real(kind=8), dimension(0:ndim), intent(inout) :: dw
110      integer :: i
111      dw=0.d0
112      do i=2*natm+1,ndim
113         IF (sf(i)==0) dw(i)=gasdev()
114      enddo
```

### 8.22.2.6 subroutine, public stoch\_mod::stoch\_oc\_unres\_vec ( real(kind=8), dimension(0:ndim), intent(inout) dW )

routine to fill the unresolved oceanic component of a vector with standard gaussian noise process values

#### Parameters

<i>dW</i>	vector to fill
-----------	----------------

Definition at line 120 of file stoch\_mod.f90.

```
120      real(kind=8), dimension(0:ndim), intent(inout) :: dw
121      integer :: i
122      dw=0.d0
123      do i=2*natm+1,ndim
124         IF (sf(i)==1) dw(i)=gasdev()
125      enddo
```

### 8.22.2.7 subroutine, public stoch\_mod::stoch\_oc\_vec ( real(kind=8), dimension(0:ndim), intent(inout) dW )

routine to fill the oceanic component of a vector with standard gaussian noise process values

#### Parameters

<i>dW</i>	vector to fill
-----------	----------------

Definition at line 99 of file stoch\_mod.f90.

```
99      real(kind=8), dimension(0:ndim), intent(inout) :: dw
100      integer :: i
101      do i=2*natm+1,ndim
102         dw(i)=gasdev()
103      enddo
```

### 8.22.2.8 subroutine, public stoch\_mod::stoch\_vec ( real(kind=8), dimension(0:ndim), intent(inout) dW )

Routine to fill a vector with standard Gaussian noise process values.

#### Parameters

<i>dW</i>	Vector to fill
-----------	----------------

Definition at line 57 of file stoch\_mod.f90.

```
57      REAL(KIND=8), DIMENSION(0:ndim), INTENT(INOUT) :: dw
58      INTEGER :: i
59      DO i=1,ndim
60          dw(i)=gasdev()
61      ENDDO
```

### 8.22.3 Variable Documentation

#### 8.22.3.1 real(kind=8) stoch\_mod::gset [private]

Definition at line 24 of file stoch\_mod.f90.

```
24      REAL(KIND=8) :: gset
```

#### 8.22.3.2 integer stoch\_mod::iset =0 [private]

Definition at line 23 of file stoch\_mod.f90.

```
23      INTEGER :: iset=0
```

## 8.23 stoch\_params Module Reference

The stochastic models parameters module.

### Functions/Subroutines

- subroutine [init\\_stoch\\_params](#)

*Stochastic parameters initialization routine.*

## Variables

- real(kind=8) **mnuti**  
*Multiplicative noise update time interval.*
- real(kind=8) **t\_trans\_stoch**  
*Transient time period of the stochastic model evolution.*
- real(kind=8) **q\_ar**  
*Atmospheric resolved component noise amplitude.*
- real(kind=8) **q\_au**  
*Atmospheric unresolved component noise amplitude.*
- real(kind=8) **q\_or**  
*Oceanic resolved component noise amplitude.*
- real(kind=8) **q\_ou**  
*Oceanic unresolved component noise amplitude.*
- real(kind=8) **dtn**  
*Square root of the timestep.*
- real(kind=8) **eps\_pert**  
*Perturbation parameter for the coupling.*
- real(kind=8) **tdelta**  
*Time separation parameter.*
- real(kind=8) **muti**  
*Memory update time interval.*
- real(kind=8) **meml**  
*Time over which the memory kernel is integrated.*
- real(kind=8) **t\_trans\_mem**  
*Transient time period to initialize the memory term.*
- character(len=4) **x\_int\_mode**  
*Integration mode for the resolved component.*
- real(kind=8) **dts**  
*Intrinsic resolved dynamics time step.*
- integer **mems**  
*Number of steps in the memory kernel integral.*
- real(kind=8) **dtsn**  
*Square root of the intrinsic resolved dynamics time step.*
- real(kind=8) **maxint**  
*Upper integration limit of the correlations.*
- character(len=4) **load\_mode**  
*Loading mode for the correlations.*
- character(len=4) **int\_corr\_mode**  
*Correlation integration mode.*
- character(len=4) **mode**  
*Stochastic mode parameter.*

### 8.23.1 Detailed Description

The stochastic models parameters module.

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#### Remarks

## 8.23.2 Function/Subroutine Documentation

### 8.23.2.1 subroutine stoch\_params::init\_stoch\_params( )

Stochastic parameters initialization routine.

Definition at line 58 of file stoch\_params.f90.

```

58      namelist /mtvparams/ mnut
59      namelist /stparams/ q_ar,q_au,q_or,q_ou,eps_pert,tdelta,t_trans_stoch
60      namelist /wlparams/ muti,meml,x_int_mode,dts,t_trans_mem
61      namelist /corr_init_mode/ load_mode,int_corr_mode,maxint
62      namelist /stoch_int_params/ mode
63
64
65
66      OPEN(8, file="stoch_params.nml", status='OLD', recl=80, delim='APOSTROPHE')
67      READ(8,nml=mtvparams)
68      READ(8,nml=wlparams)
69      READ(8,nml=stparams)
70      READ(8,nml=stoch_int_params)
71      READ(8,nml=corr_init_mode)
72      CLOSE(8)
73
74      dtm=sqrt(dt)
75      dtsn=sqrt(dts)
76      mems=ceiling(meml/muti)
77
78      q_au=q_au/tdelta
79      q_ou=q_ou/tdelta
80

```

## 8.23.3 Variable Documentation

### 8.23.3.1 real(kind=8) stoch\_params::dtn

Square root of the timestep.

Definition at line 32 of file stoch\_params.f90.

```
32      REAL(KIND=8) :: dtn           !< Square root of the timestep
```

### 8.23.3.2 real(kind=8) stoch\_params::dts

Intrinsic resolved dynamics time step.

Definition at line 40 of file stoch\_params.f90.

```
40      REAL(KIND=8) :: dts           !< Intrinsic resolved dynamics time step
```

### 8.23.3.3 real(kind=8) stoch\_params::dtsn

Square root of the intrinsic resolved dynamics time step.

Definition at line 43 of file stoch\_params.f90.

```
43      REAL(KIND=8) :: dtsn          !< Square root of the intrinsic resolved dynamics time step
```

**8.23.3.4 real(kind=8) stoch\_params::eps\_pert**

Perturbation parameter for the coupling.

Definition at line 33 of file stoch\_params.f90.

```
33    REAL(KIND=8) :: eps_pert           !< Perturbation parameter for the coupling
```

**8.23.3.5 character(len=4) stoch\_params::int\_corr\_mode**

Correlation integration mode.

Definition at line 47 of file stoch\_params.f90.

```
47    CHARACTER(LEN=4) :: int_corr_mode !< Correlation integration mode
```

**8.23.3.6 character(len=4) stoch\_params::load\_mode**

Loading mode for the correlations.

Definition at line 46 of file stoch\_params.f90.

```
46    CHARACTER(LEN=4) :: load_mode      !< Loading mode for the correlations
```

**8.23.3.7 real(kind=8) stoch\_params::maxint**

Upper integration limit of the correlations.

Definition at line 45 of file stoch\_params.f90.

```
45    REAL(KIND=8) :: maxint           !< Upper integration limit of the correlations
```

**8.23.3.8 real(kind=8) stoch\_params::mem1**

Time over which the memory kernel is integrated.

Definition at line 37 of file stoch\_params.f90.

```
37    REAL(KIND=8) :: mem1             !< Time over which the memory kernel is integrated
```

### 8.23.3.9 integer stoch\_params::mems

Number of steps in the memory kernel integral.

Definition at line 42 of file stoch\_params.f90.

```
42    INTEGER :: mems                      !< Number of steps in the memory kernel integral
```

### 8.23.3.10 real(kind=8) stoch\_params::mnuti

Multiplicative noise update time interval.

Definition at line 25 of file stoch\_params.f90.

```
25    REAL(KIND=8) :: mnuti                !< Multiplicative noise update time interval
```

### 8.23.3.11 character(len=4) stoch\_params::mode

Stochastic mode parameter.

Definition at line 49 of file stoch\_params.f90.

```
49    CHARACTER(len=4) :: mode              !< Stochastic mode parameter
```

### 8.23.3.12 real(kind=8) stoch\_params::muti

Memory update time interval.

Definition at line 36 of file stoch\_params.f90.

```
36    REAL(KIND=8) :: muti                !< Memory update time interval
```

### 8.23.3.13 real(kind=8) stoch\_params::q\_ar

Atmospheric resolved component noise amplitude.

Definition at line 28 of file stoch\_params.f90.

```
28    REAL(KIND=8) :: q_ar                !< Atmospheric resolved component noise amplitude
```

**8.23.3.14 real(kind=8) stoch\_params::q\_au**

Atmospheric unresolved component noise amplitude.

Definition at line 29 of file stoch\_params.f90.

```
29    REAL(KIND=8) :: q_au           !< Atmospheric unresolved component noise amplitude
```

**8.23.3.15 real(kind=8) stoch\_params::q\_or**

Oceanic resolved component noise amplitude.

Definition at line 30 of file stoch\_params.f90.

```
30    REAL(KIND=8) :: q_or           !< Oceanic resolved component noise amplitude
```

**8.23.3.16 real(kind=8) stoch\_params::q\_ou**

Oceanic unresolved component noise amplitude.

Definition at line 31 of file stoch\_params.f90.

```
31    REAL(KIND=8) :: q_ou           !< Oceanic unresolved component noise amplitude
```

**8.23.3.17 real(kind=8) stoch\_params::t\_trans\_mem**

Transient time period to initialize the memory term.

Definition at line 38 of file stoch\_params.f90.

```
38    REAL(KIND=8) :: t_trans_mem      !< Transient time period to initialize the memory term
```

**8.23.3.18 real(kind=8) stoch\_params::t\_trans\_stoch**

Transient time period of the stochastic model evolution.

Definition at line 27 of file stoch\_params.f90.

```
27    REAL(KIND=8) :: t_trans_stoch     !< Transient time period of the stochastic model evolution
```

### 8.23.3.19 real(kind=8) stoch\_params::tdelta

Time separation parameter.

Definition at line 34 of file stoch\_params.f90.

```
34    REAL(KIND=8) :: tdelta           !< Time separation parameter
```

### 8.23.3.20 character(len=4) stoch\_params::x\_int\_mode

Integration mode for the resolved component.

Definition at line 39 of file stoch\_params.f90.

```
39    CHARACTER(len=4) :: x_int_mode      !< Integration mode for the resolved component
```

## 8.24 tensor Module Reference

Tensor utility module.

### Data Types

- type [coolist](#)  
*Coordinate list. Type used to represent the sparse tensor.*
- type [coolist4](#)  
*4d coordinate list. Type used to represent the rank-4 sparse tensor.*
- type [coolist\\_elem](#)  
*Coordinate list element type. Elementary elements of the sparse tensors.*
- type [coolist\\_elem4](#)  
*4d coordinate list element type. Elementary elements of the 4d sparse tensors.*

### Functions/Subroutines

- subroutine, public [copy\\_coo](#) (src, dst)  
*Routine to copy a coolist.*
- subroutine, public [mat\\_to\\_coo](#) (src, dst)  
*Routine to convert a matrix to a tensor.*
- subroutine, public [sparse\\_mul3](#) (coolist\_ijk, arr\_j, arr\_k, res)  
*Sparse multiplication of a tensor with two vectors:  $\sum_{j,k=0}^{ndim} T_{i,j,k} a_j b_k$ .*
- subroutine, public [jsparse\\_mul](#) (coolist\_ijk, arr\_j, jcoo\_ij)

*Sparse multiplication of two tensors to determine the Jacobian:*

$$J_{i,j} = \sum_{k=0}^{ndim} (\mathcal{T}_{i,j,k} + \mathcal{T}_{i,k,j}) a_k.$$

*It's implemented slightly differently: for every  $\mathcal{T}_{i,j,k}$ , we add to  $J_{i,j}$  as follows:*

$$J_{i,j} = J_{i,j} + \mathcal{T}_{i,j,k} a_k J_{i,k} = J_{i,k} + \mathcal{T}_{i,j,k} a_j$$

*This version return a coolist (sparse tensor).*

- subroutine, public **jsparse\_mul\_mat** (coolist\_ijk, arr\_j, jcoo\_ij)

*Sparse multiplication of two tensors to determine the Jacobian:*

$$J_{i,j} = \sum_{k=0}^{ndim} (\mathcal{T}_{i,j,k} + \mathcal{T}_{i,k,j}) a_k.$$

*It's implemented slightly differently: for every  $\mathcal{T}_{i,j,k}$ , we add to  $J_{i,j}$  as follows:*

$$J_{i,j} = J_{i,j} + \mathcal{T}_{i,j,k} a_k J_{i,k} = J_{i,k} + \mathcal{T}_{i,j,k} a_j$$

*This version return a matrix.*

- subroutine, public **sparse\_mul2** (coolist\_ij, arr\_j, res)

*Sparse multiplication of a 2d sparse tensor with a vector:  $\sum_{j=0}^{ndim} \mathcal{T}_{i,j,k} a_j$ .*

- subroutine, public **simplify** (tensor)

*Routine to simplify a coolist (sparse tensor). For each index  $i$ , it upper triangularize the matrix*

$$\mathcal{T}_{i,j,k} \quad 0 \leq j, k \leq ndim.$$

- subroutine, public **add\_elem** (t, i, j, k, v)

*Subroutine to add element to a coolist.*

- subroutine, public **add\_check** (t, i, j, k, v, dst)

*Subroutine to add element to a coolist and check for overflow. Once the t buffer tensor is full, add it to the destination buffer.*

- subroutine, public **add\_to\_tensor** (src, dst)

*Routine to add a rank-3 tensor to another one.*

- subroutine, public **print\_tensor** (t, s)

*Routine to print a rank 3 tensor coolist.*

- subroutine, public **write\_tensor\_to\_file** (s, t)

*Load a rank-4 tensor coolist from a file definition.*

- subroutine, public **load\_tensor\_from\_file** (s, t)

*Load a rank-4 tensor coolist from a file definition.*

- subroutine, public **add\_matc\_to\_tensor** (i, src, dst)

*Routine to add a matrix to a rank-3 tensor.*

- subroutine, public **add\_matc\_to\_tensor4** (i, j, src, dst)

*Routine to add a matrix to a rank-4 tensor.*

- subroutine, public **add\_vec\_jk\_to\_tensor** (j, k, src, dst)

*Routine to add a vector to a rank-3 tensor.*

- subroutine, public **add\_vec\_ikl\_to\_tensor4\_perm** (i, k, l, src, dst)

*Routine to add a vector to a rank-4 tensor plus permutation.*

- subroutine, public **add\_vec\_ikl\_to\_tensor4** (i, k, l, src, dst)

*Routine to add a vector to a rank-4 tensor.*

- subroutine, public **add\_vec\_ijk\_to\_tensor4** (i, j, k, src, dst)

*Routine to add a vector to a rank-4 tensor.*

- subroutine, public **tensor\_to\_coo** (src, dst)

*Routine to convert a rank-3 tensor from matrix to coolist representation.*

- subroutine, public `tensor4_to_coo4` (`src, dst`)
 

*Routine to convert a rank-4 tensor from matrix to coolist representation.*
- subroutine, public `print_tensor4` (`t`)
 

*Routine to print a rank-4 tensor coolist.*
- subroutine, public `sparse_mul3_mat` (`coolist_ijk, arr_k, res`)
 

*Sparse multiplication of a rank-3 tensor coolist with a vector:  $\sum_{k=0}^{ndim} T_{i,j,k} b_k$ . Its output is a matrix.*
- subroutine, public `sparse_mul4` (`coolist_ijkl, arr_j, arr_k, arr_l, res`)
 

*Sparse multiplication of a rank-4 tensor coolist with three vectors:  $\sum_{j,k,l=0}^{ndim} T_{i,j,k,l} a_j b_k c_l$ .*
- subroutine, public `sparse_mul4_mat` (`coolist_ijkl, arr_k, arr_l, res`)
 

*Sparse multiplication of a tensor with two vectors:  $\sum_{k,l=0}^{ndim} T_{i,j,k,l} b_k c_l$ .*
- subroutine, public `sparse_mul2_j` (`coolist_ijk, arr_j, res`)
 

*Sparse multiplication of a 3d sparse tensor with a vectors:  $\sum_{j=0}^{ndim} T_{i,j,k} a_j$ .*
- subroutine, public `sparse_mul2_k` (`coolist_ijk, arr_k, res`)
 

*Sparse multiplication of a rank-3 sparse tensor coolist with a vector:  $\sum_{k=0}^{ndim} T_{i,j,k} a_k$ .*
- subroutine, public `coo_to_mat_ik` (`src, dst`)
 

*Routine to convert a rank-3 tensor coolist component into a matrix with i and k indices.*
- subroutine, public `coo_to_mat_jj` (`src, dst`)
 

*Routine to convert a rank-3 tensor coolist component into a matrix with i and j indices.*
- subroutine, public `coo_to_mat_i` (`i, src, dst`)
 

*Routine to convert a rank-3 tensor coolist component into a matrix.*
- subroutine, public `coo_to_vec_jk` (`j, k, src, dst`)
 

*Routine to convert a rank-3 tensor coolist component into a vector.*
- subroutine, public `coo_to_mat_j` (`j, src, dst`)
 

*Routine to convert a rank-3 tensor coolist component into a matrix.*
- subroutine, public `sparse_mul4_with_mat_jl` (`coolist_ijkl, mat_jl, res`)
 

*Sparse multiplication of a rank-4 tensor coolist with a matrix :  $\sum_{j,l=0}^{ndim} T_{i,j,k,l} m_{j,l}$ .*
- subroutine, public `sparse_mul4_with_mat_kl` (`coolist_ijkl, mat_kl, res`)
 

*Sparse multiplication of a rank-4 tensor coolist with a matrix :  $\sum_{j,l=0}^{ndim} T_{i,j,k,l} m_{k,l}$ .*
- subroutine, public `sparse_mul3_with_mat` (`coolist_ijk, mat_jk, res`)
 

*Sparse multiplication of a rank-3 tensor coolist with a matrix:  $\sum_{j,k=0}^{ndim} T_{i,j,k} m_{j,k}$ .*
- subroutine, public `matc_to_coo` (`src, dst`)
 

*Routine to convert a matrix to a rank-3 tensor.*
- subroutine, public `scal_mul_coo` (`s, t`)
 

*Routine to multiply a rank-3 tensor by a scalar.*
- logical function, public `tensor_empty` (`t`)
 

*Test if a rank-3 tensor coolist is empty.*
- logical function, public `tensor4_empty` (`t`)
 

*Test if a rank-4 tensor coolist is empty.*
- subroutine, public `load_tensor4_from_file` (`s, t`)
 

*Load a rank-4 tensor coolist from a file definition.*
- subroutine, public `write_tensor4_to_file` (`s, t`)
 

*Load a rank-4 tensor coolist from a file definition.*

## Variables

- real(kind=8), parameter `real_eps` = 2.2204460492503131e-16  
*Parameter to test the equality with zero.*

### 8.24.1 Detailed Description

Tensor utility module.

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#### Remarks

`coolist` is a type and also means "coordinate list"

### 8.24.2 Function/Subroutine Documentation

#### 8.24.2.1 subroutine, public `tensor::add_check` ( `type(coolist)`, `dimension(ndim)`, `intent(inout) t`, `integer, intent(in) i`, `integer, intent(in) j`, `integer, intent(in) k`, `real(kind=8), intent(in) v`, `type(coolist)`, `dimension(ndim)`, `intent(inout) dst` )

Subroutine to add element to a coolist and check for overflow. Once the `t` buffer tensor is full, add it to the destination buffer.

#### Parameters

<code>t</code>	temporary buffer tensor for the destination tensor
<code>i</code>	tensor <code>i</code> index
<code>j</code>	tensor <code>j</code> index
<code>k</code>	tensor <code>k</code> index
<code>v</code>	value to add
<code>dst</code>	destination tensor

Definition at line 332 of file `tensor.f90`.

```

332   TYPE(coolist), DIMENSION(ndim), INTENT(INOUT) :: t
333   TYPE(coolist), DIMENSION(ndim), INTENT(INOUT) :: dst
334   INTEGER, INTENT(IN) :: i,j,k
335   REAL(KIND=8), INTENT(IN) :: v
336   INTEGER :: n
337   CALL add_elem(t,i,j,k,v)
338   IF (t(i)%nelems==size(t(i)%elems)) THEN
339     CALL add_to_tensor(t,dst)
340     DO n=1,ndim
341       t(n)%nelems=0
342     ENDDO
343   ENDIF

```

#### 8.24.2.2 subroutine, public `tensor::add_elem` ( `type(coolist)`, `dimension(ndim)`, `intent(inout) t`, `integer, intent(in) i`, `integer, intent(in) j`, `integer, intent(in) k`, `real(kind=8), intent(in) v` )

Subroutine to add element to a coolist.

### Parameters

<i>t</i>	destination tensor
<i>i</i>	tensor <i>i</i> index
<i>j</i>	tensor <i>j</i> index
<i>k</i>	tensor <i>k</i> index
<i>v</i>	value to add

Definition at line 310 of file tensor.f90.

```

310      TYPE(coolist), DIMENSION(ndim), INTENT(INOUT) :: t
311      INTEGER, INTENT(IN) :: i,j,k
312      REAL(KIND=8), INTENT(IN) :: v
313      INTEGER :: n
314      IF (abs(v) .ge. real_eps) THEN
315          n=(t(i)%nelems)+1
316          t(i)%elems(n)%j=j
317          t(i)%elems(n)%k=k
318          t(i)%elems(n)%v=v
319          t(i)%nelems=n
320      END IF

```

### 8.24.2.3 subroutine, public tensor::add\_matc\_to\_tensor ( integer, intent(in) *i*, real(kind=8), dimension(ndim,ndim), intent(in) *src*, type(coolist), dimension(ndim), intent(inout) *dst* )

Routine to add a matrix to a rank-3 tensor.

### Parameters

<i>i</i>	Add to tensor component <i>i</i>
<i>src</i>	Matrix to add
<i>dst</i>	Destination tensor

Definition at line 474 of file tensor.f90.

```

474      INTEGER, INTENT(IN) :: i
475      REAL(KIND=8), DIMENSION(ndim,ndim), INTENT(IN) :: src
476      TYPE(coolist), DIMENSION(ndim), INTENT(INOUT) :: dst
477      TYPE(coolist_ele), DIMENSION(:), ALLOCATABLE :: celems
478      INTEGER :: j,k,r,n,nsrc,allocstat
479
480      nsrcc=0
481      DO j=1,ndim
482          DO k=1,ndim
483              IF (abs(src(j,k))>real_eps) nsrcc=nsrcc+1
484          END DO
485      END DO
486
487      IF (dst(i)%nelems==0) THEN
488          IF (ALLOCATED(dst(i)%elems)) THEN
489              DEALLOCATE(dst(i)%elems, stat=allocstat)
490              IF (allocstat /= 0) stop "*** Deallocation problem ! ***"
491          ENDIF
492          ALLOCATE(dst(i)%elems(nsrc), stat=allocstat)
493          IF (allocstat /= 0) stop "*** Not enough memory ! ***"
494          n=0
495      ELSE
496          n=dst(i)%nelems
497          ALLOCATE(celems(n), stat=allocstat)
498          DO j=1,n
499              celems(j)%j=dst(i)%elems(j)%j
500              celems(j)%k=dst(i)%elems(j)%k
501              celems(j)%v=dst(i)%elems(j)%v
502          ENDDO

```

```

503      DEALLOCATE(dst(i)%elems, stat=allocstat)
504      IF (allocstat /= 0) stop "*** Deallocation problem ! ***"
505      ALLOCATE(dst(i)%elems(nsrc+n), stat=allocstat)
506      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
507      DO j=1,n
508          dst(i)%elems(j)%j=celems(j)%j
509          dst(i)%elems(j)%k=celems(j)%k
510          dst(i)%elems(j)%v=celems(j)%v
511      ENDDO
512      DEALLOCATE(celems, stat=allocstat)
513      IF (allocstat /= 0) stop "*** Deallocation problem ! ***"
514  ENDIF
515  r=0
516  DO j=1,ndim
517      DO k=1,ndim
518          IF (abs(src(j,k))>real_eps) THEN
519              r=r+1
520              dst(i)%elems(n+r)%j=j
521              dst(i)%elems(n+r)%k=k
522              dst(i)%elems(n+r)%v=src(j,k)
523          ENDIF
524      ENDDO
525  END DO
526  dst(i)%nelems=nsrc+n
527

```

#### 8.24.2.4 subroutine, public tensor::add\_matc\_to\_tensor4 ( integer, intent(in) i, integer, intent(in) j, real(kind=8), dimension(ndim,ndim), intent(in) src, type(coolist4), dimension(ndim), intent(inout) dst )

Routine to add a matrix to a rank-4 tensor.

##### Parameters

<i>i</i>	Add to tensor component i,j
<i>j</i>	Add to tensor component i,j
<i>src</i>	Matrix to add
<i>dst</i>	Destination tensor

Definition at line 537 of file tensor.f90.

```

537      INTEGER, INTENT(IN) :: i,j
538      REAL(KIND=8), DIMENSION(ndim,ndim), INTENT(IN) :: src
539      TYPE(coolist4), DIMENSION(ndim), INTENT(INOUT) :: dst
540      TYPE(coolst_elem4), DIMENSION(:, :), ALLOCATABLE :: celems
541      INTEGER :: k,l,r,n,nsrc,allocstat
542
543      nsrc=0
544      DO k=1,ndim
545          DO l=1,ndim
546              IF (abs(src(k,l))>real_eps) nsrc=nsrc+1
547          END DO
548      END DO
549
550      IF (dst(i)%nelems==0) THEN
551          IF (ALLOCATED(dst(i)%elems)) THEN
552              DEALLOCATE(dst(i)%elems, stat=allocstat)
553              IF (allocstat /= 0) stop "*** Deallocation problem ! ***"
554          ENDIF
555          ALLOCATE(dst(i)%elems(nsrc), stat=allocstat)
556          IF (allocstat /= 0) stop "*** Not enough memory ! ***"
557          n=0
558      ELSE
559          n=dst(i)%nelems
560          ALLOCATE(celems(n), stat=allocstat)
561          DO k=1,n
562              celems(k)%j=dst(i)%elems(k)%j
563              celems(k)%k=dst(i)%elems(k)%k
564              celems(k)%l=dst(i)%elems(k)%l
565              celems(k)%v=dst(i)%elems(k)%v
566          ENDDO
567          DEALLOCATE(dst(i)%elems, stat=allocstat)
568          IF (allocstat /= 0) stop "*** Deallocation problem ! ***"

```

```

569      ALLOCATE (dst(i)%elems(nsrc+n), stat=allocstat)
570      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
571      DO k=1,n
572        dst(i)%elems(k)%j=celems(k)%j
573        dst(i)%elems(k)%k=celems(k)%k
574        dst(i)%elems(k)%l=celems(k)%l
575        dst(i)%elems(k)%v=celems(k)%v
576      ENDDO
577      DEALLOCATE(celems, stat=allocstat)
578      IF (allocstat /= 0) stop "*** Deallocation problem ! ***"
579    ENDIF
580    r=0
581    DO k=1,ndim
582      DO l=1,ndim
583        IF (abs(src(k,l))>real_eps) THEN
584          r=r+1
585          dst(i)%elems(n+r)%j=j
586          dst(i)%elems(n+r)%k=k
587          dst(i)%elems(n+r)%l=l
588          dst(i)%elems(n+r)%v=src(k,l)
589        ENDIF
590      ENDDO
591    END DO
592    dst(i)%nelems=nsrc+n
593

```

### 8.24.2.5 subroutine, public tensor::add\_to\_tensor ( type(coolist), dimension(ndim), intent(in) src, type(coolist), dimension(ndim), intent(inout) dst )

Routine to add a rank-3 tensor to another one.

#### Parameters

<i>src</i>	Tensor to add
<i>dst</i>	Destination tensor

Definition at line 350 of file tensor.f90.

```

350      TYPE(coolist), DIMENSION(ndim), INTENT(IN) :: src
351      TYPE(coolist), DIMENSION(ndim), INTENT(INOUT) :: dst
352      TYPE(coolist_elem), DIMENSION(:), ALLOCATABLE :: celems
353      INTEGER :: i,j,n,allocstat
354
355      DO i=1,ndim
356        IF (src(i)%nelems/=0) THEN
357          IF (dst(i)%nelems==0) THEN
358            IF (ALLOCATED(dst(i)%elems)) THEN
359              DEALLOCATE(dst(i)%elems, stat=allocstat)
360              IF (allocstat /= 0) stop "*** Deallocation problem ! ***"
361            ENDIF
362            ALLOCATE(dst(i)%elems(src(i)%nelems), stat=allocstat)
363            IF (allocstat /= 0) stop "*** Not enough memory ! ***"
364            n=0
365          ELSE
366            n=dst(i)%nelems
367            ALLOCATE(celems(n), stat=allocstat)
368            DO j=1,n
369              celems(j)%j=dst(i)%elems(j)%j
370              celems(j)%k=dst(i)%elems(j)%k
371              celems(j)%v=dst(i)%elems(j)%v
372            ENDDO
373            DEALLOCATE(dst(i)%elems, stat=allocstat)
374            IF (allocstat /= 0) stop "*** Deallocation problem ! ***"
375            ALLOCATE(dst(i)%elems(src(i)%nelems+n), stat=allocstat)
376            IF (allocstat /= 0) stop "*** Not enough memory ! ***"
377            DO j=1,n
378              dst(i)%elems(j)%j=celems(j)%j
379              dst(i)%elems(j)%k=celems(j)%k
380              dst(i)%elems(j)%v=celems(j)%v
381            ENDDO
382            DEALLOCATE(celems, stat=allocstat)
383            IF (allocstat /= 0) stop "*** Deallocation problem ! ***"
384          ENDIF
385          DO j=1,src(i)%nelems

```

```

386      dst(i)%elems(n+j)%j=src(i)%elems(j)%j
387      dst(i)%elems(n+j)%k=src(i)%elems(j)%k
388      dst(i)%elems(n+j)%v=src(i)%elems(j)%v
389    ENDDO
390    dst(i)%nelems=src(i)%nelems+
391  ENDIF
392 ENDDO
393

```

#### 8.24.2.6 subroutine, public `tensor::add_vec_ijk_to_tensor4` ( integer, intent(in) *i*, integer, intent(in) *j*, integer, intent(in) *k*, real(kind=8), dimension(ndim), intent(in) *src*, type(coolist4), dimension(ndim), intent(inout) *dst* )

Routine to add a vector to a rank-4 tensor.

##### Parameters

<i>i,j,k</i>	Add to tensor component i,j and k
<i>src</i>	Vector to add
<i>dst</i>	Destination tensor

Definition at line 785 of file `tensor.f90`.

```

785      INTEGER, INTENT(IN) :: i,j,k
786      REAL(KIND=8), DIMENSION(ndim), INTENT(IN) :: src
787      TYPE(coolist4), DIMENSION(ndim), INTENT(OUT) :: dst
788      TYPE(coolist_elem4), DIMENSION(:), ALLOCATABLE :: celems
789      INTEGER :: l,ne,r,n,nsrc,allocstat
790
791      nsrc=0
792      DO l=1,ndim
793        IF (abs(src(l))>real_eps) nsrc=nsrc+1
794      ENDDO
795
796      IF (dst(i)%nelems==0) THEN
797        IF (ALLOCATED(dst(i)%elems)) THEN
798          DEALLOCATE(dst(i)%elems, stat=allocstat)
799          IF (allocstat /= 0) stop "*** Deallocation problem ! ***"
800        ENDIF
801        ALLOCATE(dst(i)%elems(nsrc), stat=allocstat)
802        IF (allocstat /= 0) stop "*** Not enough memory ! ***"
803        n=0
804      ELSE
805        n=dst(i)%nelems
806        ALLOCATE(celems(n), stat=allocstat)
807        DO ne=1,n
808          celems(ne)%j=dst(i)%elems(ne)%j
809          celems(ne)%k=dst(i)%elems(ne)%k
810          celems(ne)%l=dst(i)%elems(ne)%l
811          celems(ne)%v=dst(i)%elems(ne)%v
812        ENDDO
813        DEALLOCATE(dst(i)%elems, stat=allocstat)
814        IF (allocstat /= 0) stop "*** Deallocation problem ! ***"
815        ALLOCATE(dst(i)%elems(nsro+n), stat=allocstat)
816        IF (allocstat /= 0) stop "*** Not enough memory ! ***"
817        ne=1,n
818        dst(i)%elems(ne)%j=celems(ne)%j
819        dst(i)%elems(ne)%k=celems(ne)%k
820        dst(i)%elems(ne)%l=celems(ne)%l
821        dst(i)%elems(ne)%v=celems(ne)%v
822      ENDDO
823      DEALLOCATE(celems, stat=allocstat)
824      IF (allocstat /= 0) stop "*** Deallocation problem ! ***"
825    ENDIF
826    r=0
827    DO l=1,ndim
828      IF (abs(src(l))>real_eps) THEN
829        r=r+1
830        dst(i)%elems(n+r)%j=j
831        dst(i)%elems(n+r)%k=k
832        dst(i)%elems(n+r)%l=l
833        dst(i)%elems(n+r)%v=src(l)
834      ENDIF
835    ENDDO
836    dst(i)%nelems=nsro+n

```

#### 8.24.2.7 subroutine, public tensor::add\_vec\_ikl\_to\_tensor4 ( integer, intent(in) *i*, integer, intent(in) *k*, integer, intent(in) *l*, real(kind=8), dimension(ndim), intent(in) *src*, type(coolist4), dimension(ndim), intent(inout) *dst* )

Routine to add a vector to a rank-4 tensor.

##### Parameters

<i>i,k,l</i>	Add to tensor component i,k and l
<i>src</i>	Vector to add
<i>dst</i>	Destination tensor

Definition at line 726 of file tensor.f90.

```

726     INTEGER, INTENT(IN) :: i,k,l
727     REAL(KIND=8), DIMENSION(ndim), INTENT(IN) :: src
728     TYPE(coolist4), DIMENSION(ndim), INTENT(INOUT) :: dst
729     TYPE(coolist_elem4), DIMENSION(:), ALLOCATABLE :: celems
730     INTEGER :: j,ne,r,n,nsrc,allocstat
731
732     nsrc=0
733     DO j=1,ndim
734       IF (abs(src(j))>real_eps) nsrc=nsrc+1
735     ENDDO
736
737     IF (dst(i)%nelems==0) THEN
738       IF (ALLOCATED(dst(i)%elems)) THEN
739         DEALLOCATE(dst(i)%elems, stat=allocstat)
740         IF (allocstat /= 0) stop "*** Deallocation problem ! ***"
741       ENDIF
742       ALLOCATE(dst(i)%elems(nsrc), stat=allocstat)
743       IF (allocstat /= 0) stop "*** Not enough memory ! ***"
744       n=0
745     ELSE
746       n=dst(i)%nelems
747       ALLOCATE(celems(n), stat=allocstat)
748       DO ne=1,n
749         celems(ne)%j=dst(i)%elems(ne)%j
750         celems(ne)%k=dst(i)%elems(ne)%k
751         celems(ne)%l=dst(i)%elems(ne)%l
752         celems(ne)%v=dst(i)%elems(ne)%v
753       ENDDO
754       DEALLOCATE(dst(i)%elems, stat=allocstat)
755       IF (allocstat /= 0) stop "*** Deallocation problem ! ***"
756       ALLOCATE(dst(i)%elems(nsnc+n), stat=allocstat)
757       IF (allocstat /= 0) stop "*** Not enough memory ! ***"
758       DO ne=1,n
759         dst(i)%elems(ne)%j=celems(ne)%j
760         dst(i)%elems(ne)%k=celems(ne)%k
761         dst(i)%elems(ne)%l=celems(ne)%l
762         dst(i)%elems(ne)%v=celems(ne)%v
763       ENDDO
764       DEALLOCATE(celems, stat=allocstat)
765       IF (allocstat /= 0) stop "*** Deallocation problem ! ***"
766     ENDIF
767     r=0
768     DO j=1,ndim
769       IF (abs(src(j))>real_eps) THEN
770         r=r+1
771         dst(i)%elems(n+r)%j=j
772         dst(i)%elems(n+r)%k=k
773         dst(i)%elems(n+r)%l=l
774         dst(i)%elems(n+r)%v=src(j)
775       ENDIF
776     ENDDO
777     dst(i)%nelems=nsnc+n

```

#### 8.24.2.8 subroutine, public tensor::add\_vec\_ikl\_to\_tensor4\_perm ( integer, intent(in) *i*, integer, intent(in) *k*, integer, intent(in) *l*, real(kind=8), dimension(ndim), intent(in) *src*, type(coolist4), dimension(ndim), intent(inout) *dst* )

Routine to add a vector to a rank-4 tensor plus permutation.

## Parameters

<i>i,k,l</i>	Add to tensor component i,k and l
<i>src</i>	Vector to add
<i>dst</i>	Destination tensor

Definition at line 657 of file tensor.f90.

```

657      INTEGER, INTENT(IN) :: i,k,l
658      REAL(KIND=8), DIMENSION(ndim), INTENT(IN) :: src
659      TYPE(coolist4), DIMENSION(ndim), INTENT(INOUT) :: dst
660      TYPE(coolist_elem4), DIMENSION(:), ALLOCATABLE :: celems
661      INTEGER :: j,ne,r,n,nsrc,allocstat
662
663      nsrc=0
664      DO j=1,ndim
665        IF (abs(src(j))>real_eps) nsrc=nsrc+1
666      ENDDO
667      nsrc=nsrc*3
668      IF (dst(i)%nelems==0) THEN
669        IF (ALLOCATED(dst(i)%elems)) THEN
670          DEALLOCATE(dst(i)%elems, stat=allocstat)
671          IF (allocstat /= 0) stop "*** Deallocation problem ! ***"
672        ENDIF
673        ALLOCATE(dst(i)%elems(nsrc), stat=allocstat)
674        IF (allocstat /= 0) stop "*** Not enough memory ! ***"
675        n=0
676      ELSE
677        n=dst(i)%nelems
678        ALLOCATE(celems(n), stat=allocstat)
679        DO ne=1,n
680          celems(ne)%j=dst(i)%elems(ne)%j
681          celems(ne)%k=dst(i)%elems(ne)%k
682          celems(ne)%l=dst(i)%elems(ne)%l
683          celems(ne)%v=dst(i)%elems(ne)%v
684        ENDDO
685        DEALLOCATE(dst(i)%elems, stat=allocstat)
686        IF (allocstat /= 0) stop "*** Deallocation problem ! ***"
687        ALLOCATE(dst(i)%elems(nsrc+n), stat=allocstat)
688        IF (allocstat /= 0) stop "*** Not enough memory ! ***"
689        DO ne=1,n
690          dst(i)%elems(ne)%j=celems(ne)%j
691          dst(i)%elems(ne)%k=celems(ne)%k
692          dst(i)%elems(ne)%l=celems(ne)%l
693          dst(i)%elems(ne)%v=celems(ne)%v
694        ENDDO
695        DEALLOCATE(celems, stat=allocstat)
696        IF (allocstat /= 0) stop "*** Deallocation problem ! ***"
697      ENDIF
698      r=0
699      DO j=1,ndim
700        IF (abs(src(j))>real_eps) THEN
701          r=r+1
702          dst(i)%elems(n+r)%j=j
703          dst(i)%elems(n+r)%k=k
704          dst(i)%elems(n+r)%l=l
705          dst(i)%elems(n+r)%v=src(j)
706          r=r+1
707          dst(i)%elems(n+r)%j=k
708          dst(i)%elems(n+r)%k=l
709          dst(i)%elems(n+r)%l=j
710          dst(i)%elems(n+r)%v=src(j)
711          r=r+1
712          dst(i)%elems(n+r)%j=l
713          dst(i)%elems(n+r)%k=j
714          dst(i)%elems(n+r)%l=k
715          dst(i)%elems(n+r)%v=src(j)
716        ENDIF
717      ENDDO
718      dst(i)%nelems=nsrc+n

```

#### 8.24.2.9 subroutine, public tensor::add\_vec\_jk\_to\_tensor ( integer, intent(in) *j*, integer, intent(in) *k*, real(kind=8), dimension(ndim), intent(in) *src*, type(coolist), dimension(ndim), intent(inout) *dst* )

Routine to add a vector to a rank-3 tensor.

**Parameters**

<i>j,k</i>	Add to tensor component j and k
<i>src</i>	Vector to add
<i>dst</i>	Destination tensor

Definition at line 602 of file tensor.f90.

```

602      INTEGER, INTENT(IN) :: j,k
603      REAL(KIND=8), DIMENSION(ndim), INTENT(IN) :: src
604      TYPE(coolist), DIMENSION(ndim), INTENT(INOUT) :: dst
605      TYPE(coolist_eleml), DIMENSION(:), ALLOCATABLE :: celems
606      INTEGER :: i,l,r,n,nsrc,allocstat
607
608      DO i=1,ndim
609         nsrc=0
610         IF (abs(src(i))>real_eps) nsrc=1
611         IF (dst(i)%nelems==0) THEN
612            IF (ALLOCATED(dst(i)%elems)) THEN
613               DEALLOCATE(dst(i)%elems, stat=allocstat)
614               IF (allocstat /= 0) stop "*** Deallocation problem ! ***"
615            ENDIF
616            ALLOCATE(dst(i)%elems(nsrc), stat=allocstat)
617            IF (allocstat /= 0) stop "*** Not enough memory ! ***"
618            n=0
619         ELSE
620            n=dst(i)%nelems
621            ALLOCATE(celems(n), stat=allocstat)
622            DO l=1,n
623               celems(l)%j=dst(i)%elems(l)%j
624               celems(l)%k=dst(i)%elems(l)%k
625               celems(l)%v=dst(i)%elems(l)%v
626            ENDDO
627            DEALLOCATE(dst(i)%elems, stat=allocstat)
628            IF (allocstat /= 0) stop "*** Deallocation problem ! ***"
629            ALLOCATE(dst(i)%elems(nsdc+n), stat=allocstat)
630            IF (allocstat /= 0) stop "*** Not enough memory ! ***"
631            DO l=1,n
632               dst(i)%elems(l)%j=celems(l)%j
633               dst(i)%elems(l)%k=celems(l)%k
634               dst(i)%elems(l)%v=celems(l)%v
635            ENDDO
636            DEALLOCATE(celems, stat=allocstat)
637            IF (allocstat /= 0) stop "*** Deallocation problem ! ***"
638         ENDIF
639         r=0
640         IF (abs(src(i))>real_eps) THEN
641            r=r+1
642            dst(i)%elems(n+r)%j=j
643            dst(i)%elems(n+r)%k=k
644            dst(i)%elems(n+r)%v=src(i)
645         ENDIF
646         dst(i)%nelems=nsrc+n
647      END DO
648
649

```

#### 8.24.2.10 subroutine, public tensor::coo\_to\_mat\_i ( integer, intent(in) *i*, type(coolist), dimension(ndim), intent(in) *src*, real(kind=8), dimension(ndim,ndim), intent(out) *dst* )

Routine to convert a rank-3 tensor coolist component into a matrix.

**Parameters**

<i>i</i>	Component to convert
<i>src</i>	Source tensor
<i>dst</i>	Destination matrix

Definition at line 1112 of file tensor.f90.

```

1112      INTEGER, INTENT(IN) :: i
1113      TYPE(coolist), DIMENSION(ndim), INTENT(IN) :: src
1114      REAL(KIND=8), DIMENSION(ndim,ndim), INTENT(OUT) :: dst
1115      INTEGER :: n
1116
1117      dst=0.d0
1118      DO n=1,src(i)%nelems
1119          dst(src(i)%elems(n)%j,src(i)%elems(n)%k)=src(i)%elems(n)%v
1120      ENDDO

```

#### 8.24.2.11 subroutine, public tensor::coo\_to\_mat\_ij ( type(coolist), dimension(ndim), intent(in) src, real(kind=8), dimension(ndim,ndim), intent(out) dst )

Routine to convert a rank-3 tensor coolist component into a matrix with i and j indices.

##### Parameters

<i>src</i>	Source tensor
<i>dst</i>	Destination matrix

Definition at line 1079 of file tensor.f90.

```

1079      TYPE(coolist), DIMENSION(ndim), INTENT(IN) :: src
1080      REAL(KIND=8), DIMENSION(ndim,ndim), INTENT(OUT) :: dst
1081      INTEGER :: i,n
1082
1083      dst=0.d0
1084      DO i=1,ndim
1085          DO n=1,src(i)%nelems
1086              dst(i,src(i)%elems(n)%j)=src(i)%elems(n)%v
1087          ENDDO
1088      ENDDO

```

#### 8.24.2.12 subroutine, public tensor::coo\_to\_mat\_ik ( type(coolist), dimension(ndim), intent(in) src, real(kind=8), dimension(ndim,ndim), intent(out) dst )

Routine to convert a rank-3 tensor coolist component into a matrix with i and k indices.

##### Parameters

<i>src</i>	Source tensor
<i>dst</i>	Destination matrix

Definition at line 1063 of file tensor.f90.

```

1063      TYPE(coolist), DIMENSION(ndim), INTENT(IN) :: src
1064      REAL(KIND=8), DIMENSION(ndim,ndim), INTENT(OUT) :: dst
1065      INTEGER :: i,n
1066
1067      dst=0.d0
1068      DO i=1,ndim
1069          DO n=1,src(i)%nelems
1070              dst(i,src(i)%elems(n)%k)=src(i)%elems(n)%v
1071          ENDDO
1072      ENDDO

```

**8.24.2.13 subroutine, public tensor::coo\_to\_mat\_j ( integer, intent(in) *j*, type(coolist), dimension(ndim), intent(in) *src*, real(kind=8), dimension(ndim,ndim), intent(out) *dst* )**

Routine to convert a rank-3 tensor coolist component into a matrix.

#### Parameters

<i>j</i>	Component to convert
<i>src</i>	Source tensor
<i>dst</i>	Destination matrix

Definition at line 1148 of file tensor.f90.

```

1148      INTEGER, INTENT(IN) :: j
1149      TYPE(coolist), DIMENSION(ndim), INTENT(IN) :: src
1150      REAL(KIND=8), DIMENSION(ndim,ndim), INTENT(OUT) :: dst
1151      INTEGER :: i,n
1152
1153      dst=0.d0
1154      DO i=1,ndim
1155        DO n=1,src(i)%nelems
1156          IF ((src(i)%elems(n)%j==j) .and. (src(i)%elems(n)%k==k)) dst(i,src(i)%elems(n)%k)=src(i)%elems(n)%v
1157        ENDDO
1158      END DO

```

**8.24.2.14 subroutine, public tensor::coo\_to\_vec\_jk ( integer, intent(in) *j*, integer, intent(in) *k*, type(coolist), dimension(ndim), intent(in) *src*, real(kind=8), dimension(ndim), intent(out) *dst* )**

Routine to convert a rank-3 tensor coolist component into a vector.

#### Parameters

<i>j</i>	Component j,k to convert
<i>k</i>	Component j,k to convert
<i>src</i>	Source tensor
<i>dst</i>	Destination vector

Definition at line 1129 of file tensor.f90.

```

1129      INTEGER, INTENT(IN) :: j,k
1130      TYPE(coolist), DIMENSION(ndim), INTENT(IN) :: src
1131      REAL(KIND=8), DIMENSION(ndim), INTENT(OUT) :: dst
1132      INTEGER :: i,n
1133
1134      dst=0.d0
1135      DO i=1,ndim
1136        DO n=1,src(i)%nelems
1137          IF (((src(i)%elems(n)%j==j) .and. (src(i)%elems(n)%k==k))) dst(i)=src(i)%elems(n)%v
1138        ENDDO
1139      ENDDO

```

**8.24.2.15 subroutine, public tensor::copy\_coo ( type(coolist), dimension(ndim), intent(in) *src*, type(coolist), dimension(ndim), intent(out) *dst* )**

Routine to copy a coolist.

### Parameters

<i>src</i>	Source coolist
<i>dst</i>	Destination coolist

### Remarks

The destination tensor have to be an empty tensor, i.e. with unallocated list of elements and nelems set to 0.

Definition at line 72 of file tensor.f90.

```

72      TYPE(coolist), DIMENSION(ndim), INTENT(IN) :: src
73      TYPE(coolist), DIMENSION(ndim), INTENT(OUT) :: dst
74      INTEGER :: i,j,allocstat
75
76      DO i=1,ndim
77          IF (dst(i)%nelems/=0) stop "*** copy_coo : Destination coolist not empty ! ***"
78          ALLOCATE(dst(i)%elems(src(i)%nelems), stat=allocstat)
79          IF (allocstat /= 0) stop "*** Not enough memory ! ***"
80          DO j=1,src(i)%nelems
81              dst(i)%elems(j)%j=src(i)%elems(j)%j
82              dst(i)%elems(j)%k=src(i)%elems(j)%k
83              dst(i)%elems(j)%v=src(i)%elems(j)%v
84          ENDDO
85          dst(i)%nelems=src(i)%nelems
86      ENDDO

```

#### 8.24.2.16 subroutine, public tensor::jsparse\_mul ( type(coolist), dimension(ndim), intent(in) coolist\_ijk, real(kind=8), dimension(0:ndim), intent(in) arr\_j, type(coolist), dimension(ndim), intent(out) jcoo\_ij )

Sparse multiplication of two tensors to determine the Jacobian:

$$J_{i,j} = \sum_{k=0}^{ndim} (\mathcal{T}_{i,j,k} + \mathcal{T}_{i,k,j}) a_k.$$

It's implemented slightly differently: for every  $\mathcal{T}_{i,j,k}$ , we add to  $J_{i,j}$  as follows:

$$J_{i,j} = J_{i,j} + \mathcal{T}_{i,j,k} a_k J_{i,k} = J_{i,k} + \mathcal{T}_{i,j,k} a_j$$

This version return a coolist (sparse tensor).

### Parameters

<i>coolist_ijk</i>	a coordinate list (sparse tensor) of which index 2 or 3 will be contracted.
<i>arr_j</i>	the vector to be contracted with index 2 and then index 3 of ffi_coo_ijk
<i>jcoo_ij</i>	a coolist (sparse tensor) to store the result of the contraction

Definition at line 153 of file tensor.f90.

```

153      TYPE(coolist), DIMENSION(ndim), INTENT(IN):: coolist_ijk
154      TYPE(coolist), DIMENSION(ndim), INTENT(OUT):: jcoo_ij
155      REAL(KIND=8), DIMENSION(0:ndim), INTENT(IN)  :: arr_j
156      REAL(KIND=8) :: v
157      INTEGER :: i,j,k,n,nj,allocstat
158      DO i=1,ndim
159          IF (jcoo_ij(i)%nelems/=0) stop "*** jsparse_mul : Destination coolist not empty ! ***"
160          nj=2*coolist_ijk(i)%nelems

```

```

161      ALLOCATE(jcoo_ij(i)%elems(nj), stat=allocstat)
162      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
163      nj=0
164      DO n=1,coolist_ijk(i)%nelems
165          j=coolist_ijk(i)%elems(n)%j
166          k=coolist_ijk(i)%elems(n)%k
167          v=coolist_ijk(i)%elems(n)%v
168          IF (j /=0) THEN
169              nj=nj+1
170              jcoo_ij(i)%elems(nj)%j=j
171              jcoo_ij(i)%elems(nj)%k=0
172              jcoo_ij(i)%elems(nj)%v=v*arr_j(k)
173          END IF
174          IF (k /=0) THEN
175              nj=nj+1
176              jcoo_ij(i)%elems(nj)%j=k
177              jcoo_ij(i)%elems(nj)%k=0
178              jcoo_ij(i)%elems(nj)%v=v*arr_j(j)
179          END IF
180      END DO
181      jcoo_ij(i)%nelems=nj
182  END DO

```

#### 8.24.2.17 subroutine, public tensor::jsparse\_mul\_mat ( type(coolist), dimension(ndim), intent(in) coolist\_ijk, real(kind=8), dimension(0:ndim), intent(in) arr\_j, real(kind=8), dimension(ndim,ndim), intent(out) jcoo\_ij )

Sparse multiplication of two tensors to determine the Jacobian:

$$J_{i,j} = \sum_{k=0}^{ndim} (\mathcal{T}_{i,j,k} + \mathcal{T}_{i,k,j}) a_k.$$

It's implemented slightly differently: for every  $\mathcal{T}_{i,j,k}$ , we add to  $J_{i,j}$  as follows:

$$J_{i,j} = J_{i,j} + \mathcal{T}_{i,j,k} a_k J_{i,k} = J_{i,k} + \mathcal{T}_{i,j,k} a_j$$

This version return a matrix.

#### Parameters

<i>coolist_ijk</i>	a coordinate list (sparse tensor) of which index 2 or 3 will be contracted.
<i>arr_j</i>	the vector to be contracted with index 2 and then index 3 of ffi_coo_ijk
<i>jcoo_ij</i>	a matrix to store the result of the contraction

Definition at line 196 of file tensor.f90.

```

196      TYPE(coolist), DIMENSION(ndim), INTENT(IN):: coolist_ijk
197      REAL(KIND=8), DIMENSION(ndim,ndim), INTENT(OUT):: jcoo_ij
198      REAL(KIND=8), DIMENSION(0:ndim), INTENT(IN) :: arr_j
199      REAL(KIND=8) :: v
200      INTEGER :: i,j,k,n
201      jcoo_ij=0.d0
202      DO i=1,ndim
203          DO n=1,coolist_ijk(i)%nelems
204              j=coolist_ijk(i)%elems(n)%j
205              k=coolist_ijk(i)%elems(n)%k
206              v=coolist_ijk(i)%elems(n)%v
207              IF (j /=0) jcoo_ij(i,j)=jcoo_ij(i,j)+v*arr_j(k)
208              IF (k /=0) jcoo_ij(i,k)=jcoo_ij(i,k)+v*arr_j(j)
209          END DO
210      END DO

```

```
8.24.2.18 subroutine, public tensor::load_tensor4_from_file ( character (len=*) s, intent(in) s, type(coolist4), dimension(ndim),
intent(out) t )
```

Load a rank-4 tensor coolist from a file definition.

### Parameters

<i>s</i>	Filename of the tensor definition file
<i>t</i>	The loaded coolist

### Remarks

The destination tensor have to be an empty tensor, i.e. with unallocated list of elements and nelems set to 0.

Definition at line 1322 of file tensor.f90.

```

1322      CHARACTER (LEN=*), INTENT(IN) :: s
1323      TYPE(coolist4), DIMENSION(ndim), INTENT(OUT) :: t
1324      INTEGER :: i,ir,j,k,l,n,allocstat
1325      REAL(KIND=8) :: v
1326      OPEN(30,file=s,status='old')
1327      DO i=1,ndim
1328          READ(30,*) ir,n
1329          IF (n /= 0) THEN
1330              ALLOCATE(t(i)%elems(n), stat=allocstat)
1331              IF (allocstat /= 0) stop "*** Not enough memory ! ***"
1332              t(i)%nelems=n
1333          ENDIF
1334          DO n=1,t(i)%nelems
1335              READ(30,*) ir,j,k,l,v
1336              t(i)%elems(n)%j=j
1337              t(i)%elems(n)%k=k
1338              t(i)%elems(n)%l=l
1339              t(i)%elems(n)%v=v
1340          ENDDO
1341      END DO
1342      CLOSE(30)

```

### 8.24.2.19 subroutine, public tensor::load\_tensor\_from\_file ( character (len=\*), intent(in) *s*, type(coolist), dimension(ndim), intent(out) *t* )

Load a rank-4 tensor coolist from a file definition.

### Parameters

<i>s</i>	Filename of the tensor definition file
<i>t</i>	The loaded coolist

### Remarks

The destination tensor have to be an empty tensor, i.e. with unallocated list of elements and nelems set to 0.

Definition at line 445 of file tensor.f90.

```

445      CHARACTER (LEN=*), INTENT(IN) :: s
446      TYPE(coolist), DIMENSION(ndim), INTENT(OUT) :: t
447      INTEGER :: i,ir,j,k,n,allocstat
448      REAL(KIND=8) :: v
449      OPEN(30,file=s,status='old')
450      DO i=1,ndim
451          READ(30,*) ir,n
452          IF (n /= 0) THEN
453              ALLOCATE(t(i)%elems(n), stat=allocstat)
454              IF (allocstat /= 0) stop "*** Not enough memory ! ***"
455              t(i)%nelems=n
456          ENDIF

```

```

457      DO n=1,t(i)%nelems
458        READ(30,*) ir,j,k,v
459        t(i)%elems(n)%j=j
460        t(i)%elems(n)%k=k
461        t(i)%elems(n)%v=v
462      ENDDO
463    END DO
464  CLOSE(30)

```

#### 8.24.2.20 subroutine, public tensor::mat\_to\_coo ( real(kind=8), dimension(0:ndim,0:ndim), intent(in) *src*, type(coolist), dimension(ndim), intent(out) *dst* )

Routine to convert a matrix to a tensor.

##### Parameters

<i>src</i>	Source matrix
<i>dst</i>	Destination tensor

##### Remarks

The destination tensor have to be an empty tensor, i.e. with unallocated list of elements and nelems set to 0.

Definition at line 94 of file tensor.f90.

```

94      REAL(KIND=8), DIMENSION(0:ndim,0:ndim), INTENT(IN) :: src
95      TYPE(coolist), DIMENSION(ndim), INTENT(OUT) :: dst
96      INTEGER :: i,j,n,allocstat
97      DO i=1,ndim
98        n=0
99        DO j=1,ndim
100          IF (abs(src(i,j))>real_eps) n=n+1
101        ENDDO
102        IF (n/=0) THEN
103          IF (dst(i)%nelems/=0) stop "*** mat_to_coo : Destination coolist not empty ! ***"
104          ALLOCATE(dst(i)%elems(n), stat=allocstat)
105          IF (allocstat /= 0) stop "*** Not enough memory ! ***"
106          n=0
107          DO j=1,ndim
108            IF (abs(src(i,j))>real_eps) THEN
109              n=n+1
110              dst(i)%elems(n)%j=j
111              dst(i)%elems(n)%k=0
112              dst(i)%elems(n)%v=src(i,j)
113            ENDIF
114          ENDDO
115        ENDIF
116        dst(i)%nelems=n
117      ENDDO

```

#### 8.24.2.21 subroutine, public tensor::matc\_to\_coo ( real(kind=8), dimension(ndim,ndim), intent(in) *src*, type(coolist), dimension(ndim), intent(out) *dst* )

Routine to convert a matrix to a rank-3 tensor.

##### Parameters

<i>src</i>	Source matrix
<i>dst</i>	Destination tensor

## Remarks

The destination tensor have to be an empty tensor, i.e. with unallocated list of elements and nelems set to 0.  
The j component will be set to 0.

Definition at line 1244 of file tensor.f90.

```

1244      REAL(KIND=8), DIMENSION(ndim,ndim), INTENT(IN) :: src
1245      TYPE(coolist), DIMENSION(ndim), INTENT(OUT) :: dst
1246      INTEGER :: i,j,n,allocstat
1247      DO i=1,ndim
1248          n=0
1249          DO j=1,ndim
1250              IF (abs(src(i,j))>real_eps) n=n+1
1251          ENDDO
1252          IF (n/=0) THEN
1253              IF (dst(i)%nelems/=0) stop "*** mat_to_coo : Destination coolist not empty ! ***"
1254              ALLOCATE(dst(i)%elems(n), stat=allocstat)
1255              IF (allocstat /= 0) stop "*** Not enough memory ! ***"
1256              n=0
1257              DO j=1,ndim
1258                  IF (abs(src(i,j))>real_eps) THEN
1259                      n=n+1
1260                      dst(i)%elems(n)%j=0
1261                      dst(i)%elems(n)%k=j
1262                      dst(i)%elems(n)%v=src(i,j)
1263                  ENDIF
1264              ENDDO
1265          ENDIF
1266          dst(i)%nelems=n
1267      ENDDO

```

### 8.24.2.22 subroutine, public tensor::print\_tensor ( type(coolist), dimension(ndim), intent(in) t, character, intent(in), optional s )

Routine to print a rank 3 tensor coolist.

## Parameters

<i>t</i>	coolist to print
----------	------------------

Definition at line 399 of file tensor.f90.

```

399      USE util, only: str
400      TYPE(coolist), DIMENSION(ndim), INTENT(IN) :: t
401      CHARACTER, INTENT(IN), OPTIONAL :: s
402      CHARACTER :: r
403      INTEGER :: i,n,j,k
404      IF (PRESENT(s)) THEN
405          r=s
406      ELSE
407          r="t"
408      END IF
409      DO i=1,ndim
410          n=1,t(i)%nelems
411          j=t(i)%elems(n)%j
412          k=t(i)%elems(n)%k
413          IF( abs(t(i)%elems(n)%v) .GE. real_eps) THEN
414              write(*,"(A,ES12.5)") r//"[//trim(str(i))//]"//trim(str(j)) //
415              & //"[//trim(str(k))//]" = ",t(i)%elems(n)%v
416          END IF
417          END DO
418      END DO

```

### 8.24.2.23 subroutine, public tensor::print\_tensor4 ( type(coolist4), dimension(ndim), intent(in) t )

Routine to print a rank-4 tensor coolist.

**Parameters**

<i>t</i>	coolist to print
----------	------------------

Definition at line 922 of file tensor.f90.

```

922      USE util, only: str
923      TYPE(coolist4), DIMENSION(ndim), INTENT(IN) :: t
924      INTEGER :: i,n,j,k,l
925      DO i=1,ndim
926          DO n=1,t(i)%nelems
927              j=t(i)%elems(n)%j
928              k=t(i)%elems(n)%k
929              l=t(i)%elems(n)%l
930              IF( abs(t(i)%elems(n)%v) .GE. real_eps) THEN
931                  write(*,"(A,ES12.5)") "tensor[""/trim(str(i))//"][""/trim(str(j)) &
932                  & //"][""/trim(str(k))//"][""/trim(str(l))//"] = ",t(i)%elems(n)%v
933              END IF
934          END DO
935      END DO

```

#### 8.24.2.24 subroutine, public tensor::scal\_mul\_coo ( real(kind=8), intent(in) s, type(coolist), dimension(ndim), intent(inout) t )

Routine to multiply a rank-3 tensor by a scalar.

**Parameters**

<i>s</i>	The scalar
<i>t</i>	The tensor

Definition at line 1274 of file tensor.f90.

```

1274      REAL(KIND=8), INTENT(IN) :: s
1275      TYPE(coolist), DIMENSION(ndim), INTENT(INOUT) :: t
1276      INTEGER :: i,li,n
1277      DO i=1,ndim
1278          n=t(i)%nelems
1279          DO li=1,n
1280              t(i)%elems(li)%v=s*t(i)%elems(li)%v
1281          ENDDO
1282      ENDDO

```

#### 8.24.2.25 subroutine, public tensor::simplify ( type(coolist), dimension(ndim), intent(inout) tensor )

Routine to simplify a coolist (sparse tensor). For each index *i*, it upper triangularize the matrix

$$\mathcal{T}_{i,j,k} \quad 0 \leq j, k \leq ndim.$$

**Parameters**

<i>tensor</i>	a coordinate list (sparse tensor) which will be simplified.
---------------	---

Definition at line 238 of file tensor.f90.

```

238  TYPE(coolist), DIMENSION(ndim), INTENT(INOUT) :: tensor
239  INTEGER :: i,j,k
240  INTEGER :: li,lii,liii,n
241  DO i= 1,ndim
242    n=tensor(i)%nelems
243    DO li=n,2,-1
244      j=tensor(i)%elems(li)%j
245      k=tensor(i)%elems(li)%k
246      DO lii=li-1,1,-1
247        IF (((j==tensor(i)%elems(lii)%j).AND.(k==tensor(i)%
248          &%elems(lii)%k)).OR.((j==tensor(i)%elems(lii)%k).AND.(k==
249          tensor(i)%elems(lii)%j))) THEN
250          ! Found another entry with the same i,j,k: merge both into
251          ! the one listed first (of those two).
252          tensor(i)%elems(lii)%v=tensor(i)%elems(lii)%v+tensor(i)%elems(li)%v
253          IF (j>k) THEN
254            tensor(i)%elems(lii)%j=tensor(i)%elems(li)%k
255            tensor(i)%elems(lii)%k=tensor(i)%elems(li)%j
256          ENDIF
257
258          ! Shift the rest of the items one place down.
259          DO liii=li+1,n
260            tensor(i)%elems(lilli-1)%j=tensor(i)%elems(lilli)%j
261            tensor(i)%elems(lilli-1)%k=tensor(i)%elems(lilli)%k
262            tensor(i)%elems(lilli-1)%v=tensor(i)%elems(lilli)%v
263          END DO
264          tensor(i)%nelems=tensor(i)%nelems-1
265          ! Here we should stop because the li no longer points to the
266          ! original i,j,k element
267          EXIT
268        ENDIF
269      ENDDO
270    n=tensor(i)%nelems
271    DO li=1,n
272      ! Clear new "almost" zero entries and shift rest of the items one place down.
273      ! Make sure not to skip any entries while shifting!
274      DO WHILE (abs(tensor(i)%elems(li)%v) < real_eps)
275        DO liii=li+1,n
276          tensor(i)%elems(lilli-1)%j=tensor(i)%elems(lilli)%j
277          tensor(i)%elems(lilli-1)%k=tensor(i)%elems(lilli)%k
278          tensor(i)%elems(lilli-1)%v=tensor(i)%elems(lilli)%v
279        ENDDO
280        tensor(i)%nelems=tensor(i)%nelems-1
281        if (li > tensor(i)%nelems) THEN
282          EXIT
283        ENDIF
284      ENDDO
285    ENDDO
286
287    n=tensor(i)%nelems
288    DO li=1,n
289      ! Upper triangularize
290      j=tensor(i)%elems(li)%j
291      k=tensor(i)%elems(li)%k
292      IF (j>k) THEN
293        tensor(i)%elems(li)%j=k
294        tensor(i)%elems(li)%k=j
295      ENDIF
296    ENDDO
297
298  ENDDO
299

```

#### 8.24.2.26 subroutine, public tensor::sparse\_mul2 ( type(coolist), dimension(ndim), intent(in) coolist\_ij, real(kind=8), dimension(0:ndim), intent(in) arr\_j, real(kind=8), dimension(0:ndim), intent(out) res )

Sparse multiplication of a 2d sparse tensor with a vector:  $\sum_{j=0}^{ndim} \mathcal{T}_{i,j,k} a_j$ .

##### Parameters

<i>coolist_ij</i>	a coordinate list (sparse tensor) of which index 2 will be contracted.
<i>arr_j</i>	the vector to be contracted with index 2 of coolist_ijk
<i>res</i>	vector (buffer) to store the result of the contraction

### Remarks

Note that it is NOT safe to pass `arr_j` as a result buffer, as this operation does multiple passes.

Definition at line 221 of file `tensor.f90`.

```

221   TYPE(coolist), DIMENSION(ndim), INTENT(IN):: coolist_ij
222   REAL(KIND=8), DIMENSION(0:ndim), INTENT(IN) :: arr_j
223   REAL(KIND=8), DIMENSION(0:ndim), INTENT(OUT) :: res
224   INTEGER :: i,j,n
225   res=0.d0
226   DO i=1,ndim
227     DO n=1,coolist_ij(i)%nelems
228       j=coolist_ij(i)%elems(n)%j
229       res(i) = res(i) + coolist_ij(i)%elems(n)%v * arr_j(j)
230     END DO
231   END DO

```

**8.24.2.27 subroutine, public `tensor::sparse_mul2_j` ( type(`coolist`), dimension(`ndim`), intent(`in`) `coolist_ijk`, `real(kind=8)`, dimension(`0:ndim`), intent(`in`) `arr_j`, `real(kind=8)`, dimension(`0:ndim`), intent(`out`) `res` )**

Sparse multiplication of a 3d sparse tensor with a vectors:  $\sum_{j=0}^{ndim} T_{i,j,k} a_j$ .

### Parameters

<code>coolist_ijk</code>	a coordinate list (sparse tensor) of which index 2 will be contracted.
<code>arr_j</code>	the vector to be contracted with index 2 of <code>coolist_ijk</code>
<code>res</code>	vector (buffer) to store the result of the contraction

### Remarks

Note that it is NOT safe to pass `arr_j` as a result buffer, as this operation does multiple passes.

Definition at line 1024 of file `tensor.f90`.

```

1024   TYPE(coolist), DIMENSION(ndim), INTENT(IN):: coolist_ijk
1025   REAL(KIND=8), DIMENSION(0:ndim), INTENT(IN) :: arr_j
1026   REAL(KIND=8), DIMENSION(0:ndim), INTENT(OUT) :: res
1027   INTEGER :: i,j,n
1028   res=0.d0
1029   DO i=1,ndim
1030     DO n=1,coolist_ijk(i)%nelems
1031       j=coolist_ijk(i)%elems(n)%j
1032       res(i) = res(i) + coolist_ijk(i)%elems(n)%v * arr_j(j)
1033     END DO
1034   END DO

```

**8.24.2.28 subroutine, public `tensor::sparse_mul2_k` ( type(`coolist`), dimension(`ndim`), intent(`in`) `coolist_ijk`, `real(kind=8)`, dimension(`0:ndim`), intent(`in`) `arr_k`, `real(kind=8)`, dimension(`0:ndim`), intent(`out`) `res` )**

Sparse multiplication of a rank-3 sparse tensor `coolist` with a vector:  $\sum_{k=0}^{ndim} T_{i,j,k} a_k$ .

### Parameters

<i>coolist_ijk</i>	a coordinate list (sparse tensor) of which index k will be contracted.
<i>arr_k</i>	the vector to be contracted with index k of coolist_ijk
<i>res</i>	vector (buffer) to store the result of the contraction

### Remarks

Note that it is NOT safe to pass *arr\_k* as a result buffer, as this operation does multiple passes.

Definition at line 1045 of file tensor.f90.

```

1045      TYPE(coolist), DIMENSION(ndim), INTENT(IN):: coolist_ijk
1046      REAL(KIND=8), DIMENSION(0:ndim), INTENT(IN) :: arr_k
1047      REAL(KIND=8), DIMENSION(0:ndim), INTENT(OUT) :: res
1048      INTEGER :: i,k,n
1049      res=0.d0
1050      DO i=1,ndim
1051        DO n=1,coolist_ijk(i)%nelems
1052          k=coolist_ijk(i)%elems(n)%k
1053          res(i) = res(i) + coolist_ijk(i)%elems(n)%v * arr_k(k)
1054        END DO
1055      END DO

```

**8.24.2.29 subroutine, public tensor::sparse\_mul3 ( type(coolist), dimension(ndim), intent(in) *coolist\_ijk*, real(kind=8), dimension(0:ndim), intent(in) *arr\_j*, real(kind=8), dimension(0:ndim), intent(in) *arr\_k*, real(kind=8), dimension(0:ndim), intent(out) *res* )**

Sparse multiplication of a tensor with two vectors:  $\sum_{j,k=0}^{ndim} T_{i,j,k} a_j b_k$ .

### Parameters

<i>coolist_ijk</i>	a coordinate list (sparse tensor) of which index 2 and 3 will be contracted.
<i>arr_j</i>	the vector to be contracted with index 2 of coolist_ijk
<i>arr_k</i>	the vector to be contracted with index 3 of coolist_ijk
<i>res</i>	vector (buffer) to store the result of the contraction

### Remarks

Note that it is NOT safe to pass *arr\_j*/*arr\_k* as a result buffer, as this operation does multiple passes.

Definition at line 129 of file tensor.f90.

```

129      TYPE(coolist), DIMENSION(ndim), INTENT(IN):: coolist_ijk
130      REAL(KIND=8), DIMENSION(0:ndim), INTENT(IN) :: arr_j, arr_k
131      REAL(KIND=8), DIMENSION(0:ndim), INTENT(OUT) :: res
132      INTEGER :: i,j,k,n
133      res=0.d0
134      DO i=1,ndim
135        DO n=1,coolist_ijk(i)%nelems
136          j=coolist_ijk(i)%elems(n)%j
137          k=coolist_ijk(i)%elems(n)%k
138          res(i) = res(i) + coolist_ijk(i)%elems(n)%v * arr_j(j)*arr_k(k)
139        END DO
140      END DO

```

8.24.2.30 subroutine, public `tensor::sparse_mul3_mat` ( `type(coolist)`, `dimension(ndim)`, `intent(in) coolist_ijk`, `real(kind=8)`,  
`dimension(0:ndim), intent(in) arr_k`, `real(kind=8)`, `dimension(ndim,ndim), intent(out) res` )

Sparse multiplication of a rank-3 tensor `coolist` with a vector:  $\sum_{k=0}^{ndim} T_{i,j,k} b_k$ . Its output is a matrix.

#### Parameters

<code>coolist_ijk</code>	a coolist (sparse tensor) of which index k will be contracted.
<code>arr_k</code>	the vector to be contracted with index k of <code>coolist_ijk</code>
<code>res</code>	matrix (buffer) to store the result of the contraction

#### Remarks

Note that it is NOT safe to pass `arr_k` as a result buffer, as this operation does multiple passes.

Definition at line 948 of file `tensor.f90`.

```

948      TYPE(coolist), DIMENSION(ndim), INTENT(IN):: coolist_ijk
949      REAL(KIND=8), DIMENSION(0:ndim), INTENT(IN) :: arr_k
950      REAL(KIND=8), DIMENSION(ndim,ndim), INTENT(OUT) :: res
951      INTEGER :: i,j,k,n
952      res=0.d0
953      DO i=1,ndim
954        DO n=1,coolist_ijk(i)%nelems
955          j=coolist_ijk(i)%elems(n)%j
956          IF (j /= 0) THEN
957            k=coolist_ijk(i)%elems(n)%k
958            res(i,j) = res(i,j) + coolist_ijk(i)%elems(n)%v * arr_k(k)
959          ENDIF
960        END DO
961      END DO

```

8.24.2.31 subroutine, public `tensor::sparse_mul3_with_mat` ( `type(coolist)`, `dimension(ndim)`, `intent(in) coolist_ijk`,  
`real(kind=8)`, `dimension(ndim,ndim), intent(in) mat_jk`, `real(kind=8)`, `dimension(0:ndim), intent(out) res` )

Sparse multiplication of a rank-3 tensor `coolist` with a matrix:  $\sum_{j,k=0}^{ndim} T_{i,j,k} m_{j,k}$ .

#### Parameters

<code>coolist_ijk</code>	a coolist (sparse tensor) of which index j and k will be contracted.
<code>mat_jk</code>	the matrix to be contracted with index j and k of <code>coolist_ijk</code>
<code>res</code>	vector (buffer) to store the result of the contraction

#### Remarks

Note that it is NOT safe to pass `mat_jk` as a result buffer, as this operation does multiple passes.

Definition at line 1220 of file `tensor.f90`.

```

1220   TYPE(coolist), DIMENSION(ndim), INTENT(IN):: coolist_ijk
1221   REAL(KIND=8), DIMENSION(ndim,ndim), INTENT(IN) :: mat_jk
1222   REAL(KIND=8), DIMENSION(0:ndim), INTENT(OUT) :: res
1223   INTEGER i,j,k,n
1224
1225   res=0.d0
1226   DO i=1,ndim
1227     DO n=1,coolist_ijk(i)%nelems
1228       j=coolist_ijk(i)%elems(n)%j
1229       k=coolist_ijk(i)%elems(n)%k
1230
1231       res(i) = res(i) + coolist_ijk(i)%elems(n)%v * mat_jk(j,k)
1232     ENDDO
1233   END DO
1234

```

**8.24.2.32 subroutine, public tensor::sparse\_mul4 ( type(coolist4), dimension(ndim), intent(in) coolist\_ijkl, real(kind=8), dimension(0:ndim), intent(in) arr\_j, real(kind=8), dimension(0:ndim), intent(in) arr\_k, real(kind=8), dimension(0:ndim), intent(in) arr\_l, real(kind=8), dimension(0:ndim), intent(out) res )**

Sparse multiplication of a rank-4 tensor coolist with three vectors:  $\sum_{j,k,l=0}^{ndim} \mathcal{T}_{i,j,k,l} a_j b_k c_l$ .

#### Parameters

<i>coolist_ijkl</i>	a coolist (sparse tensor) of which index j, k and l will be contracted.
<i>arr_j</i>	the vector to be contracted with index j of coolist_ijkl
<i>arr_k</i>	the vector to be contracted with index k of coolist_ijkl
<i>arr_l</i>	the vector to be contracted with index l of coolist_ijkl
<i>res</i>	vector (buffer) to store the result of the contraction

#### Remarks

Note that it is NOT safe to pass *arr\_j/arr\_k/arr\_l* as a result buffer, as this operation does multiple passes.

Definition at line 974 of file tensor.f90.

```

974   TYPE(coolist4), DIMENSION(ndim), INTENT(IN):: coolist_ijkl
975   REAL(KIND=8), DIMENSION(0:ndim), INTENT(IN) :: arr_j, arr_k, arr_l
976   REAL(KIND=8), DIMENSION(0:ndim), INTENT(OUT) :: res
977   INTEGER :: i,j,k,n,l
978   res=0.d0
979   DO i=1,ndim
980     DO n=1,coolist_ijkl(i)%nelems
981       j=coolist_ijkl(i)%elems(n)%j
982       k=coolist_ijkl(i)%elems(n)%k
983       l=coolist_ijkl(i)%elems(n)%l
984       res(i) = res(i) + coolist_ijkl(i)%elems(n)%v * arr_j(j)*arr_k(k)*arr_l(l)
985     END DO
986   END DO

```

**8.24.2.33 subroutine, public tensor::sparse\_mul4\_mat ( type(coolist4), dimension(ndim), intent(in) coolist\_ijkl, real(kind=8), dimension(0:ndim), intent(in) arr\_k, real(kind=8), dimension(0:ndim), intent(in) arr\_l, real(kind=8), dimension(ndim,ndim), intent(out) res )**

Sparse multiplication of a tensor with two vectors:  $\sum_{k,l=0}^{ndim} \mathcal{T}_{i,j,k,l} b_k c_l$ .

### Parameters

<i>coolist_ijkl</i>	a coordinate list (sparse tensor) of which index 3 and 4 will be contracted.
<i>arr_k</i>	the vector to be contracted with index 3 of coolist_ijkl
<i>arr_l</i>	the vector to be contracted with index 4 of coolist_ijkl
<i>res</i>	matrix (buffer) to store the result of the contraction

### Remarks

Note that it is NOT safe to pass *arr\_k/arr\_l* as a result buffer, as this operation does multiple passes.

Definition at line 998 of file tensor.f90.

```

998      TYPE(coolist4), DIMENSION(ndim), INTENT(IN):: coolist_ijkl
999      REAL(KIND=8), DIMENSION(0:ndim), INTENT(IN) :: arr_k, arr_l
1000     REAL(KIND=8), DIMENSION(ndim,ndim), INTENT(OUT) :: res
1001     INTEGER :: i,j,k,n,l
1002     res=0.d0
1003     DO i=1,ndim
1004       DO n=1,coolist_ijkl(i)%nelems
1005         j=coolist_ijkl(i)%elems(n)%j
1006         IF (j /= 0) THEN
1007           k=coolist_ijkl(i)%elems(n)%k
1008           l=coolist_ijkl(i)%elems(n)%l
1009           res(i,j) = res(i,j) + coolist_ijkl(i)%elems(n)%v * arr_k(k) * arr_l(l)
1010         ENDIF
1011       END DO
1012     END DO

```

**8.24.2.34 subroutine, public tensor::sparse\_mul4\_with\_mat\_jl ( type(coolist4), dimension(ndim), intent(in) *coolist\_ijkl*, real(kind=8), dimension(ndim,ndim), intent(in) *mat\_jl*, real(kind=8), dimension(ndim,ndim), intent(out) *res* )**

Sparse multiplication of a rank-4 tensor coolist with a matrix :  $\sum_{j,l=0}^{ndim} T_{i,j,k,l} m_{j,l}$ .

### Parameters

<i>coolist_ijkl</i>	a coolist (sparse tensor) of which index j and l will be contracted.
<i>mat_jl</i>	the matrix to be contracted with indices j and l of coolist_ijkl
<i>res</i>	matrix (buffer) to store the result of the contraction

### Remarks

Note that it is NOT safe to pass *mat\_jl* as a result buffer, as this operation does multiple passes.

Definition at line 1169 of file tensor.f90.

```

1169      TYPE(coolist4), DIMENSION(ndim), INTENT(IN):: coolist_ijkl
1170      REAL(KIND=8), DIMENSION(ndim,ndim), INTENT(IN) :: mat_jl
1171      REAL(KIND=8), DIMENSION(ndim,ndim), INTENT(OUT) :: res
1172      INTEGER i,j,k,l,n
1173
1174      res=0.d0
1175      DO i=1,ndim
1176        DO n=1,coolist_ijkl(i)%nelems
1177          j=coolist_ijkl(i)%elems(n)%j

```

```

1178      k=coolist_ijkl(i)%elems(n)%k
1179      l=coolist_ijkl(i)%elems(n)%l
1180
1181      res(i,k) = res(i,k) + coolist_ijkl(i)%elems(n)%v * mat_jl(j,l)
1182    ENDDO
1183  END DO
1184

```

**8.24.2.35 subroutine, public tensor::sparse\_mul4\_with\_mat\_kl ( type(coolist4), dimension(ndim), intent(in) coolist\_ijkl, real(kind=8), dimension(ndim,ndim), intent(in) mat\_kl, real(kind=8), dimension(ndim,ndim), intent(out) res )**

Sparse multiplication of a rank-4 tensor coolist with a matrix : 
$$\sum_{j,l=0}^{ndim} T_{i,j,k,l} m_{k,l}.$$

#### Parameters

<i>coolist_ijkl</i>	a coolist (sparse tensor) of which index k and l will be contracted.
<i>mat_kl</i>	the matrix to be contracted with indices k and l of coolist_ijkl
<i>res</i>	matrix (buffer) to store the result of the contraction

#### Remarks

Note that it is NOT safe to pass *mat\_kl* as a result buffer, as this operation does multiple passes.

Definition at line 1194 of file tensor.f90.

```

1194  TYPE(coolist4), DIMENSION(ndim), INTENT(IN):: coolist_ijkl
1195  REAL(KIND=8), DIMENSION(ndim,ndim), INTENT(IN) :: mat_kl
1196  REAL(KIND=8), DIMENSION(ndim,ndim), INTENT(OUT) :: res
1197  INTEGER i,j,k,l,n
1198
1199  res=0.d0
1200  DO i=1,ndim
1201    DO n=1,coolist_ijkl(i)%nelems
1202      j=coolist_ijkl(i)%elems(n)%j
1203      k=coolist_ijkl(i)%elems(n)%k
1204      l=coolist_ijkl(i)%elems(n)%l
1205
1206      res(i,j) = res(i,j) + coolist_ijkl(i)%elems(n)%v * mat_kl(k,l)
1207    ENDDO
1208  END DO
1209

```

**8.24.2.36 logical function, public tensor::tensor4\_empty ( type(coolist4), dimension(ndim), intent(in) t )**

Test if a rank-4 tensor coolist is empty.

#### Parameters

<i>t</i>	rank-4 tensor coolist to be tested
----------	------------------------------------

Definition at line 1304 of file tensor.f90.

```
1304  TYPE(coolist4), DIMENSION(ndim), INTENT(IN) :: t
```

```

1305      LOGICAL :: tensor4_empty
1306      INTEGER :: i
1307      tensor4_empty=.true.
1308      DO i=1,ndim
1309          IF (t(i)%nelems /= 0) THEN
1310              tensor4_empty=.false.
1311          RETURN
1312      ENDIF
1313  END DO
1314  RETURN

```

#### 8.24.2.37 subroutine, public tensor::tensor4\_to\_coo4 ( real(kind=8), dimension(ndim,0:ndim,0:ndim,0:ndim), intent(in) src, type(coolist4), dimension(ndim), intent(out) dst )

Routine to convert a rank-4 tensor from matrix to coolist representation.

##### Parameters

<i>src</i>	Source matrix
<i>dst</i>	Destination coolist

##### Remarks

The destination coolist have to be an empty one, i.e. with unallocated list of elements and nelems set to 0.

Definition at line 883 of file tensor.f90.

```

883      REAL(KIND=8), DIMENSION(ndim,0:ndim,0:ndim,0:ndim), INTENT(IN) :: src
884      TYPE(coolist4), DIMENSION(ndim), INTENT(OUT) :: dst
885      INTEGER :: i,j,k,l,n,allocstat
886
887      DO i=1,ndim
888          n=0
889          DO j=0,ndim
890              DO k=0,ndim
891                  DO l=0,ndim
892                      IF (abs(src(i,j,k,l))>real_eps) n=n+1
893                  ENDDO
894              ENDDO
895          IF (n/=0) THEN
896              IF (dst(i)%nelems/=0) stop "*** tensor_to_coo : Destination coolist not empty ! ***"
897              ALLOCATE(dst(i)%elems(n), stat=allocstat)
898              IF (allocstat /= 0) stop "*** Not enough memory ! ***"
899              n=0
900              DO j=0,ndim
901                  DO k=0,ndim
902                      DO l=0,ndim
903                          IF (abs(src(i,j,k,l))>real_eps) THEN
904                              n=n+1
905                              dst(i)%elems(n)%j=j
906                              dst(i)%elems(n)%k=k
907                              dst(i)%elems(n)%l=l
908                              dst(i)%elems(n)%v=src(i,j,k,l)
909                          ENDIF
910                      ENDDO
911                  ENDDO
912              ENDDO
913          ENDIF
914          dst(i)%nelems=n
915      ENDDO

```

#### 8.24.2.38 logical function, public tensor::tensor\_empty ( type(coolist), dimension(ndim), intent(in) t )

Test if a rank-3 tensor coolist is empty.

## Parameters

<i>t</i>	rank-3 tensor coolist to be tested
----------	------------------------------------

Definition at line 1288 of file tensor.f90.

```

1288      TYPE(coolist), DIMENSION(ndim), INTENT(IN) :: t
1289      LOGICAL :: tensor_empty
1290      INTEGER :: i
1291      tensor_empty=.true.
1292      DO i=1,ndim
1293         IF (t(i)%nelems /= 0) THEN
1294            tensor_empty=.false.
1295            RETURN
1296         ENDIF
1297      END DO
1298      RETURN

```

**8.24.2.39 subroutine, public tensor::tensor\_to\_coo ( real(kind=8), dimension(ndim,0:ndim,0:ndim), intent(in) *src*, type(coolist), dimension(ndim), intent(out) *dst* )**

Routine to convert a rank-3 tensor from matrix to coolist representation.

## Parameters

<i>src</i>	Source matrix
<i>dst</i>	Destination coolist

## Remarks

The destination coolist have to be an empty one, i.e. with unallocated list of elements and nelems set to 0.

Definition at line 847 of file tensor.f90.

```

847      REAL(KIND=8), DIMENSION(ndim,0:ndim,0:ndim), INTENT(IN) :: src
848      TYPE(coolist), DIMENSION(ndim), INTENT(OUT) :: dst
849      INTEGER :: i,j,k,n,allocstat
850
851      DO i=1,ndim
852         n=0
853         DO j=0,ndim
854            DO k=0,ndim
855               IF (abs(src(i,j,k))>real_eps) n=n+1
856            ENDDO
857         ENDDO
858         IF (n/=0) THEN
859            IF (dst(i)%nelems/=0) stop "*** tensor_to_coo : Destination coolist not empty ! ***"
860            ALLOCATE(dst(i)%elems(n), stat=allocstat)
861            IF (allocstat /= 0) stop "*** Not enough memory ! ***"
862            n=0
863            DO j=0,ndim
864               DO k=0,ndim
865                  IF (abs(src(i,j,k))>real_eps) THEN
866                     n=n+1
867                     dst(i)%elems(n)%j=j
868                     dst(i)%elems(n)%k=k
869                     dst(i)%elems(n)%v=src(i,j,k)
870                  ENDIF
871               ENDDO
872            ENDDO
873            IF (dst(i)%nelems<n) stop "*** tensor_to_coo : Destination coolist not empty ! ***"
874            dst(i)%nelems=n
875         ENDDO

```

**8.24.2.40 subroutine, public tensor::write\_tensor4\_to\_file ( character (len=\*), intent(in) s, type(coolist4), dimension(ndim), intent(in) t )**

Load a rank-4 tensor coolist from a file definition.

#### Parameters

s	Destination filename
t	The coolist to write

Definition at line 1349 of file tensor.f90.

```

1349      CHARACTER (LEN=*), INTENT(IN) :: s
1350      TYPE(coolist4), DIMENSION(ndim), INTENT(IN) :: t
1351      INTEGER :: i,j,k,l,n
1352      OPEN(30,file=s)
1353      DO i=1,ndim
1354        WRITE(30,*) i,t(i)%nelems
1355        DO n=1,t(i)%nelems
1356          j=t(i)%elems(n)%j
1357          k=t(i)%elems(n)%k
1358          l=t(i)%elems(n)%l
1359          WRITE(30,*) i,j,k,l,t(i)%elems(n)%v
1360        END DO
1361      END DO
1362      CLOSE(30)

```

**8.24.2.41 subroutine, public tensor::write\_tensor\_to\_file ( character (len=\*), intent(in) s, type(coolist), dimension(ndim), intent(in) t )**

Load a rank-4 tensor coolist from a file definition.

#### Parameters

s	Destination filename
t	The coolist to write

Definition at line 425 of file tensor.f90.

```

425      CHARACTER (LEN=*), INTENT(IN) :: s
426      TYPE(coolist), DIMENSION(ndim), INTENT(IN) :: t
427      INTEGER :: i,j,k,n
428      OPEN(30,file=s)
429      DO i=1,ndim
430        WRITE(30,*) i,t(i)%nelems
431        DO n=1,t(i)%nelems
432          j=t(i)%elems(n)%j
433          k=t(i)%elems(n)%k
434          WRITE(30,*) i,j,k,t(i)%elems(n)%v
435        END DO
436      END DO
437      CLOSE(30)

```

## 8.24.3 Variable Documentation

**8.24.3.1 real(kind=8), parameter tensor::real\_eps = 2.2204460492503131e-16**

Parameter to test the equality with zero.

Definition at line 50 of file tensor.f90.

```
50  REAL(KIND=8), PARAMETER :: real_eps = 2.2204460492503131e-16
```

## 8.25 tl\_ad\_integrator Module Reference

Tangent Linear (TL) and Adjoint (AD) model versions of MAOOAM. Integrators module.

### Functions/Subroutines

- subroutine, public [init\\_tl\\_ad\\_integrator](#)  
*Routine to initialise the integration buffers.*
- subroutine, public [ad\\_step](#) (y, ystar, t, dt, res)  
*Routine to perform an integration step (Heun algorithm) of the adjoint model. The incremented time is returned.*
- subroutine, public [tl\\_step](#) (y, ystar, t, dt, res)  
*Routine to perform an integration step (Heun algorithm) of the tangent linear model. The incremented time is returned.*

### Variables

- real(kind=8), dimension(:), allocatable [buf\\_y1](#)  
*Buffer to hold the intermediate position (Heun algorithm) of the tangent linear model.*
- real(kind=8), dimension(:), allocatable [buf\\_f0](#)  
*Buffer to hold tendencies at the initial position of the tangent linear model.*
- real(kind=8), dimension(:), allocatable [buf\\_f1](#)  
*Buffer to hold tendencies at the intermediate position of the tangent linear model.*
- real(kind=8), dimension(:), allocatable [buf\\_ka](#)  
*Buffer to hold tendencies in the RK4 scheme for the tangent linear model.*
- real(kind=8), dimension(:), allocatable [buf\\_kb](#)  
*Buffer to hold tendencies in the RK4 scheme for the tangent linear model.*

### 8.25.1 Detailed Description

Tangent Linear (TL) and Adjoint (AD) model versions of MAOOAM. Integrators module.

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#### Remarks

This module actually contains the Heun algorithm routines. The user can modify it according to its preferred integration scheme. For higher-order schemes, additional buffers will probably have to be defined.

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#### Remarks

This module actually contains the RK4 algorithm routines. The user can modify it according to its preferred integration scheme. For higher-order schemes, additional buffers will probably have to be defined.

### 8.25.2 Function/Subroutine Documentation

#### 8.25.2.1 subroutine public tl\_ad\_integrator::ad\_step ( real(kind=8), dimension(0:ndim), intent(in) y, real(kind=8), dimension(0:ndim), intent(in) ystar, real(kind=8), intent(inout) t, real(kind=8), intent(in) dt, real(kind=8), dimension(0:ndim), intent(out) res )

Routine to perform an integration step (Heun algorithm) of the adjoint model. The incremented time is returned.

Routine to perform an integration step (RK4 algorithm) of the adjoint model. The incremented time is returned.

#### Parameters

<i>y</i>	Initial point.
<i>ystar</i>	Adjoint model at the point <i>ystar</i> .
<i>t</i>	Actual integration time
<i>dt</i>	Integration timestep.
<i>res</i>	Final point after the step.

Definition at line 61 of file rk2\_tl\_ad\_integrator.f90.

```

61      REAL(KIND=8), DIMENSION(0:ndim), INTENT(IN) :: y,ystar
62      REAL(KIND=8), INTENT(INOUT) :: t
63      REAL(KIND=8), INTENT(IN) :: dt
64      REAL(KIND=8), DIMENSION(0:ndim), INTENT(OUT) :: res
65
66      CALL ad(t,ystar,y,buf_f0)
67      buf_y1 = y+dt*buf_f0
68      CALL ad(t+dt,ystar,buf_y1,buf_f1)
69      res=y+0.5*(buf_f0+buf_f1)*dt
70      t=t+dt

```

#### 8.25.2.2 subroutine public tl\_ad\_integrator::init\_tl\_ad\_integrator( )

Routine to initialise the integration buffers.

Routine to initialise the TL-AD integration buffers.

Definition at line 41 of file rk2\_tl\_ad\_integrator.f90.

```

41      INTEGER :: allocstat
42      ALLOCATE(buf_y1(0:ndim),buf_f0(0:ndim),buf_f1(0:ndim),stat=allocstat)
43      IF (allocstat /= 0) stop "*** Not enough memory ! ***"

```

#### 8.25.2.3 subroutine public tl\_ad\_integrator::tl\_step ( real(kind=8), dimension(0:ndim), intent(in) *y*, real(kind=8), dimension(0:ndim), intent(in) *ystar*, real(kind=8), intent(inout) *t*, real(kind=8), intent(in) *dt*, real(kind=8), dimension(0:ndim), intent(out) *res* )

Routine to perform an integration step (Heun algorithm) of the tangent linear model. The incremented time is returned.

Routine to perform an integration step (RK4 algorithm) of the tangent linear model. The incremented time is returned.

#### Parameters

<i>y</i>	Initial point.
<i>ystar</i>	Adjoint model at the point <i>ystar</i> .
<i>t</i>	Actual integration time
<i>dt</i>	Integration timestep.
<i>res</i>	Final point after the step.

Definition at line 86 of file rk2\_tl\_ad\_integrator.f90.

```

86      REAL(KIND=8), DIMENSION(0:ndim), INTENT(IN) :: y,ystar
87      REAL(KIND=8), INTENT(INOUT) :: t
88      REAL(KIND=8), INTENT(IN) :: dt
89      REAL(KIND=8), DIMENSION(0:ndim), INTENT(OUT) :: res
90
91      CALL tl(t,ystar,y,buf_f0)
92      buf_y1 = y+dt*buf_f0
93      CALL tl(t+dt,ystar,buf_y1,buf_f1)
94      res=y+0.5*(buf_f0+buf_f1)*dt
95      t=t+dt

```

### 8.25.3 Variable Documentation

#### 8.25.3.1 real(kind=8), dimension(:), allocatable tl\_ad\_integrator::buf\_f0 [private]

Buffer to hold tendencies at the initial position of the tangent linear model.

Definition at line 31 of file rk2\_tl\_ad\_integrator.f90.

```

31      REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: buf_f0 !< Buffer to hold tendencies at the initial position
                                                 of the tangent linear model

```

#### 8.25.3.2 real(kind=8), dimension(:), allocatable tl\_ad\_integrator::buf\_f1 [private]

Buffer to hold tendencies at the intermediate position of the tangent linear model.

Definition at line 32 of file rk2\_tl\_ad\_integrator.f90.

```

32      REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: buf_f1 !< Buffer to hold tendencies at the intermediate
                                                 position of the tangent linear model

```

#### 8.25.3.3 real(kind=8), dimension(:), allocatable tl\_ad\_integrator::buf\_ka [private]

Buffer to hold tendencies in the RK4 scheme for the tangent linear model.

Definition at line 33 of file rk4\_tl\_ad\_integrator.f90.

```

33      REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: buf_ka !< Buffer to hold tendencies in the RK4 scheme for the
                                                 tangent linear model

```

#### 8.25.3.4 real(kind=8), dimension(:), allocatable tl\_ad\_integrator::buf\_kb [private]

Buffer to hold tendencies in the RK4 scheme for the tangent linear model.

Definition at line 34 of file rk4\_tl\_ad\_integrator.f90.

```

34      REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: buf_kb !< Buffer to hold tendencies in the RK4 scheme for the
                                                 tangent linear model

```

### 8.25.3.5 real(kind=8), dimension(:), allocatable tl\_ad\_integrator::buf\_y1 [private]

Buffer to hold the intermediate position (Heun algorithm) of the tangent linear model.

Buffer to hold the intermediate position of the tangent linear model.

Definition at line 30 of file rk2\_tl\_ad\_integrator.f90.

```
30  REAL(KIND=8), DIMENSION(:), ALLOCATABLE :: buf_y1 !< Buffer to hold the intermediate position (Heun
   algorithm) of the tangent linear model
```

## 8.26 tl\_ad\_tensor Module Reference

Tangent Linear (TL) and Adjoint (AD) model versions of MAOOAM. Tensors definition module.

### Functions/Subroutines

- type(coolist) function, dimension(ndim) **jacobian** (ystar)
 

*Compute the Jacobian of MAOOAM in point ystar.*
- real(kind=8) function, dimension(ndim, ndim), public **jacobian\_mat** (ystar)
 

*Compute the Jacobian of MAOOAM in point ystar.*
- subroutine, public **init\_tltensor**

*Routine to initialize the TL tensor.*
- subroutine **compute\_tltensor** (func)
 

*Routine to compute the TL tensor from the original MAOOAM one.*
- subroutine **tl\_add\_count** (i, j, k, v)
 

*Subroutine used to count the number of TL tensor entries.*
- subroutine **tl\_coeff** (i, j, k, v)
 

*Subroutine used to compute the TL tensor entries.*
- subroutine, public **init\_adtensor**

*Routine to initialize the AD tensor.*
- subroutine **compute\_adtensor** (func)
 

*Subroutine to compute the AD tensor from the original MAOOAM one.*
- subroutine **ad\_add\_count** (i, j, k, v)
 

*Subroutine used to count the number of AD tensor entries.*
- subroutine **ad\_coeff** (i, j, k, v)
 

*Subroutine used to compute the AD tensor entries from the TL tensor.*
- subroutine, public **init\_adtensor\_ref**

*Alternate method to initialize the AD tensor from the TL tensor.*
- subroutine **compute\_adtensor\_ref** (func)
 

*Alternate subroutine to compute the AD tensor from the TL one.*
- subroutine **ad\_add\_count\_ref** (i, j, k, v)
 

*Alternate subroutine used to count the number of AD tensor entries from the TL tensor.*
- subroutine **ad\_coeff\_ref** (i, j, k, v)
 

*Alternate subroutine used to compute the AD tensor entries from the TL tensor.*
- subroutine, public **ad** (t, ystar, delтай, buf)
 

*Tendencies for the AD of MAOOAM in point ystar for perturbation delтай.*
- subroutine, public **tl** (t, ystar, delтай, buf)
 

*Tendencies for the TL of MAOOAM in point ystar for perturbation delтай.*

## Variables

- real(kind=8), parameter `real_eps` = 2.2204460492503131e-16  
*Epsilon to test equality with 0.*
- integer, dimension(:), allocatable `count_elems`  
*Vector used to count the tensor elements.*
- type(`coolist`), dimension(:), allocatable, public `tltensor`  
*Tensor representation of the Tangent Linear tendencies.*
- type(`coolist`), dimension(:), allocatable, public `adtensor`  
*Tensor representation of the Adjoint tendencies.*

### 8.26.1 Detailed Description

Tangent Linear (TL) and Adjoint (AD) model versions of MAOOAM. Tensors definition module.

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#### Remarks

The routines of this module should be called only after `params::init_params()` and `aotensor_def::init_← aotensor()` have been called !

### 8.26.2 Function/Subroutine Documentation

#### 8.26.2.1 subroutine, public `tl_ad_tensor::ad` ( `real(kind=8), intent(in) t, real(kind=8), dimension(0:ndim), intent(in) ystar,` `real(kind=8), dimension(0:ndim), intent(in) deltay, real(kind=8), dimension(0:ndim), intent(out) buf` )

Tendencies for the AD of MAOOAM in point `ystar` for perturbation `deltay`.

#### Parameters

<code>t</code>	time
<code>ystar</code>	vector with the variables (current point in trajectory)
<code>deltay</code>	vector with the perturbation of the variables at time <code>t</code>
<code>buf</code>	vector (buffer) to store derivatives.

Definition at line 384 of file `tl_ad_tensor.f90`.

```
384      REAL (KIND=8), INTENT(IN) :: t
385      REAL (KIND=8), DIMENSION(0:ndim), INTENT(IN) :: ystar,deltay
386      REAL (KIND=8), DIMENSION(0:ndim), INTENT(OUT) :: buf
387      CALL sparse_mul3(adtensor,deltay,ystar,buf)
```

#### 8.26.2.2 subroutine `tl_ad_tensor::ad_add_count` ( `integer, intent(in) i, integer, intent(in) j, integer, intent(in) k, real(kind=8), intent(in) v` ) [private]

Subroutine used to count the number of AD tensor entries.

### Parameters

<i>i</i>	tensor <i>i</i> index
<i>j</i>	tensor <i>j</i> index
<i>k</i>	tensor <i>k</i> index
<i>v</i>	value that will be added

Definition at line 243 of file tl\_ad\_tensor.f90.

```
243      INTEGER, INTENT(IN) :: i,j,k
244      REAL(KIND=8), INTENT(IN) :: v
245      IF ((abs(v) .ge. real_eps).AND.(i /= 0)) THEN
246          IF (k /= 0) count_elems(k)=count_elems(k)+1
247          IF (j /= 0) count_elems(j)=count_elems(j)+1
248      ENDIF
```

### 8.26.2.3 subroutine tl\_ad\_tensor::ad\_add\_count\_ref ( integer, intent(in) *i*, integer, intent(in) *j*, integer, intent(in) *k*, real(kind=8), intent(in) *v* ) [private]

Alternate subroutine used to count the number of AD tensor entries from the TL tensor.

### Parameters

<i>i</i>	tensor <i>i</i> index
<i>j</i>	tensor <i>j</i> index
<i>k</i>	tensor <i>k</i> index
<i>v</i>	value that will be added

Definition at line 346 of file tl\_ad\_tensor.f90.

```
346      INTEGER, INTENT(IN) :: i,j,k
347      REAL(KIND=8), INTENT(IN) :: v
348      IF ((abs(v) .ge. real_eps).AND.(j /= 0)) count_elems(j)=count_elems(j)+1
```

### 8.26.2.4 subroutine tl\_ad\_tensor::ad\_coeff ( integer, intent(in) *i*, integer, intent(in) *j*, integer, intent(in) *k*, real(kind=8), intent(in) *v* ) [private]

### Parameters

<i>i</i>	tensor <i>i</i> index
<i>j</i>	tensor <i>j</i> index
<i>k</i>	tensor <i>k</i> index
<i>v</i>	value to add

Definition at line 257 of file tl\_ad\_tensor.f90.

```
257      INTEGER, INTENT(IN) :: i,j,k
258      REAL(KIND=8), INTENT(IN) :: v
259      INTEGER :: n
```

```

260      IF (.NOT. ALLOCATED(adtensor)) stop "*** ad_coeff routine : tensor not yet allocated ***"
261      IF ((abs(v) .ge. real_eps).AND.(i /=0)) THEN
262          IF (k /=0) THEN
263              IF (.NOT. ALLOCATED(adtensor(k)%elems)) stop "*** ad_coeff routine : tensor not yet allocated
264                  ***
265                  n=(adtensor(k)%nelems)+1
266                  adtensor(k)%elems(n)%j=i
267                  adtensor(k)%elems(n)%k=j
268                  adtensor(k)%elems(n)%v=v
269                  adtensor(k)%nelems=n
270          END IF
271          IF (j /=0) THEN
272              IF (.NOT. ALLOCATED(adtensor(j)%elems)) stop "*** ad_coeff routine : tensor not yet allocated
273                  ***
274                  n=(adtensor(j)%nelems)+1
275                  adtensor(j)%elems(n)%j=i
276                  adtensor(j)%elems(n)%k=k
277                  adtensor(j)%elems(n)%v=v
278                  adtensor(j)%nelems=n
277      END IF
278  END IF

```

### 8.26.2.5 subroutine tl\_ad\_tensor::ad\_coeff\_ref ( integer, intent(in) i, integer, intent(in) j, integer, intent(in) k, real(kind=8), intent(in) v ) [private]

Alternate subroutine used to compute the AD tensor entries from the TL tensor.

#### Parameters

<i>i</i>	tensor <i>i</i> index
<i>j</i>	tensor <i>j</i> index
<i>k</i>	tensor <i>k</i> index
<i>v</i>	value to add

Definition at line 358 of file tl\_ad\_tensor.f90.

```

358      INTEGER, INTENT(IN) :: i,j,k
359      REAL(KIND=8), INTENT(IN) :: v
360      INTEGER :: n
361      IF (.NOT. ALLOCATED(adtensor)) stop "*** ad_coeff_ref routine : tensor not yet allocated ***"
362      IF ((abs(v) .ge. real_eps).AND.(j /=0)) THEN
363          IF (.NOT. ALLOCATED(adtensor(j)%elems)) stop "*** ad_coeff_ref routine : tensor not yet allocated
364          ***
365          n=(adtensor(j)%nelems)+1
366          adtensor(j)%elems(n)%j=i
367          adtensor(j)%elems(n)%k=k
368          adtensor(j)%elems(n)%v=v
369          adtensor(j)%nelems=n
369      END IF

```

### 8.26.2.6 subroutine tl\_ad\_tensor::compute\_adtensor ( external func ) [private]

Subroutine to compute the AD tensor from the original MAOOAM one.

#### Parameters

<i>func</i>	subroutine used to do the computation
-------------	---------------------------------------

Definition at line 217 of file tl\_ad\_tensor.f90.

### 8.26.2.7 subroutine tl\_ad\_tensor::compute\_adtensor\_ref ( external *func* ) [private]

Alternate subroutine to compute the AD tensor from the TL one.

#### Parameters

<i>func</i>	subroutine used to do the computation
-------------	---------------------------------------

Definition at line 318 of file tl\_ad\_tensor.f90.

### 8.26.2.8 subroutine tl\_ad\_tensor::compute\_tltensor ( external *func* ) [private]

Routine to compute the TL tensor from the original MAOOAM one.

#### Parameters

<i>func</i>	subroutine used to do the computation
-------------	---------------------------------------

Definition at line 121 of file tl\_ad\_tensor.f90.

### 8.26.2.9 subroutine, public tl\_ad\_tensor::init\_adtensor ( )

Routine to initialize the AD tensor.

Definition at line 193 of file tl\_ad\_tensor.f90.

```

193      INTEGER :: i
194      INTEGER :: allocstat
195      ALLOCATE(adtensor(ndim),count_elems(ndim), stat=allocstat)
196      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
197      count_elems=0
198      CALL compute_adtensor(ad_add_count)
199
200      DO i=1,ndim
201          ALLOCATE(adtensor(i)%elems(count_elems(i)), stat=allocstat)
202          IF (allocstat /= 0) stop "*** Not enough memory ! ***"
203      END DO
204
205      DEALLOCATE(count_elems, stat=allocstat)
206      IF (allocstat /= 0) stop "*** Deallocation problem ! ***"
207
208      CALL compute_adtensor(ad_coeff)
209
210      CALL simplify(adtensor)
211

```

### 8.26.2.10 subroutine, public tl\_ad\_tensor::init\_adtensor\_ref ( )

Alternate method to initialize the AD tensor from the TL tensor.

## Remarks

The `tltensor` must be initialised before using this method.

Definition at line 294 of file `tl_ad_tensor.f90`.

```

294      INTEGER :: i
295      INTEGER :: allocstat
296      ALLOCATE(adtensor(ndim),count_elems(ndim), stat=allocstat)
297      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
298      count_elems=0
299      CALL compute_adtensor_ref(ad_add_count_ref)
300
301      DO i=1,ndim
302          ALLOCATE(adtensor(i)%elems(count_elems(i)), stat=allocstat)
303          IF (allocstat /= 0) stop "*** Not enough memory ! ***"
304      END DO
305
306      DEALLOCATE(count_elems, stat=allocstat)
307      IF (allocstat /= 0) stop "*** Deallocation problem ! ***"
308
309      CALL compute_adtensor_ref(ad_coeff_ref)
310
311      CALL simplify(adtensor)
312

```

### 8.26.2.11 subroutine, public `tl_ad_tensor::init_tltensor( )`

Routine to initialize the TL tensor.

Definition at line 97 of file `tl_ad_tensor.f90`.

```

97      INTEGER :: i
98      INTEGER :: allocstat
99      ALLOCATE(tltensor(ndim),count_elems(ndim), stat=allocstat)
100     IF (allocstat /= 0) stop "*** Not enough memory ! ***"
101     count_elems=0
102     CALL compute_tltensor(tl_add_count)
103
104     DO i=1,ndim
105         ALLOCATE(tltensor(i)%elems(count_elems(i)), stat=allocstat)
106         IF (allocstat /= 0) stop "*** Not enough memory ! ***"
107     END DO
108
109     DEALLOCATE(count_elems, stat=allocstat)
110     IF (allocstat /= 0) stop "*** Deallocation problem ! ***"
111
112     CALL compute_tltensor(tl_coeff)
113
114     CALL simplify(tltensor)
115

```

### 8.26.2.12 type(coolist) function, dimension(ndim) `tl_ad_tensor::jacobian( real(kind=8), dimension(0:ndim), intent(in) ystar ) [private]`

Compute the Jacobian of MAOOAM in point `ystar`.

#### Parameters

<code>ystar</code>	array with variables in which the jacobian should be evaluated.
--------------------	---

**Returns**

Jacobian in coolist-form (table of tuples {i,j,0,value})

Definition at line 75 of file tl\_ad\_tensor.f90.

```
75      REAL(KIND=8), DIMENSION(0:ndim), INTENT(IN) :: ystar
76      TYPE(coolist), DIMENSION(ndim) :: jacobian
77      CALL jsparse_mul(aotensor,ystar,jacobian)
```

### 8.26.2.13 real(kind=8) function, dimension(ndim,ndim), public tl\_ad\_tensor::jacobian\_mat ( real(kind=8), dimension(0:ndim), intent(in) ystar )

Compute the Jacobian of MAOOAM in point ystar.

**Parameters**

<i>ystar</i>	array with variables in which the jacobian should be evaluated.
--------------	---

**Returns**

Jacobian in matrix form

Definition at line 84 of file tl\_ad\_tensor.f90.

```
84      REAL(KIND=8), DIMENSION(0:ndim), INTENT(IN) :: ystar
85      REAL(KIND=8), DIMENSION(ndim,ndim) :: jacobian_mat
86      CALL jsparse_mul_mat(aotensor,ystar,jacobian_mat)
```

### 8.26.2.14 subroutine, public tl\_ad\_tensor::tl ( real(kind=8), intent(in) t, real(kind=8), dimension(0:ndim), intent(in) ystar, real(kind=8), dimension(0:ndim), intent(in) deltay, real(kind=8), dimension(0:ndim), intent(out) buf )

Tendencies for the TL of MAOOAM in point ystar for perturbation deltay.

**Parameters**

<i>t</i>	time
<i>ystar</i>	vector with the variables (current point in trajectory)
<i>deltay</i>	vector with the perturbation of the variables at time t
<i>buf</i>	vector (buffer) to store derivatives.

Definition at line 396 of file tl\_ad\_tensor.f90.

```
396      REAL(KIND=8), INTENT(IN) :: t
397      REAL(KIND=8), DIMENSION(0:ndim), INTENT(IN) :: ystar,deltay
398      REAL(KIND=8), DIMENSION(0:ndim), INTENT(OUT) :: buf
399      CALL sparse_mul3(tl_tensor,deltay,ystar,buf)
```

---

8.26.2.15 subroutine `tl_ad_tensor::tl_add_count` ( integer, intent(in)  $i$ , integer, intent(in)  $j$ , integer, intent(in)  $k$ , real(kind=8), intent(in)  $v$  ) [private]

Subroutine used to count the number of TL tensor entries.

#### Parameters

$i$	tensor $i$ index
$j$	tensor $j$ index
$k$	tensor $k$ index
$v$	value that will be added

Definition at line 147 of file `tl_ad_tensor.f90`.

```

147      INTEGER, INTENT(IN) :: i,j,k
148      REAL(KIND=8), INTENT(IN) :: v
149      IF (abs(v) .ge. real_eps) THEN
150          IF (j /= 0) count_elems(i)=count_elems(i)+1
151          IF (k /= 0) count_elems(i)=count_elems(i)+1
152      ENDIF

```

---

8.26.2.16 subroutine `tl_ad_tensor::tl_coeff` ( integer, intent(in)  $i$ , integer, intent(in)  $j$ , integer, intent(in)  $k$ , real(kind=8), intent(in)  $v$  ) [private]

Subroutine used to compute the TL tensor entries.

#### Parameters

$i$	tensor $i$ index
$j$	tensor $j$ index
$k$	tensor $k$ index
$v$	value to add

Definition at line 161 of file `tl_ad_tensor.f90`.

---

```

161      INTEGER, INTENT(IN) :: i,j,k
162      REAL(KIND=8), INTENT(IN) :: v
163      INTEGER :: n
164      IF (.NOT. ALLOCATED(tltensor)) stop "*** tl_coeff routine : tensor not yet allocated ***"
165      IF (.NOT. ALLOCATED(tltensor(i)%elems)) stop "*** tl_coeff routine : tensor not yet allocated
***"
166      IF (abs(v) .ge. real_eps) THEN
167          IF (j /= 0) THEN
168              n=(tltensor(i)%nelems)+1
169              tltensor(i)%elems(n)%j=j
170              tltensor(i)%elems(n)%k=k
171              tltensor(i)%elems(n)%v=v
172              tltensor(i)%nelems=n
173          END IF
174          IF (k /= 0) THEN
175              n=(tltensor(i)%nelems)+1
176              tltensor(i)%elems(n)%j=k
177              tltensor(i)%elems(n)%k=j
178              tltensor(i)%elems(n)%v=v
179              tltensor(i)%nelems=n
180          END IF
181      END IF

```

### 8.26.3 Variable Documentation

#### 8.26.3.1 type(coolist), dimension(:), allocatable, public tl\_ad\_tensor::adtensor

Tensor representation of the Adjoint tendencies.

Definition at line 44 of file tl\_ad\_tensor.f90.

```
44      TYPE(coolist), DIMENSION(:), ALLOCATABLE, PUBLIC :: adtensor
```

#### 8.26.3.2 integer, dimension(:), allocatable tl\_ad\_tensor::count\_elems [private]

Vector used to count the tensor elements.

Definition at line 38 of file tl\_ad\_tensor.f90.

```
38      INTEGER, DIMENSION(:), ALLOCATABLE :: count_elems
```

#### 8.26.3.3 real(kind=8), parameter tl\_ad\_tensor::real\_eps = 2.2204460492503131e-16 [private]

Epsilon to test equality with 0.

Definition at line 35 of file tl\_ad\_tensor.f90.

```
35      REAL(KIND=8), PARAMETER :: real_eps = 2.2204460492503131e-16
```

#### 8.26.3.4 type(coolist), dimension(:), allocatable, public tl\_ad\_tensor::tltensor

Tensor representation of the Tangent Linear tendencies.

Definition at line 41 of file tl\_ad\_tensor.f90.

```
41      TYPE(coolist), DIMENSION(:), ALLOCATABLE, PUBLIC :: tltensor
```

## 8.27 util Module Reference

Utility module.

## Functions/Subroutines

- character(len=20) function, public `str` (k)  
*Convert an integer to string.*
- character(len=40) function, public `rstr` (x, fm)  
*Convert a real to string with a given format.*
- integer function, dimension(size(s)), public `isin` (c, s)  
*Determine if a character is in a string and where.*
- subroutine, public `init_random_seed` ()  
*Random generator initialization routine.*
- subroutine, public `piksrt` (k, arr, par)  
*Simple card player sorting function.*
- subroutine, public `init_one` (A)  
*Initialize a square matrix A as a unit matrix.*
- real(kind=8) function, public `mat_trace` (A)
- real(kind=8) function, public `mat_contract` (A, B)
- subroutine, public `choldc` (a, p)
- subroutine, public `printmat` (A)
- subroutine, public `cprintmat` (A)
- real(kind=8) function, dimension(size(a, 1), size(a, 2)), public `invmat` (A)
- subroutine, public `triu` (A, T)
- subroutine, public `diag` (A, d)
- subroutine, public `cdiag` (A, d)
- integer function, public `floordiv` (i, j)
- subroutine, public `reduce` (A, Ared, n, ind, rind)
- subroutine, public `ireduce` (A, Ared, n, ind, rind)
- subroutine, public `vector_outer` (u, v, A)

### 8.27.1 Detailed Description

Utility module.

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### 8.27.2 Function/Subroutine Documentation

#### 8.27.2.1 subroutine, public util::cdiag ( complex(kind=16), dimension(:,:), intent(in) A, complex(kind=16), dimension(:), intent(out) d )

Definition at line 269 of file util.f90.

```

269      COMPLEX(KIND=16), DIMENSION(:,:), INTENT(IN) :: a
270      COMPLEX(KIND=16), DIMENSION(:), INTENT(OUT) :: d
271      INTEGER :: i
272
273      DO i=1,SIZE(a,1)
274          d(i)=a(i,i)
275      END DO

```

### 8.27.2.2 subroutine, public util::choldc ( real(kind=8), dimension(:,:) a, real(kind=8), dimension(:) p )

Definition at line 176 of file util.f90.

```

176      REAL(KIND=8), DIMENSION(:,:) :: a
177      REAL(KIND=8), DIMENSION(:) :: p
178      INTEGER :: n
179      INTEGER :: i,j,k
180      REAL(KIND=8) :: sum
181      n=size(a,1)
182      DO i=1,n
183          DO j=i,n
184              sum=a(i,j)
185              DO k=i-1,1,-1
186                  sum=sum-a(i,k)*a(j,k)
187              END DO
188              IF (i.eq.j) THEN
189                  IF (sum.le.0.) stop 'choldc failed'
190                  p(i)=sqrt(sum)
191              ELSE
192                  a(j,i)=sum/p(i)
193              ENDIF
194          END DO
195      END DO
196      RETURN

```

### 8.27.2.3 subroutine, public util::cprintmat ( complex(kind=16), dimension(:,:), intent(in) A )

Definition at line 209 of file util.f90.

```

209      COMPLEX(KIND=16), DIMENSION(:,:), INTENT(IN) :: a
210      INTEGER :: i
211
212      DO i=1,SIZE(a,1)
213          print*, a(i,:)
214      END DO

```

### 8.27.2.4 subroutine, public util::diag ( real(kind=8), dimension(:, :), intent(in) A, real(kind=8), dimension(:), intent(out) d )

Definition at line 259 of file util.f90.

```

259      REAL(KIND=8), DIMENSION(:, :), INTENT(IN) :: a
260      REAL(KIND=8), DIMENSION(:), INTENT(OUT) :: d
261      INTEGER :: i
262
263      DO i=1,SIZE(a,1)
264          d(i)=a(i,i)
265      END DO

```

### 8.27.2.5 integer function, public util::floordiv ( integer i, integer j )

Definition at line 280 of file util.f90.

```

280      INTEGER :: i,j,floordiv
281      floordiv=int(floor(real(i)/real(j)))
282      RETURN

```

### 8.27.2.6 subroutine, public util::init\_one ( real(kind=8), dimension(:, :, ), intent(inout) A )

Initialize a square matrix A as a unit matrix.

Definition at line 139 of file util.f90.

```

139      REAL(KIND=8), DIMENSION(:, :, ), INTENT(INOUT) :: a
140      INTEGER :: i,n
141      n=size(a,1)
142      a=0.0d0
143      DO i=1,n
144          a(i,i)=1.0d0
145      END DO
146

```

### 8.27.2.7 subroutine, public util::init\_random\_seed ( )

Random generator initialization routine.

Definition at line 64 of file util.f90.

### 8.27.2.8 real(kind=8) function, dimension(size(a,1),size(a,2)), public util::invmat ( real(kind=8), dimension(:, :, ), intent(in) A )

Definition at line 218 of file util.f90.

```

218      REAL(KIND=8), DIMENSION(:, :, ), INTENT(IN) :: a
219      REAL(KIND=8), DIMENSION(SIZE(A, 1),SIZE(A, 2)) :: ainv
220
221      REAL(KIND=8), DIMENSION(SIZE(A,1)) :: work ! work array for LAPACK
222      INTEGER, DIMENSION(SIZE(A,1)) :: ipiv ! pivot indices
223      INTEGER :: n, info
224
225      ! Store A in Ainv to prevent it from being overwritten by LAPACK
226      ainv = a
227      n = size(a,1)
228
229      ! DGETRF computes an LU factorization of a general M-by-N matrix A
230      ! using partial pivoting with row interchanges.
231      CALL dgetrf(n, n, ainv, n, ipiv, info)
232
233      IF (info /= 0) THEN
234          stop 'Matrix is numerically singular!'
235      ENDIF
236
237      ! DGETRI computes the inverse of a matrix using the LU factorization
238      ! computed by DGETRF.
239      CALL dgetri(n, ainv, n, ipiv, work, n, info)
240
241      IF (info /= 0) THEN
242          stop 'Matrix inversion failed!'
243      ENDIF

```

### 8.27.2.9 subroutine, public util::ireduce ( real(kind=8), dimension(:, :, ), intent(out) A, real(kind=8), dimension(:, :, ), intent(in) Ared, integer, intent(in) n, integer, intent(in) ind, integer, dimension(:, ), intent(in) rind )

Definition at line 314 of file util.f90.

```

314      REAL(KIND=8), DIMENSION(:, :, ), INTENT(OUT) :: a
315      REAL(KIND=8), DIMENSION(:, :, ), INTENT(IN) :: ared
316      INTEGER, INTENT(IN) :: n
317      INTEGER, DIMENSION(:, ), INTENT(IN) :: ind,rind
318      INTEGER :: i,j
319      a=0.d0
320      DO i=1,n
321          DO j=1,n
322              a(ind(i),ind(j))=ared(i,j)
323          END DO
324      END DO

```

## 8.27.2.10 integer function, dimension(size(s)), public util::isin ( character, intent(in) c, character, dimension(:), intent(in) s )

Determine if a character is in a string and where.

**Remarks**

: return positions in a vector if found and 0 vector if not found

Definition at line 47 of file util.f90.

```

47      CHARACTER, INTENT(IN) :: c
48      CHARACTER, DIMENSION(:), INTENT(IN) :: s
49      INTEGER, DIMENSION(size(s)) :: isin
50      INTEGER :: i,j
51
52      isin=0
53      j=0
54      DO i=size(s),1,-1
55          IF (c==s(i)) THEN
56              j=j+1
57              isin(j)=i
58          END IF
59      END DO

```

## 8.27.2.11 real(kind=8) function, public util::mat\_contract ( real(kind=8), dimension(:, :) A, real(kind=8), dimension(:, :) B )

Definition at line 162 of file util.f90.

```

162      REAL(KIND=8), DIMENSION(:, :) :: a,b
163      REAL(KIND=8) :: mat_contract
164      INTEGER :: i,j,n
165      n=size(a,1)
166      mat_contract=0.d0
167      DO i=1,n
168          DO j=1,n
169              mat_contract=mat_contract+a(i,j)*b(i,j)
170          END DO
171      ENDDO
172      RETURN

```

## 8.27.2.12 real(kind=8) function, public util::mat\_trace ( real(kind=8), dimension(:, :) A )

Definition at line 150 of file util.f90.

```

150      REAL(KIND=8), DIMENSION(:, :) :: a
151      REAL(KIND=8) :: mat_trace
152      INTEGER :: i,n
153      n=size(a,1)
154      mat_trace=0.d0
155      DO i=1,n
156          mat_trace=mat_trace+a(i,i)
157      END DO
158      RETURN

```

### 8.27.2.13 subroutine, public util::piksrt ( integer, intent(in) k, integer, dimension(k), intent(inout) arr, integer, intent(out) par )

Simple card player sorting function.

Definition at line 118 of file util.f90.

```

118      INTEGER, INTENT(IN) :: k
119      INTEGER, DIMENSION(k), INTENT(INOUT) :: arr
120      INTEGER, INTENT(OUT) :: par
121      INTEGER :: i,j,a,b
122
123      par=1
124
125      DO j=2,k
126          a=arr(j)
127          DO i=j-1,1,-1
128              IF(arr(i).le.a) EXIT
129              arr(i+1)=arr(i)
130              par=-par
131          END DO
132          arr(i+1)=a
133      ENDDO
134      RETURN

```

### 8.27.2.14 subroutine, public util::printmat ( real(kind=8), dimension(:,,:), intent(in) A )

Definition at line 200 of file util.f90.

```

200      REAL(KIND=8), DIMENSION(:,,:), INTENT(IN) :: a
201      INTEGER :: i
202
203      DO i=1,SIZE(a,1)
204          print*, a(i,:)
205      ENDDO

```

### 8.27.2.15 subroutine, public util::reduce ( real(kind=8), dimension(:,,:), intent(in) A, real(kind=8), dimension(:,,:), intent(out) Ared, integer, intent(out) n, integer, dimension(:,), intent(out) ind, integer, dimension(:,), intent(out) rind )

Definition at line 286 of file util.f90.

```

286      REAL(KIND=8), DIMENSION(:,,:), INTENT(IN) :: a
287      REAL(KIND=8), DIMENSION(:,,:), INTENT(OUT) :: ared
288      INTEGER, INTENT(OUT) :: n
289      INTEGER, DIMENSION(:,), INTENT(OUT) :: ind,rind
290      LOGICAL, DIMENSION(SIZE(A,1)) :: sel
291      INTEGER :: i,j
292
293      ind=0
294      rind=0
295      sel=.false.
296      n=0
297      DO i=1,SIZE(a,1)
298          IF (any(a(i,:)/=0)) THEN
299              n=n+1
300              sel(i)=.true.
301              ind(n)=i
302              rind(i)=n
303          ENDIF
304      ENDDO
305      ared=0.d0
306      DO i=1,SIZE(a,1)
307          DO j=1,SIZE(a,1)
308              IF (sel(i).and.sel(j)) ared(rind(i),rind(j))=a(i,j)
309          ENDDO
310      ENDDO

```

### 8.27.2.16 character(len=40) function, public util::rstr ( real(kind=8), intent(in) x, character(len=20), intent(in) fm )

Convert a real to string with a given format.

Definition at line 38 of file util.f90.

```
38      REAL(KIND=8), INTENT(IN) :: x
39      CHARACTER(len=20), INTENT(IN) :: fm
40      WRITE (rstr, trim(adjustl(fm))) x
41      rstr = adjustl(rstr)
```

### 8.27.2.17 character(len=20) function, public util::str ( integer, intent(in) k )

Convert an integer to string.

Definition at line 31 of file util.f90.

```
31      INTEGER, INTENT(IN) :: k
32      WRITE (str, *) k
33      str = adjustl(str)
```

### 8.27.2.18 subroutine, public util::triu ( real(kind=8), dimension(:, :), intent(in) A, real(kind=8), dimension(:, :), intent(out) T )

Definition at line 247 of file util.f90.

```
247      REAL(KIND=8), DIMENSION(:, :, :), INTENT(IN) :: a
248      REAL(KIND=8), DIMENSION(:, :, :), INTENT(OUT) :: t
249      INTEGER i,j
250      t=0.d0
251      DO i=1,SIZE(a,1)
252          DO j=i,SIZE(a,1)
253              t(i,j)=a(i,j)
254          END DO
255      END DO
```

### 8.27.2.19 subroutine, public util::vector\_outer ( real(kind=8), dimension(:, :), intent(in) u, real(kind=8), dimension(:, :), intent(in) v, real(kind=8), dimension(:, :), intent(out) A )

Definition at line 328 of file util.f90.

```
328      REAL(KIND=8), DIMENSION(:, :), INTENT(IN) :: u,v
329      REAL(KIND=8), DIMENSION(:, :, :), INTENT(OUT) :: a
330      INTEGER :: i, j
331
332      a=0.d0
333      DO i=1,SIZE(u)
334          DO j=1,SIZE(v)
335              a(i,j)=u(i)*v(j)
336          ENDDO
337      ENDDO
```

## 8.28 wl\_tensor Module Reference

The WL tensors used to integrate the model.

## Functions/Subroutines

- subroutine, public `init_wl_tensor`

*Subroutine to initialise the WL tensor.*

## Variables

- `real(kind=8), dimension(:), allocatable, public m11`  
*First component of the M1 term.*
- `type(coolist), dimension(:), allocatable, public m12`  
*Second component of the M1 term.*
- `real(kind=8), dimension(:), allocatable, public m13`  
*Third component of the M1 term.*
- `real(kind=8), dimension(:), allocatable, public m1tot`  
*Total  $M_1$  vector.*
- `type(coolist), dimension(:), allocatable, public m21`  
*First tensor of the M2 term.*
- `type(coolist), dimension(:), allocatable, public m22`  
*Second tensor of the M2 term.*
- `type(coolist), dimension(:,:,), allocatable, public l1`  
*First linear tensor.*
- `type(coolist), dimension(:,:,), allocatable, public l2`  
*Second linear tensor.*
- `type(coolist), dimension(:,:,), allocatable, public l4`  
*Fourth linear tensor.*
- `type(coolist), dimension(:,:,), allocatable, public l5`  
*Fifth linear tensor.*
- `type(coolist), dimension(:,:,), allocatable, public ltot`  
*Total linear tensor.*
- `type(coolist), dimension(:,:,), allocatable, public b1`  
*First quadratic tensor.*
- `type(coolist), dimension(:,:,), allocatable, public b2`  
*Second quadratic tensor.*
- `type(coolist), dimension(:,:,), allocatable, public b3`  
*Third quadratic tensor.*
- `type(coolist), dimension(:,:,), allocatable, public b4`  
*Fourth quadratic tensor.*
- `type(coolist), dimension(:,:,), allocatable, public b14`  
*Joint 1st and 4th tensors.*
- `type(coolist), dimension(:,:,), allocatable, public b23`  
*Joint 2nd and 3rd tensors.*
- `type(coolist4), dimension(:,:,), allocatable, public mtot`  
*Tensor for the cubic terms.*
- `real(kind=8), dimension(:), allocatable dumb_vec`  
*Dummy vector.*
- `real(kind=8), dimension(:,:,), allocatable dumb_mat1`  
*Dummy matrix.*
- `real(kind=8), dimension(:,:,), allocatable dumb_mat2`  
*Dummy matrix.*
- `real(kind=8), dimension(:,:,), allocatable dumb_mat3`

*Dummy matrix.*

- `real(kind=8), dimension(:, :, :), allocatable dumb_mat4`

*Dummy matrix.*

- `logical, public m12def`
- `logical, public m21def`
- `logical, public m22def`
- `logical, public ldef`
- `logical, public b14def`
- `logical, public b23def`
- `logical, public mdef`

*Boolean to (de)activate the computation of the terms.*

## 8.28.1 Detailed Description

The WL tensors used to integrate the model.

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### Remarks

## 8.28.2 Function/Subroutine Documentation

### 8.28.2.1 subroutine, public wl\_tensor::init\_wl\_tensor( )

Subroutine to initialise the WL tensor.

Definition at line 94 of file `WL_tensor.f90`.

```

94      INTEGER :: allocstat,i,j,k,m
95
96      print*, 'Initializing the decomposition tensors...'
97      CALL init_dec_tensor
98      print*, "Initializing the correlation matrices and tensors..."
99      CALL init_corr_tensor
100
101      !M1 part
102      print*, "Computing the M1 terms..."
103
104      ALLOCATE(m11(0:ndim), m12(ndim), m13(0:ndim), mltot(0:ndim),
105      stat=allocstat)
106      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
107      ALLOCATE(dumb_mat1(ndim,ndim), dumb_mat2(ndim,ndim), stat=allocstat)
108      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
109      ALLOCATE(dumb_mat3(ndim,ndim), dumb_mat4(ndim,ndim), stat=allocstat)
110      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
111      ALLOCATE(dumb_vec(ndim), stat=allocstat)
112      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
113
114      !M11
115      m11=0.d0
116      ! CALL coo_to_mat_ik(Lxy,dumb_mat1)
117      ! M11(1:ndim)=matmul(dumb_mat1,mean_full(1:ndim))
118
119      !M12
120
121      !M12

```

```

122 ! dumb_mat2=0.D0
123 ! DO i=1,ndim
124 !   CALL coo_to_mat_i(i,Bxxy,dumb_mat1)
125 !   dumb_mat2(i,:)=matmul(dumb_mat1,mean_full(1:ndim))
126 ! ENDDO
127 ! CALL matc_to_coo(dumb_mat2,M12)
128
129 m12def=.not.tensor_empty(m12)
130
131 !M13
132 m13=0.d0
133 CALL sparse_mul3_with_mat(bxxy,corr_i_full,m13)
134
135 !M1tot
136 mltot=0.d0
137 mltot=m11+m13
138
139 print*, "Computing the M2 terms..."
140 ALLOCATE(m21(ndim), m22(ndim), stat=allocstat)
141 IF (allocstat /= 0) stop "*** Not enough memory ! ***"
142
143 !M21
144 CALL copy_coo(lxy,m21)
145 CALL add_to_tensor(bxxy,m21)
146
147 m21def=.not.tensor_empty(m21)
148
149 !M22
150 CALL copy_coo(bxxy,m22)
151
152 m22def=.not.tensor_empty(m22)
153
154 !M3 tensor
155 print*, "Computing the M3 terms..."
156 ! Linear terms
157 print*, "Computing the L subterms..."
158 ALLOCATE(l1(ndim,mems), l2(ndim,mems), l4(ndim,mems), l5(ndim,mems),
159 stat=allocstat)
160 IF (allocstat /= 0) stop "*** Not enough memory ! ***"
161
162 !L1
163 CALL coo_to_mat_ik(lxy,dumb_mat1)
164 CALL coo_to_mat_ik(lxy,dumb_mat2)
165 DO m=1,mems
166 ! CALL coo_to_mat_ik(dy(:,m),dumb_mat3)
167 ! dumb_mat4=matmul(dumb_mat2,matmul(transpose(dumb_mat3),dumb_mat1))
168 ! CALL matc_to_coo(dumb_mat4,l1(:,m))
169 ENDDO
170
171 !L2
172 DO m=1,mems
173 ! dumb_mat4=0.d0
174 ! DO i=1,ndim
175 !   CALL coo_to_mat_i(i,bxxy,dumb_mat1)
176 !   CALL sparse_mul4_with_mat_k1(ydyy(:,m),dumb_mat1,dumb_mat2)
177 !   DO j=1,ndim
178 !     CALL coo_to_mat_j(j,Byxy,dumb_mat1)
179 !     dumb_mat4(i,j)=mat_trace(matmul(dumb_mat1,dumb_mat2))
180 !   ENDDO
181 ! END DO
182 ! CALL matc_to_coo(dumb_mat4,l2(:,m))
183 ENDDO
184
185 !L4
186 ! DO m=1,mems
187 !   dumb_mat4=0.D0
188 !   DO i=1,ndim
189 !     CALL coo_to_mat_i(i,Bxxy,dumb_mat1)
190 !     CALL sparse_mul3_with_mat(dYY(:,m),dumb_mat1,dumb_vec) ! Bxxy*dYY
191 !     CALL coo_to_mat_ik(Lxy,dumb_mat1)
192 !     dumb_mat4(i,:)=matmul(transpose(dumb_mat1),dumb_vec)
193 !   ENDDO
194 !   CALL matc_to_coo(dumb_mat4,L4(:,m))
195 ! ENDDO
196
197 !L5
198 ! CALL coo_to_mat_ik(Lxy,dumb_mat1)
199 ! DO m=1,mems
200 !   dumb_mat4=0.D0
201 !   DO i=1,ndim
202 !     CALL sparse_mul3_mat(YdY(:,m),dumb_mat1(i,:),dumb_mat2)
203 !     DO j=1,ndim
204 !       CALL coo_to_mat_j(j,Byxy,dumb_mat3)
205 !       dumb_mat4(i,j)=mat_trace(matmul(dumb_mat3,dumb_mat2))
206 !     ENDDO
207 !   END DO

```

```

208      !     CALL matc_to_coo(dumb_mat4,L5(:,m))
209      ! ENDDO
210
211      !Ltot
212
213      ALLOCATE(ltot(ndim,mems), stat=allocstat)
214      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
215
216      DO m=1,mems
217          CALL add_to_tensor(l1(:,m),ltot(:,m))
218          CALL add_to_tensor(l2(:,m),ltot(:,m))
219          CALL add_to_tensor(l4(:,m),ltot(:,m))
220          CALL add_to_tensor(l5(:,m),ltot(:,m))
221      ENDDO
222
223      ldef=.not.tensor_empty(ltot)
224
225      print*, "Computing the B terms..."
226      ALLOCATE(b1(ndim,mems), b2(ndim,mems), b3(ndim,mems), b4(ndim,mems),
227      stat=allocstat)
228      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
229
230      ! B1
231      CALL coo_to_mat_ik(lxy,dumb_mat1)
232      dumb_mat1=transpose(dumb_mat1)
233      DO m=1,mems
234          CALL coo_to_mat_ik(dy(:,m),dumb_mat2)
235          dumb_mat2=matmul(dumb_mat2,dumb_mat1)
236          DO j=1,ndim
237              DO k=1,ndim
238                  CALL coo_to_vec_jk(j,k,byxx,dumb_vec)
239                  dumb_vec=matmul(dumb_vec,dumb_mat2)
240                  CALL add_vec_jk_to_tensor(j,k,dumb_vec,b1(:,m))
241          ENDDO
242      ENDDO
243
244      ! B2
245      CALL coo_to_mat_ik(lyx,dumb_mat3)
246      dumb_mat3=transpose(dumb_mat3)
247      DO m=1,mems
248          DO i=1,ndim
249              CALL coo_to_mat_i(i,bxxy,dumb_mat1)
250              CALL coo_to_mat_ik(dy(:,m),dumb_mat2)
251              dumb_mat1=matmul(dumb_mat2,transpose(dumb_mat1))
252              dumb_mat1=matmul(dumb_mat3,dumb_mat1)
253              CALL add_matc_to_tensor(i,dumb_mat1,b2(:,m))
254      ENDDO
255  ENDDO
256
257      ! B3
258      ! DO m=1,mems
259      !     DO i=1,ndim
260      !         CALL coo_to_mat_i(i,Bxxy,dumb_mat1)
261      !         dumb_mat4=0.D0
262      !         DO j=1,ndim
263      !             CALL coo_to_mat_j(j,YdY(:,m),dumb_mat2)
264      !             CALL coo_to_mat_i(j,Byxy,dumb_mat3)
265      !             dumb_mat2=matmul(dumb_mat3,dumb_mat2)
266      !             dumb_mat4=dumb_mat4+dumb_mat2
267      !         ENDDO
268      !         dumb_mat4=matmul(dumb_mat4,transpose(dumb_mat1))
269      !         CALL add_matc_to_tensor(i,dumb_mat4,B3(:,m))
270      !     ENDDO
271  ! ENDDO
272
273      ! B4
274      ! DO m=1,mems
275      !     DO i=1,ndim
276      !         CALL coo_to_mat_i(i,Bxyy,dumb_mat1)
277      !         CALL sparse_mul3_with_mat(dYY(:,m),dumb_mat1,dumb_vec) ! Bxyy*dYY
278      !         DO j=1,ndim
279      !             CALL coo_to_mat_j(j,Byxx,dumb_mat1)
280      !             dumb_mat4(j,:)=matmul(transpose(dumb_mat1),dumb_vec)
281      !         ENDDO
282      !         CALL add_matc_to_tensor(i,dumb_mat4,B4(:,m))
283      !     ENDDO
284  ! ENDDO
285
286      ALLOCATE(b14(ndim,mems), b23(ndim,mems), stat=allocstat)
287      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
288
289      DO m=1,mems
290          CALL add_to_tensor(b1(:,m),b14(:,m))
291          CALL add_to_tensor(b2(:,m),b23(:,m))
292          CALL add_to_tensor(b4(:,m),b14(:,m))
293          CALL add_to_tensor(b3(:,m),b23(:,m))

```

```

294      ENDDO
295
296      b14def=.not.tensor_empty(b14)
297      b23def=.not.tensor_empty(b23)
298
299      !M
300
301      print*, "Computing the M term..."
302
303      ALLOCATE(mtot(ndim,mems), stat=allocstat)
304      IF (allocstat /= 0) stop "*** Not enough memory ! ***"
305
306      DO m=1,mems
307          DO i=1,ndim
308              CALL coo_to_mat_i(i,bxxy,dumb_mat1)
309              CALL coo_to_mat_ik(dy(:,m),dumb_mat2)
310              dumb_mat1=matmul(dumb_mat2,transpose(dumb_mat1))
311              DO j=1,ndim
312                  DO k=1,ndim
313                      CALL coo_to_vec_jk(j,k,byxx,dumb_vec)
314                      dumb_vec=matmul(dumb_vec,dumb_mat1)
315                      CALL add_vec_ijk_to_tensor4(i,j,k,dumb_vec,mtot(:,m))
316                  ENDDO
317              END DO
318          END DO
319      END DO
320
321      mdef=.not.tensor4_empty(mtot)
322
323
324      DEALLOCATE(dumb_mat1, dumb_mat2, stat=allocstat)
325      IF (allocstat /= 0) stop "*** Problem to deallocate ! ***"
326
327      DEALLOCATE(dumb_mat3, dumb_mat4, stat=allocstat)
328      IF (allocstat /= 0) stop "*** Problem to deallocate ! ***"
329
330      DEALLOCATE(dumb_vec, stat=allocstat)
331      IF (allocstat /= 0) stop "*** Problem to deallocate ! ***"
332
333

```

### 8.28.3 Variable Documentation

#### 8.28.3.1 type(coolist), dimension(:,:), allocatable, public wl\_tensor::b1

First quadratic tensor.

Definition at line 60 of file WL\_tensor.f90.

```
60      TYPE(coolist), DIMENSION(:,:), ALLOCATABLE, PUBLIC :: b1      !< First quadratic tensor
```

#### 8.28.3.2 type(coolist), dimension(:,:), allocatable, public wl\_tensor::b14

Joint 1st and 4th tensors.

Definition at line 64 of file WL\_tensor.f90.

```
64      TYPE(coolist), DIMENSION(:,:), ALLOCATABLE, PUBLIC :: b14    !< Joint 1st and 4th tensors
```

#### 8.28.3.3 logical, public wl\_tensor::b14def

Definition at line 75 of file WL\_tensor.f90.

#### 8.28.3.4 type(coolist), dimension(:, :, ), allocatable, public wl\_tensor::b2

Second quadratic tensor.

Definition at line 61 of file WL\_tensor.f90.

```
61   TYPE(coolist), DIMENSION(:, :, ), ALLOCATABLE, PUBLIC :: b2      !< Second quadratic tensor
```

#### 8.28.3.5 type(coolist), dimension(:, :, ), allocatable, public wl\_tensor::b23

Joint 2nd and 3rd tensors.

Definition at line 65 of file WL\_tensor.f90.

```
65   TYPE(coolist), DIMENSION(:, :, ), ALLOCATABLE, PUBLIC :: b23     !< Joint 2nd and 3rd tensors
```

#### 8.28.3.6 logical, public wl\_tensor::b23def

Definition at line 75 of file WL\_tensor.f90.

#### 8.28.3.7 type(coolist), dimension(:, :, ), allocatable, public wl\_tensor::b3

Third quadratic tensor.

Definition at line 62 of file WL\_tensor.f90.

```
62   TYPE(coolist), DIMENSION(:, :, ), ALLOCATABLE, PUBLIC :: b3      !< Third quadratic tensor
```

#### 8.28.3.8 type(coolist), dimension(:, :, ), allocatable, public wl\_tensor::b4

Fourth quadratic tensor.

Definition at line 63 of file WL\_tensor.f90.

```
63   TYPE(coolist), DIMENSION(:, :, ), ALLOCATABLE, PUBLIC :: b4      !< Fourth quadratic tensor
```

#### 8.28.3.9 real(kind=8), dimension(:, :, ), allocatable wl\_tensor::dumb\_mat1 [private]

Dummy matrix.

Definition at line 70 of file WL\_tensor.f90.

```
70   REAL(KIND=8), DIMENSION(:, :, ), ALLOCATABLE :: dumb_mat1 !< Dummy matrix
```

**8.28.3.10 real(kind=8), dimension(:, :, allocatable wl\_tensor::dumb\_mat2 [private]**

Dummy matrix.

Definition at line 71 of file WL\_tensor.f90.

```
71    REAL(KIND=8), DIMENSION(:, :, ), ALLOCATABLE :: dumb_mat2 !< Dummy matrix
```

**8.28.3.11 real(kind=8), dimension(:, :, allocatable wl\_tensor::dumb\_mat3 [private]**

Dummy matrix.

Definition at line 72 of file WL\_tensor.f90.

```
72    REAL(KIND=8), DIMENSION(:, :, ), ALLOCATABLE :: dumb_mat3 !< Dummy matrix
```

**8.28.3.12 real(kind=8), dimension(:, :, allocatable wl\_tensor::dumb\_mat4 [private]**

Dummy matrix.

Definition at line 73 of file WL\_tensor.f90.

```
73    REAL(KIND=8), DIMENSION(:, :, ), ALLOCATABLE :: dumb_mat4 !< Dummy matrix
```

**8.28.3.13 real(kind=8), dimension(:, allocatable wl\_tensor::dumb\_vec [private]**

Dummy vector.

Definition at line 69 of file WL\_tensor.f90.

```
69    REAL(KIND=8), DIMENSION(:, ), ALLOCATABLE :: dumb_vec !< Dummy vector
```

**8.28.3.14 type(coolist), dimension(:, :, allocatable, public wl\_tensor::l1**

First linear tensor.

Definition at line 53 of file WL\_tensor.f90.

```
53    TYPE(coolist), DIMENSION(:, :, ), ALLOCATABLE, PUBLIC :: l1      !< First linear tensor
```

### 8.28.3.15 type(coolist), dimension(:, :, ), allocatable, public wl\_tensor::l2

Second linear tensor.

Definition at line 54 of file WL\_tensor.f90.

```
54    TYPE(coolist), DIMENSION(:, :, ), ALLOCATABLE, PUBLIC :: l2      !< Second linear tensor
```

### 8.28.3.16 type(coolist), dimension(:, :, ), allocatable, public wl\_tensor::l4

Fourth linear tensor.

Definition at line 55 of file WL\_tensor.f90.

```
55    TYPE(coolist), DIMENSION(:, :, ), ALLOCATABLE, PUBLIC :: l4      !< Fourth linear tensor
```

### 8.28.3.17 type(coolist), dimension(:, :, ), allocatable, public wl\_tensor::l5

Fifth linear tensor.

Definition at line 56 of file WL\_tensor.f90.

```
56    TYPE(coolist), DIMENSION(:, :, ), ALLOCATABLE, PUBLIC :: l5      !< Fifth linear tensor
```

### 8.28.3.18 logical, public wl\_tensor::ldef

Definition at line 75 of file WL\_tensor.f90.

### 8.28.3.19 type(coolist), dimension(:, :, ), allocatable, public wl\_tensor::ltot

Total linear tensor.

Definition at line 57 of file WL\_tensor.f90.

```
57    TYPE(coolist), DIMENSION(:, :, ), ALLOCATABLE, PUBLIC :: ltot   !< Total linear tensor
```

### 8.28.3.20 real(kind=8), dimension(:), allocatable, public wl\_tensor::m11

First component of the M1 term.

Definition at line 42 of file WL\_tensor.f90.

```
42    REAL(KIND=8), DIMENSION(:), ALLOCATABLE, PUBLIC :: m11      !< First component of the M1 term
```

### 8.28.3.21 type(coolist), dimension(:), allocatable, public wl\_tensor::m12

Second component of the M1 term.

Definition at line 43 of file WL\_tensor.f90.

```
43    TYPE(coolist), DIMENSION(:), ALLOCATABLE, PUBLIC :: m12    !< Second component of the M1 term
```

### 8.28.3.22 logical, public wl\_tensor::m12def

Definition at line 75 of file WL\_tensor.f90.

```
75    LOGICAL, PUBLIC :: m12def,m21def,m22def,ldef,b14def,b23def,mdef !< Boolean to (de)activate the
   computation of the terms
```

### 8.28.3.23 real(kind=8), dimension(:), allocatable, public wl\_tensor::m13

Third component of the M1 term.

Definition at line 44 of file WL\_tensor.f90.

```
44    REAL(KIND=8), DIMENSION(:), ALLOCATABLE, PUBLIC :: m13    !< Third component of the M1 term
```

### 8.28.3.24 real(kind=8), dimension(:), allocatable, public wl\_tensor::m1tot

Total  $M_1$  vector.

Definition at line 45 of file WL\_tensor.f90.

```
45    REAL(KIND=8), DIMENSION(:), ALLOCATABLE, PUBLIC :: mltot  !< Total \f$M_1\f$ vector
```

### 8.28.3.25 type(coolist), dimension(:), allocatable, public wl\_tensor::m21

First tensor of the M2 term.

Definition at line 48 of file WL\_tensor.f90.

```
48    TYPE(coolist), DIMENSION(:), ALLOCATABLE, PUBLIC :: m21    !< First tensor of the M2 term
```

### 8.28.3.26 logical, public wl\_tensor::m21def

Definition at line 75 of file WL\_tensor.f90.

**8.28.3.27 type(coolist), dimension(:,), allocatable, public wl\_tensor::m22**

Second tensor of the M2 term.

Definition at line 49 of file WL\_tensor.f90.

```
49   TYPE(coolist), DIMENSION(:,), ALLOCATABLE, PUBLIC :: m22 !< Second tensor of the M2 term
```

**8.28.3.28 logical, public wl\_tensor::m22def**

Definition at line 75 of file WL\_tensor.f90.

**8.28.3.29 logical, public wl\_tensor::mdef**

Boolean to (de)activate the computation of the terms.

Definition at line 75 of file WL\_tensor.f90.

**8.28.3.30 type(coolist4), dimension(:,:,), allocatable, public wl\_tensor::mtot**

Tensor for the cubic terms.

Definition at line 67 of file WL\_tensor.f90.

```
67   TYPE(coolist4), DIMENSION(:,:), ALLOCATABLE, PUBLIC :: mtot !< Tensor for the cubic terms
```



# Chapter 9

## Data Type Documentation

### 9.1 inprod\_analytic::atm\_tensors Type Reference

Type holding the atmospheric inner products tensors.

#### Private Attributes

- procedure([calculate\\_a](#)), pointer, nopass **a**
- procedure([calculate\\_b](#)), pointer, nopass **b**
- procedure([calculate\\_c\\_atm](#)), pointer, nopass **c**
- procedure([calculate\\_d](#)), pointer, nopass **d**
- procedure([calculate\\_g](#)), pointer, nopass **g**
- procedure([calculate\\_s](#)), pointer, nopass **s**

#### 9.1.1 Detailed Description

Type holding the atmospheric inner products tensors.

Definition at line 53 of file inprod\_analytic.f90.

#### 9.1.2 Member Data Documentation

##### 9.1.2.1 procedure([calculate\\_a](#)), pointer, nopass inprod\_analytic::atm\_tensors::a [private]

Definition at line 54 of file inprod\_analytic.f90.

```
54      PROCEDURE (calculate_a), POINTER, NOPASS :: a
```

##### 9.1.2.2 procedure([calculate\\_b](#)), pointer, nopass inprod\_analytic::atm\_tensors::b [private]

Definition at line 55 of file inprod\_analytic.f90.

```
55      PROCEDURE (calculate_b), POINTER, NOPASS :: b
```

### 9.1.2.3 procedure(**calculate\_c\_atm**), pointer, nopass in**prod\_analytic**::**atm\_tensors**::**c** [private]

Definition at line 56 of file `inprod_analytic.f90`.

```
56      PROCEDURE(calculate_c_atm), POINTER, NOPASS :: c
```

### 9.1.2.4 procedure(**calculate\_d**), pointer, nopass in**prod\_analytic**::**atm\_tensors**::**d** [private]

Definition at line 57 of file `inprod_analytic.f90`.

```
57      PROCEDURE(calculate_d), POINTER, NOPASS :: d
```

### 9.1.2.5 procedure(**calculate\_g**), pointer, nopass in**prod\_analytic**::**atm\_tensors**::**g** [private]

Definition at line 58 of file `inprod_analytic.f90`.

```
58      PROCEDURE(calculate_g), POINTER, NOPASS :: g
```

### 9.1.2.6 procedure(**calculate\_s**), pointer, nopass in**prod\_analytic**::**atm\_tensors**::**s** [private]

Definition at line 59 of file `inprod_analytic.f90`.

```
59      PROCEDURE(calculate_s), POINTER, NOPASS :: s
```

The documentation for this type was generated from the following file:

- [inprod\\_analytic.f90](#)

## 9.2 in**prod\_analytic**::**atm\_wavenum** Type Reference

Atmospheric bloc specification type.

### Private Attributes

- character **typ**
- integer **m** =0
- integer **p** =0
- integer **h** =0
- real(kind=8) **nx** =0.
- real(kind=8) **ny** =0.

### 9.2.1 Detailed Description

Atmospheric bloc specification type.

Definition at line 40 of file inprod\_analytic.f90.

### 9.2.2 Member Data Documentation

#### 9.2.2.1 integer inprod\_analytic::atm\_wavenum::h =0 [private]

Definition at line 42 of file inprod\_analytic.f90.

#### 9.2.2.2 integer inprod\_analytic::atm\_wavenum::m =0 [private]

Definition at line 42 of file inprod\_analytic.f90.

```
42      INTEGER :: m=0, p=0, h=0
```

#### 9.2.2.3 real(kind=8) inprod\_analytic::atm\_wavenum::nx =0. [private]

Definition at line 43 of file inprod\_analytic.f90.

```
43      REAL (KIND=8) :: nx=0., ny=0.
```

#### 9.2.2.4 real(kind=8) inprod\_analytic::atm\_wavenum::ny =0. [private]

Definition at line 43 of file inprod\_analytic.f90.

#### 9.2.2.5 integer inprod\_analytic::atm\_wavenum::p =0 [private]

Definition at line 42 of file inprod\_analytic.f90.

#### 9.2.2.6 character inprod\_analytic::atm\_wavenum::typ [private]

Definition at line 41 of file inprod\_analytic.f90.

```
41      CHARACTER :: typ
```

The documentation for this type was generated from the following file:

- [inprod\\_analytic.f90](#)

## 9.3 tensor::coolist Type Reference

Coordinate list. Type used to represent the sparse tensor.

### Public Attributes

- type([coolist\\_elem](#)), dimension(:), allocatable [elems](#)  
*Lists of elements [tensor::coolist\\_elem](#).*
- integer [nelems](#) = 0  
*Number of elements in the list.*

### 9.3.1 Detailed Description

Coordinate list. Type used to represent the sparse tensor.

Definition at line 38 of file [tensor.f90](#).

### 9.3.2 Member Data Documentation

#### 9.3.2.1 type([coolist\\_elem](#)), dimension(:), allocatable [tensor::coolist::elems](#)

Lists of elements [tensor::coolist\\_elem](#).

Definition at line 39 of file [tensor.f90](#).

```
39      TYPE(coolist\_elem), DIMENSION(:), ALLOCATABLE :: elems !< Lists of elements
tensor::coolist\_elem
```

#### 9.3.2.2 integer [tensor::coolist::nelems](#) = 0

Number of elements in the list.

Definition at line 40 of file [tensor.f90](#).

```
40      INTEGER :: nelems = 0 !< Number of elements in the list.
```

The documentation for this type was generated from the following file:

- [tensor.f90](#)

## 9.4 tensor::coolist4 Type Reference

4d coordinate list. Type used to represent the rank-4 sparse tensor.

## Public Attributes

- type([coolist\\_elem4](#)), dimension(:), allocatable `elems`
- integer `nelems` = 0

### 9.4.1 Detailed Description

4d coordinate list. Type used to represent the rank-4 sparse tensor.

Definition at line 44 of file `tensor.f90`.

### 9.4.2 Member Data Documentation

#### 9.4.2.1 type([coolist\\_elem4](#)), dimension(:), allocatable `tensor::coolist4::elems`

Definition at line 45 of file `tensor.f90`.

```
45      TYPE(coolist\_elem4), DIMENSION(:), ALLOCATABLE :: elems
```

#### 9.4.2.2 integer `tensor::coolist4::nelems` = 0

Definition at line 46 of file `tensor.f90`.

```
46      INTEGER :: nelems = 0
```

The documentation for this type was generated from the following file:

- [tensor.f90](#)

## 9.5 tensor::coolist\_elem Type Reference

Coordinate list element type. Elementary elements of the sparse tensors.

## Private Attributes

- integer `j`  
*Index j of the element.*
- integer `k`  
*Index k of the element.*
- real(kind=8) `v`  
*Value of the element.*

### 9.5.1 Detailed Description

Coordinate list element type. Elementary elements of the sparse tensors.

Definition at line 25 of file tensor.f90.

### 9.5.2 Member Data Documentation

#### 9.5.2.1 integer tensor::coolist\_elem::j [private]

Index  $j$  of the element.

Definition at line 26 of file tensor.f90.

```
26      INTEGER :: j !< Index \f$j\f$ of the element
```

#### 9.5.2.2 integer tensor::coolist\_elem::k [private]

Index  $k$  of the element.

Definition at line 27 of file tensor.f90.

```
27      INTEGER :: k !< Index \f$k\f$ of the element
```

#### 9.5.2.3 real(kind=8) tensor::coolist\_elem::v [private]

Value of the element.

Definition at line 28 of file tensor.f90.

```
28      REAL(KIND=8) :: v !< Value of the element
```

The documentation for this type was generated from the following file:

- [tensor.f90](#)

## 9.6 tensor::coolist\_elem4 Type Reference

4d coordinate list element type. Elementary elements of the 4d sparse tensors.

## Private Attributes

- integer j
- integer k
- integer l
- real(kind=8) v

### 9.6.1 Detailed Description

4d coordinate list element type. Elementary elements of the 4d sparse tensors.

Definition at line 32 of file tensor.f90.

### 9.6.2 Member Data Documentation

#### 9.6.2.1 integer tensor::coolist\_elem4::j [private]

Definition at line 33 of file tensor.f90.

```
33      INTEGER :: j,k,l
```

#### 9.6.2.2 integer tensor::coolist\_elem4::k [private]

Definition at line 33 of file tensor.f90.

#### 9.6.2.3 integer tensor::coolist\_elem4::l [private]

Definition at line 33 of file tensor.f90.

#### 9.6.2.4 real(kind=8) tensor::coolist\_elem4::v [private]

Definition at line 34 of file tensor.f90.

```
34      REAL (KIND=8) :: v
```

The documentation for this type was generated from the following file:

- [tensor.f90](#)

## 9.7 inprod\_analytic::ocean\_tensors Type Reference

Type holding the oceanic inner products tensors.

## Private Attributes

- procedure([calculate\\_k](#)), pointer, nopass **k**
- procedure([calculate\\_m](#)), pointer, nopass **m**
- procedure([calculate\\_c\\_oc](#)), pointer, nopass **c**
- procedure([calculate\\_n](#)), pointer, nopass **n**
- procedure([calculate\\_o](#)), pointer, nopass **o**
- procedure([calculate\\_w](#)), pointer, nopass **w**

### 9.7.1 Detailed Description

Type holding the oceanic inner products tensors.

Definition at line 63 of file `inprod_analytic.f90`.

### 9.7.2 Member Data Documentation

#### 9.7.2.1 procedure([calculate\\_c\\_oc](#)), pointer, nopass `inprod_analytic::ocean_tensors::c` [private]

Definition at line 66 of file `inprod_analytic.f90`.

```
66      PROCEDURE(calculate_c_oc), POINTER, NOPASS :: c
```

#### 9.7.2.2 procedure([calculate\\_k](#)), pointer, nopass `inprod_analytic::ocean_tensors::k` [private]

Definition at line 64 of file `inprod_analytic.f90`.

```
64      PROCEDURE(calculate_k), POINTER, NOPASS :: k
```

#### 9.7.2.3 procedure([calculate\\_m](#)), pointer, nopass `inprod_analytic::ocean_tensors::m` [private]

Definition at line 65 of file `inprod_analytic.f90`.

```
65      PROCEDURE(calculate_m), POINTER, NOPASS :: m
```

#### 9.7.2.4 procedure([calculate\\_n](#)), pointer, nopass `inprod_analytic::ocean_tensors::n` [private]

Definition at line 67 of file `inprod_analytic.f90`.

```
67      PROCEDURE(calculate_n), POINTER, NOPASS :: n
```

**9.7.2.5 procedure(calculate\_o), pointer, nopass inprod\_analytic::ocean\_tensors::o [private]**

Definition at line 68 of file inprod\_analytic.f90.

```
68      PROCEDURE(calculate_o), POINTER, NOPASS :: o
```

**9.7.2.6 procedure(calculate\_w), pointer, nopass inprod\_analytic::ocean\_tensors::w [private]**

Definition at line 69 of file inprod\_analytic.f90.

```
69      PROCEDURE(calculate_w), POINTER, NOPASS :: w
```

The documentation for this type was generated from the following file:

- [inprod\\_analytic.f90](#)

## 9.8 inprod\_analytic::ocean\_wavenum Type Reference

Oceanic bloc specification type.

### Private Attributes

- integer [p](#)
- integer [h](#)
- real(kind=8) [nx](#)
- real(kind=8) [ny](#)

### 9.8.1 Detailed Description

Oceanic bloc specification type.

Definition at line 47 of file inprod\_analytic.f90.

### 9.8.2 Member Data Documentation

**9.8.2.1 integer inprod\_analytic::ocean\_wavenum::h [private]**

Definition at line 48 of file inprod\_analytic.f90.

**9.8.2.2 real(kind=8) inprod\_analytic::ocean\_wavenum::nx [private]**

Definition at line 49 of file inprod\_analytic.f90.

```
49      REAL(KIND=8) :: nx, ny
```

9.8.2.3 real(kind=8) inprod\_analytic::ocean\_wavenum::ny [private]

Definition at line 49 of file inprod\_analytic.f90.

9.8.2.4 integer inprod\_analytic::ocean\_wavenum::p [private]

Definition at line 48 of file inprod\_analytic.f90.

48        INTEGER :: p,h

The documentation for this type was generated from the following file:

- [inprod\\_analytic.f90](#)

# Chapter 10

## File Documentation

### 10.1 aotensor\_def.f90 File Reference

#### Modules

- module `aotensor_def`

*The equation tensor for the coupled ocean-atmosphere model with temperature which allows for an extensible set of modes in the ocean and in the atmosphere.*

#### Functions/Subroutines

- integer function `aotensor_def::psi` (i)

*Translate the  $\psi_{o,i}$  coefficients into effective coordinates.*

- integer function `aotensor_def::theta` (i)

*Translate the  $\theta_{a,i}$  coefficients into effective coordinates.*

- integer function `aotensor_def::a` (i)

*Translate the  $\psi_{o,i}$  coefficients into effective coordinates.*

- integer function `aotensor_def::t` (i)

*Translate the  $\delta T_{o,i}$  coefficients into effective coordinates.*

- integer function `aotensor_def::kdelta` (i, j)

*Kronecker delta function.*

- subroutine `aotensor_def::coeff` (i, j, k, v)

*Subroutine to add element in the `aotensor`  $\mathcal{T}_{i,j,k}$  structure.*

- subroutine `aotensor_def::add_count` (i, j, k, v)

*Subroutine to count the elements of the `aotensor`  $\mathcal{T}_{i,j,k}$ . Add +1 to `count_elems(i)` for each value that is added to the tensor i-th component.*

- subroutine `aotensor_def::compute_aotensor` (func)

*Subroutine to compute the tensor `aotensor`.*

- subroutine, public `aotensor_def::init_aotensor`

*Subroutine to initialise the `aotensor` tensor.*

## Variables

- integer, dimension(:), allocatable `aotensor_def::count_elems`  
*Vector used to count the tensor elements.*
- real(kind=8), parameter `aotensor_def::real_eps = 2.2204460492503131e-16`  
*Epsilon to test equality with 0.*
- type(coolist), dimension(:), allocatable, public `aotensor_def::aotensor`  
 $T_{i,j,k}$  - *Tensor representation of the tendencies.*

## 10.2 corr\_tensor.f90 File Reference

### Modules

- module `corr_tensor`  
*Module to compute the correlations and derivatives used to compute the memory term of the WL parameterization.*

### Functions/Subroutines

- subroutine, public `corr_tensor::init_corr_tensor`  
*Subroutine to initialise the correlations tensors.*

### Variables

- type(coolist), dimension(:, :, :), allocatable, public `corr_tensor::yy`  
*Coolist holding the  $\langle Y \otimes Y^s \rangle$  terms.*
- type(coolist), dimension(:, :, :), allocatable, public `corr_tensor::dy`  
*Coolist holding the  $\langle \partial_Y \otimes Y^s \rangle$  terms.*
- type(coolist), dimension(:, :, :), allocatable, public `corr_tensor::ydy`  
*Coolist holding the  $\langle Y \otimes \partial_Y \otimes Y^s \rangle$  terms.*
- type(coolist), dimension(:, :, :), allocatable, public `corr_tensor::dyy`  
*Coolist holding the  $\langle \partial_Y \otimes Y^s \otimes Y^s \rangle$  terms.*
- type(coolist4), dimension(:, :, :), allocatable, public `corr_tensor::yddy`  
*Coolist holding the  $\langle Y \otimes \partial_Y \otimes Y^s \otimes Y^s \rangle$  terms.*
- real(kind=8), dimension(:), allocatable `corr_tensor::dumb_vec`  
*Dumb vector to be used in the calculation.*
- real(kind=8), dimension(:, :, :), allocatable `corr_tensor::dumb_mat1`  
*Dumb matrix to be used in the calculation.*
- real(kind=8), dimension(:, :, :), allocatable `corr_tensor::dumb_mat2`  
*Dumb matrix to be used in the calculation.*
- real(kind=8), dimension(:, :, :), allocatable `corr_tensor::expm`  
*Matrix holding the product  $\text{inv\_corr\_i} * \text{corr\_ij}$  at time  $s$ .*

## 10.3 corrmod.f90 File Reference

### Modules

- module `corrmod`  
*Module to initialize the correlation matrix of the unresolved variables.*

## Functions/Subroutines

- subroutine, public `corrmod::init_corr`  
*Subroutine to initialise the computation of the correlation.*
- subroutine `corrmod::corrcomp_from_def` (*s*)  
*Subroutine to compute the correlation of the unresolved variables  $\langle Y \otimes Y^s \rangle$  at time *s* from the definition given inside the module.*
- subroutine `corrmod::corrcomp_from_spline` (*s*)  
*Subroutine to compute the correlation of the unresolved variables  $\langle Y \otimes Y^s \rangle$  at time *s* from the spline representation.*
- subroutine `corrmod::splint` (*xa*, *ya*, *y2a*, *n*, *x*, *y*)  
*Routine to compute the spline representation parameters.*
- real(kind=8) function `corrmod::fs` (*s*, *p*)  
*Exponential fit function.*
- subroutine `corrmod::corrcomp_from_fit` (*s*)  
*Subroutine to compute the correlation of the unresolved variables  $\langle Y \otimes Y^s \rangle$  at time *s* from the exponential representation.*

## Variables

- real(kind=8), dimension(:), allocatable, public `corrmod::mean`  
*Vector holding the mean of the unresolved dynamics (reduced version)*
- real(kind=8), dimension(:), allocatable, public `corrmod::mean_full`  
*Vector holding the mean of the unresolved dynamics (full version)*
- real(kind=8), dimension(:, :), allocatable, public `corrmod::corr_i_full`  
*Covariance matrix of the unresolved variables (full version)*
- real(kind=8), dimension(:, :), allocatable, public `corrmod::inv_corr_i_full`  
*Inverse of the covariance matrix of the unresolved variables (full version)*
- real(kind=8), dimension(:, :), allocatable, public `corrmod::corr_i`  
*Covariance matrix of the unresolved variables (reduced version)*
- real(kind=8), dimension(:, :), allocatable, public `corrmod::inv_corr_i`  
*Inverse of the covariance matrix of the unresolved variables (reduced version)*
- real(kind=8), dimension(:, :, :), allocatable, public `corrmod::corr_ij`  
*Matrix holding the correlation matrix at a given time.*
- real(kind=8), dimension(:, :, :, :), allocatable `corrmod::y2`  
*Vector holding coefficient of the spline and exponential correlation representation.*
- real(kind=8), dimension(:, :, :, :), allocatable `corrmod::ya`  
*Vector holding coefficient of the spline and exponential correlation representation.*
- real(kind=8), dimension(:, :), allocatable `corrmod::xa`  
*Vector holding coefficient of the spline and exponential correlation representation.*
- integer `corrmod::nspl`  
*Integers needed by the spline representation of the correlation.*
- integer `corrmod::klo`
- integer `corrmod::khi`
- procedure(`corrcomp_from_spline`), pointer, public `corrmod::corrcomp`  
*Pointer to the correlation computation routine.*

## 10.4 dec\_tensor.f90 File Reference

### Modules

- module `dec_tensor`  
*The resolved-unresolved components decomposition of the tensor.*

## Functions/Subroutines

- subroutine `dec_tensor::suppress_and` (t, cst, v1, v2)  
*Subroutine to suppress from the tensor  $t_{ijk}$  components satisfying  $SF(j)=v1$  and  $SF(k)=v2$ .*
- subroutine `dec_tensor::suppress_or` (t, cst, v1, v2)  
*Subroutine to suppress from the tensor  $t_{ijk}$  components satisfying  $SF(j)=v1$  or  $SF(k)=v2$ .*
- subroutine `dec_tensor::reorder` (t, cst, v)  
*Subroutine to reorder the tensor  $t_{ijk}$  components : if  $SF(j)=v$  then it return  $t_{ikj}$ .*
- subroutine `dec_tensor::init_sub_tensor` (t, cst, v)  
*Subroutine that suppress all the components of a tensor  $t_{ijk}$  where if  $SF(i)=v$ .*
- subroutine, public `dec_tensor::init_dec_tensor`  
*Subroutine that initialize and compute the decomposed tensors.*

## Variables

- type(coolist), dimension(:), allocatable, public `dec_tensor::ff_tensor`  
*Tensor holding the part of the unresolved tensor involving only unresolved variables.*
- type(coolist), dimension(:), allocatable, public `dec_tensor::sf_tensor`  
*Tensor holding the part of the resolved tensor involving unresolved variables.*
- type(coolist), dimension(:), allocatable, public `dec_tensor::ss_tensor`  
*Tensor holding the part of the resolved tensor involving only resolved variables.*
- type(coolist), dimension(:), allocatable, public `dec_tensor::fs_tensor`  
*Tensor holding the part of the unresolved tensor involving resolved variables.*
- type(coolist), dimension(:), allocatable, public `dec_tensor::hx`  
*Tensor holding the constant part of the resolved tendencies.*
- type(coolist), dimension(:), allocatable, public `dec_tensor::lxx`  
*Tensor holding the linear part of the resolved tendencies involving the resolved variables.*
- type(coolist), dimension(:), allocatable, public `dec_tensor::lxy`  
*Tensor holding the linear part of the resolved tendencies involving the unresolved variables.*
- type(coolist), dimension(:), allocatable, public `dec_tensor::bxxx`  
*Tensor holding the quadratic part of the resolved tendencies involving resolved variables.*
- type(coolist), dimension(:), allocatable, public `dec_tensor::bxxy`  
*Tensor holding the quadratic part of the resolved tendencies involving both resolved and unresolved variables.*
- type(coolist), dimension(:), allocatable, public `dec_tensor::bxyy`  
*Tensor holding the quadratic part of the resolved tendencies involving unresolved variables.*
- type(coolist), dimension(:), allocatable, public `dec_tensor::hy`  
*Tensor holding the constant part of the unresolved tendencies.*
- type(coolist), dimension(:), allocatable, public `dec_tensor::lyx`  
*Tensor holding the linear part of the unresolved tendencies involving the resolved variables.*
- type(coolist), dimension(:), allocatable, public `dec_tensor::lyy`  
*Tensor holding the linear part of the unresolved tendencies involving the unresolved variables.*
- type(coolist), dimension(:), allocatable, public `dec_tensor::byxx`  
*Tensor holding the quadratic part of the unresolved tendencies involving resolved variables.*
- type(coolist), dimension(:), allocatable, public `dec_tensor::byxy`  
*Tensor holding the quadratic part of the unresolved tendencies involving both resolved and unresolved variables.*
- type(coolist), dimension(:), allocatable, public `dec_tensor::byyy`  
*Tensor holding the quadratic part of the unresolved tendencies involving unresolved variables.*
- type(coolist), dimension(:), allocatable, public `dec_tensor::ss_tl_tensor`  
*Tensor of the tangent linear model tendencies of the resolved component alone.*
- type(coolist), dimension(:), allocatable `dec_tensor::dumb`  
*Dumb coolist to make the computations.*

## 10.5 doc/def\_doc.md File Reference

### 10.6 doc/gen\_doc.md File Reference

### 10.7 doc/sto\_doc.md File Reference

### 10.8 doc/tl\_ad\_doc.md File Reference

### 10.9 ic\_def.f90 File Reference

#### Modules

- module [ic\\_def](#)

*Module to load the initial condition.*

#### Functions/Subroutines

- subroutine, public [ic\\_def::load\\_ic](#)

*Subroutine to load the initial condition if IC.nml exists. If it does not, then write IC.nml with 0 as initial condition.*

#### Variables

- logical [ic\\_def::exists](#)

*Boolean to test for file existence.*

- real(kind=8), dimension(:), allocatable, public [ic\\_def::ic](#)

*Initial condition vector.*

## 10.10 inprod\_analytic.f90 File Reference

#### Data Types

- type [inprod\\_analytic::atm\\_wavenum](#)

*Atmospheric bloc specification type.*

- type [inprod\\_analytic::ocean\\_wavenum](#)

*Oceanic bloc specification type.*

- type [inprod\\_analytic::atm\\_tensors](#)

*Type holding the atmospheric inner products tensors.*

- type [inprod\\_analytic::ocean\\_tensors](#)

*Type holding the oceanic inner products tensors.*

## Modules

- module `inprod_analytic`

*Inner products between the truncated set of basis functions for the ocean and atmosphere streamfunction fields. These are partly calculated using the analytical expressions from Cehelsky, P., & Tung, K. K. : Theories of multiple equilibria and weather regimes-A critical reexamination. Part II: Baroclinic two-layer models. Journal of the atmospheric sciences, 44(21), 3282-3303, 1987.*

## Functions/Subroutines

- real(kind=8) function `inprod_analytic::b1` ( $P_i, P_j, P_k$ )  
*Cehelsky & Tung Helper functions.*
- real(kind=8) function `inprod_analytic::b2` ( $P_i, P_j, P_k$ )  
*Cehelsky & Tung Helper functions.*
- real(kind=8) function `inprod_analytic::delta` ( $r$ )  
*Integer Dirac delta function.*
- real(kind=8) function `inprod_analytic::flambda` ( $r$ )  
*"Odd or even" function*
- real(kind=8) function `inprod_analytic::s1` ( $P_j, P_k, M_j, H_k$ )  
*Cehelsky & Tung Helper functions.*
- real(kind=8) function `inprod_analytic::s2` ( $P_j, P_k, M_j, H_k$ )  
*Cehelsky & Tung Helper functions.*
- real(kind=8) function `inprod_analytic::s3` ( $P_j, P_k, H_j, H_k$ )  
*Cehelsky & Tung Helper functions.*
- real(kind=8) function `inprod_analytic::s4` ( $P_j, P_k, H_j, H_k$ )  
*Cehelsky & Tung Helper functions.*
- real(kind=8) function `inprod_analytic::calculate_a` ( $i, j$ )  
*Eigenvalues of the Laplacian (atmospheric)*
- real(kind=8) function `inprod_analytic::calculate_b` ( $i, j, k$ )  
*Streamfunction advection terms (atmospheric)*
- real(kind=8) function `inprod_analytic::calculate_c_atm` ( $i, j$ )  
*Beta term for the atmosphere.*
- real(kind=8) function `inprod_analytic::calculate_d` ( $i, j$ )  
*Forcing of the ocean on the atmosphere.*
- real(kind=8) function `inprod_analytic::calculate_g` ( $i, j, k$ )  
*Temperature advection terms (atmospheric)*
- real(kind=8) function `inprod_analytic::calculate_s` ( $i, j$ )  
*Forcing (thermal) of the ocean on the atmosphere.*
- real(kind=8) function `inprod_analytic::calculate_k` ( $i, j$ )  
*Forcing of the atmosphere on the ocean.*
- real(kind=8) function `inprod_analytic::calculate_m` ( $i, j$ )  
*Forcing of the ocean fields on the ocean.*
- real(kind=8) function `inprod_analytic::calculate_n` ( $i, j$ )  
*Beta term for the ocean.*
- real(kind=8) function `inprod_analytic::calculate_o` ( $i, j, k$ )  
*Temperature advection term (passive scalar)*
- real(kind=8) function `inprod_analytic::calculate_c_oc` ( $i, j, k$ )  
*Streamfunction advection terms (oceanic)*
- real(kind=8) function `inprod_analytic::calculate_w` ( $i, j$ )  
*Short-wave radiative forcing of the ocean.*
- subroutine, public `inprod_analytic::init_inprod`  
*Initialisation of the inner product.*

## Variables

- type(atm\_wavenum), dimension(:), allocatable, public [inprod\\_analytic::awavenum](#)  
*Atmospheric blocs specification.*
- type(ocean\_wavenum), dimension(:), allocatable, public [inprod\\_analytic::owavenum](#)  
*Oceanic blocs specification.*
- type(atm\_tensors), public [inprod\\_analytic::atmos](#)  
*Atmospheric tensors.*
- type(ocean\_tensors), public [inprod\\_analytic::ocean](#)  
*Oceanic tensors.*

## 10.11 int\_comp.f90 File Reference

### Modules

- module [int\\_comp](#)  
*Utility module containing the routines to perform the integration of functions.*

### Functions/Subroutines

- subroutine, public [int\\_comp::integrate](#) (func, ss)  
*Routine to compute integrals of function from O to #maxint.*
- subroutine [int\\_comp::qromb](#) (func, a, b, ss)  
*Romberg integration routine.*
- subroutine [int\\_comp::qromo](#) (func, a, b, ss, choose)  
*Romberg integration routine on an open interval.*
- subroutine [int\\_comp::polint](#) (xa, ya, n, x, y, dy)  
*Polynomial interpolation routine.*
- subroutine [int\\_comp::trapzd](#) (func, a, b, s, n)  
*Trapezoidal rule integration routine.*
- subroutine [int\\_comp::midpnt](#) (func, a, b, s, n)  
*Midpoint rule integration routine.*
- subroutine [int\\_comp::midexp](#) (funk, aa, bb, s, n)  
*Midpoint routine for bb infinite with funk decreasing infinitely rapidly at infinity.*

## 10.12 int\_corr.f90 File Reference

### Modules

- module [int\\_corr](#)  
*Module to compute or load the integrals of the correlation matrices.*

## Functions/Subroutines

- subroutine, public `int_corr::init_corrint`  
*Subroutine to initialise the integrated matrices and tensors.*
- real(kind=8) function `int_corr::func_ij` (s)  
*Function that returns the component  $oi$  and  $oj$  of the correlation matrix at time  $s$ .*
- real(kind=8) function `int_corr::func_ijkl` (s)  
*Function that returns the component  $oi,oj,ok$  and  $ol$  of the outer product of the correlation matrix with itself at time  $s$ .*
- subroutine, public `int_corr::comp_corrint`  
*Routine that actually compute or load the integrals.*

## Variables

- integer `int_corr::oi`
- integer `int_corr::oj`
- integer `int_corr::ok`
- integer `int_corr::ol`  
*Integers that specify the matrices and tensor component considered as a function of time.*
- real(kind=8), parameter `int_corr::real_eps` = 2.2204460492503131e-16  
*Small epsilon constant to determine equality with zero.*
- real(kind=8), dimension(:, :), allocatable, public `int_corr::corrint`  
*Matrix holding the integral of the correlation matrix.*
- type(coolist4), dimension(:, :), allocatable, public `int_corr::corr2int`  
*Tensor holding the integral of the correlation outer product with itself.*

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### Functions

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## 10.14 maooom.f90 File Reference

### Functions/Subroutines

- program `maooom`

*Fortran 90 implementation of the modular arbitrary-order ocean-atmosphere model MAOOAM.*

### 10.14.1 Function/Subroutine Documentation

#### 10.14.1.1 program maooram( )

Fortran 90 implementation of the modular arbitrary-order ocean-atmosphere model MAOOAM.

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Definition at line 13 of file maooram.f90.

## 10.15 maooram\_MTV.f90 File Reference

### Functions/Subroutines

- program [maooram\\_mtv](#)

*Fortran 90 implementation of the modular arbitrary-order ocean-atmosphere model MAOOAM - MTV parameterization.*

### 10.15.1 Function/Subroutine Documentation

#### 10.15.1.1 program maooram\_mtv( )

Fortran 90 implementation of the modular arbitrary-order ocean-atmosphere model MAOOAM - MTV parameterization.

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Definition at line 12 of file maooram\_MTV.f90.

## 10.16 maooram\_stoch.f90 File Reference

### Functions/Subroutines

- program [maooram\\_stoch](#)

*Fortran 90 implementation of the stochastic modular arbitrary-order ocean-atmosphere model MAOOAM.*

### 10.16.1 Function/Subroutine Documentation

#### 10.16.1.1 program maoam\_stoch( )

Fortran 90 implementation of the stochastic modular arbitrary-order ocean-atmosphere model MAOOAM.

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##### Remarks

There are four dynamics modes:

- full: generate the full dynamics
- unres: generate the intrinsic unresolved dynamics
- qfst: generate dynamics given by the quadratic terms of the unresolved tendencies
- reso: use the resolved dynamics alone

Definition at line 24 of file maoam\_stoch.f90.

## 10.17 maoam\_WL.f90 File Reference

### Functions/Subroutines

- program maoam\_wl

*Fortran 90 implementation of the modular arbitrary-order ocean-atmosphere model MAOOAM - WL parameterization.*

#### 10.17.1 Function/Subroutine Documentation

##### 10.17.1.1 program maoam\_wl( )

Fortran 90 implementation of the modular arbitrary-order ocean-atmosphere model MAOOAM - WL parameterization.

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Definition at line 12 of file maoam\_WL.f90.

## 10.18 MAR.f90 File Reference

### Modules

- module mar

*Multidimensional Autoregressive module to generate the correlation for the WL parameterization.*

## Functions/Subroutines

- subroutine, public `mar::init_mar`  
*Subroutine to initialise the MAR.*
- subroutine, public `mar::mar_step` (x)  
*Routine to generate one step of the MAR.*
- subroutine, public `mar::mar_step_red` (xred)  
*Routine to generate one step of the reduce MAR.*
- subroutine `mar::stoch_vec` (dW)

## Variables

- real(kind=8), dimension(:, :, :), allocatable, public `mar::q`  
*Square root of the noise covariance matrix.*
- real(kind=8), dimension(:, :, :), allocatable, public `mar::qred`  
*Reduce version of Q.*
- real(kind=8), dimension(:, :, :), allocatable, public `mar::rred`  
*Covariance matrix of the noise.*
- real(kind=8), dimension(:, :, :, :), allocatable, public `mar::w`  
*W\_i matrix.*
- real(kind=8), dimension(:, :, :, :), allocatable, public `mar::wred`  
*Reduce W\_i matrix.*
- real(kind=8), dimension(:, :), allocatable `mar::buf_y`
- real(kind=8), dimension(:, :), allocatable `mar::dw`
- integer, public `mar::ms`  
*order of the MAR*

## 10.19 memory.f90 File Reference

### Modules

- module `memory`  
*Module that compute the memory term  $M_3$  of the WL parameterization.*

## Functions/Subroutines

- subroutine, public `memory::init_memory`  
*Subroutine to initialise the memory.*
- subroutine, public `memory::compute_m3` (y, dt, dtn, savey, save\_ev, evolve, inter, h\_int)  
*Compute the integrand of  $M_3$  at each time in the past and integrate to get the memory term.*
- subroutine, public `memory::test_m3` (y, dt, dtn, h\_int)  
*Routine to test the #compute\_M3 routine.*

## Variables

- real(kind=8), dimension(:, :, :), allocatable `memory::x`  
*Array storing the previous state of the system.*
- real(kind=8), dimension(:, :, :), allocatable `memory::xs`  
*Array storing the resolved time evolution of the previous state of the system.*
- real(kind=8), dimension(:, :, :), allocatable `memory::zs`  
*Dummy array to replace Xs in case where the evolution is not stored.*
- real(kind=8), dimension(:, :), allocatable `memory::buf_m`  
*Dummy vector.*
- real(kind=8), dimension(:, :), allocatable `memory::buf_m3`  
*Dummy vector to store the  $M_3$  integrand.*
- integer `memory::t_index`  
*Integer storing the time index (current position in the arrays)*
- procedure(ss\_step), pointer `memory::step`  
*Procedural pointer pointing on the resolved dynamics step routine.*

## 10.20 MTV\_int\_tensor.f90 File Reference

### Modules

- module `mtv_int_tensor`  
*The MTV tensors used to integrate the MTV model.*

### Functions/Subroutines

- subroutine, public `mtv_int_tensor::init_mtv_int_tensor`  
*Subroutine to initialise the MTV tensor.*

### Variables

- real(kind=8), dimension(:, :), allocatable, public `mtv_int_tensor::h1`  
*First constant vector.*
- real(kind=8), dimension(:, :), allocatable, public `mtv_int_tensor::h2`  
*Second constant vector.*
- real(kind=8), dimension(:, :), allocatable, public `mtv_int_tensor::h3`  
*Third constant vector.*
- real(kind=8), dimension(:, :), allocatable, public `mtv_int_tensor::htot`  
*Total constant vector.*
- type(coolist), dimension(:, :), allocatable, public `mtv_int_tensor::l1`  
*First linear tensor.*
- type(coolist), dimension(:, :), allocatable, public `mtv_int_tensor::l2`  
*Second linear tensor.*
- type(coolist), dimension(:, :), allocatable, public `mtv_int_tensor::l3`  
*Third linear tensor.*
- type(coolist), dimension(:, :), allocatable, public `mtv_int_tensor::ltot`  
*Total linear tensor.*
- type(coolist), dimension(:, :), allocatable, public `mtv_int_tensor::b1`

*First quadratic tensor.*

- type(coolist), dimension(:), allocatable, public `mtv_int_tensor::b2`

*Second quadratic tensor.*

- type(coolist), dimension(:), allocatable, public `mtv_int_tensor::btot`

*Total quadratic tensor.*

- type(coolist4), dimension(:), allocatable, public `mtv_int_tensor::mtot`

*Tensor for the cubic terms.*

- real(kind=8), dimension(:, :, :), allocatable, public `mtv_int_tensor::q1`

*Constant terms for the state-dependent noise covariance matrix.*

- real(kind=8), dimension(:, :, :), allocatable, public `mtv_int_tensor::q2`

*Constant terms for the state-independent noise covariance matrix.*

- type(coolist), dimension(:), allocatable, public `mtv_int_tensor::utot`

*Linear terms for the state-dependent noise covariance matrix.*

- type(coolist4), dimension(:), allocatable, public `mtv_int_tensor::vtot`

*Quadratic terms for the state-dependent noise covariance matrix.*

- real(kind=8), dimension(:, :, :), allocatable `mtv_int_tensor::dumb_vec`

*Dummy vector.*

- real(kind=8), dimension(:, :, :), allocatable `mtv_int_tensor::dumb_mat1`

*Dummy matrix.*

- real(kind=8), dimension(:, :, :), allocatable `mtv_int_tensor::dumb_mat2`

*Dummy matrix.*

- real(kind=8), dimension(:, :, :), allocatable `mtv_int_tensor::dumb_mat3`

*Dummy matrix.*

- real(kind=8), dimension(:, :, :), allocatable `mtv_int_tensor::dumb_mat4`

*Dummy matrix.*

## 10.21 MTV\_sigma\_tensor.f90 File Reference

### Modules

- module `sigma`

*The MTV noise sigma matrices used to integrate the MTV model.*

### Functions/Subroutines

- subroutine, public `sigma::init_sigma` (`mult`, `Q1fill`)

*Subroutine to initialize the sigma matices.*

- subroutine, public `sigma::compute_mult_sigma` (`y`)

*Routine to actualize the matrix  $\sigma_1$  based on the state  $y$  of the MTV system.*

## Variables

- real(kind=8), dimension(:, :, :), allocatable, public **sigma::sig1**  
 $\sigma_1(X)$  state-dependent noise matrix
- real(kind=8), dimension(:, :, :), allocatable, public **sigma::sig2**  
 $\sigma_2$  state-independent noise matrix
- real(kind=8), dimension(:, :, :), allocatable, public **sigma::sig1r**  
Reduced  $\sigma_1(X)$  state-dependent noise matrix.
- real(kind=8), dimension(:, :, :), allocatable **sigma::dumb\_mat1**  
Dummy matrix.
- real(kind=8), dimension(:, :, :), allocatable **sigma::dumb\_mat2**  
Dummy matrix.
- real(kind=8), dimension(:, :, :), allocatable **sigma::dumb\_mat3**  
Dummy matrix.
- real(kind=8), dimension(:, :, :), allocatable **sigma::dumb\_mat4**  
Dummy matrix.
- integer, dimension(:), allocatable **sigma::ind1**
- integer, dimension(:), allocatable **sigma::rind1**
- integer, dimension(:), allocatable **sigma::ind2**
- integer, dimension(:), allocatable **sigma::rind2**  
Reduction indices.
- integer **sigma::n1**
- integer **sigma::n2**

## 10.22 params.f90 File Reference

### Modules

- module **params**  
The model parameters module.

### Functions/Subroutines

- subroutine, private **params::init\_nml**  
Read the basic parameters and mode selection from the namelist.
- subroutine **params::init\_params**  
Parameters initialisation routine.

### Variables

- real(kind=8) **params::n**  
 $n = 2L_y/L_x$  - Aspect ratio
- real(kind=8) **params::phi0**  
Latitude in radian.
- real(kind=8) **params::rra**  
Earth radius.
- real(kind=8) **params::sig0**  
 $\sigma_0$  - Non-dimensional static stability of the atmosphere.

- real(kind=8) **params::k**  
*Bottom atmospheric friction coefficient.*
- real(kind=8) **params::kp**  
*k' - Internal atmospheric friction coefficient.*
- real(kind=8) **params::r**  
*Frictional coefficient at the bottom of the ocean.*
- real(kind=8) **params::d**  
*Mechanical coupling parameter between the ocean and the atmosphere.*
- real(kind=8) **params::f0**  
 $f_0$  - Coriolis parameter
- real(kind=8) **params::gp**  
 $g'$  Reduced gravity
- real(kind=8) **params::h**  
*Depth of the active water layer of the ocean.*
- real(kind=8) **params::phi0\_npi**  
*Latitude exprimed in fraction of pi.*
- real(kind=8) **params::lambda**  
 $\lambda$  - Sensible + turbulent heat exchange between the ocean and the atmosphere.
- real(kind=8) **params::co**  
 $C_a$  - Constant short-wave radiation of the ocean.
- real(kind=8) **params::go**  
 $\gamma_o$  - Specific heat capacity of the ocean.
- real(kind=8) **params::ca**  
 $C_a$  - Constant short-wave radiation of the atmosphere.
- real(kind=8) **params::to0**  
 $T_o^0$  - Stationary solution for the 0-th order ocean temperature.
- real(kind=8) **params::ta0**  
 $T_a^0$  - Stationary solution for the 0-th order atmospheric temperature.
- real(kind=8) **params::epsa**  
 $\epsilon_a$  - Emissivity coefficient for the grey-body atmosphere.
- real(kind=8) **params::ga**  
 $\gamma_a$  - Specific heat capacity of the atmosphere.
- real(kind=8) **params::rr**  
 $R$  - Gas constant of dry air
- real(kind=8) **params::scale**  
 $L_y = L \pi$  - The characteristic space scale.
- real(kind=8) **params::pi**  
 $\pi$
- real(kind=8) **params::lr**  
 $L_R$  - Rossby deformation radius
- real(kind=8) **params::g**  
 $\gamma$
- real(kind=8) **params::rp**  
 $r'$  - Frictional coefficient at the bottom of the ocean.
- real(kind=8) **params::dp**  
 $d'$  - Non-dimensional mechanical coupling parameter between the ocean and the atmosphere.
- real(kind=8) **params::kd**  
 $k_d$  - Non-dimensional bottom atmospheric friction coefficient.
- real(kind=8) **params::kdp**  
 $k'_d$  - Non-dimensional internal atmospheric friction coefficient.
- real(kind=8) **params::cpo**

- real(kind=8) **params::lpo**  
 $C'_a$  - Non-dimensional constant short-wave radiation of the ocean.
- real(kind=8) **params::cpa**  
 $\lambda'_o$  - Non-dimensional sensible + turbulent heat exchange from ocean to atmosphere.
- real(kind=8) **params::lpa**  
 $C'_a$  - Non-dimensional constant short-wave radiation of the atmosphere.
- real(kind=8) **params::sbpo**  
 $\lambda'_a$  - Non-dimensional sensible + turbulent heat exchange from atmosphere to ocean.
- real(kind=8) **params::sbpa**  
 $\sigma'_{B,o}$  - Long wave radiation lost by ocean to atmosphere & space.
- real(kind=8) **params::lsbpo**  
 $\sigma'_{B,a}$  - Long wave radiation from atmosphere absorbed by ocean.
- real(kind=8) **params::lsbpa**  
 $S'_{B,o}$  - Long wave radiation from ocean absorbed by atmosphere.
- real(kind=8) **params::l**  
 $L$  - Domain length scale
- real(kind=8) **params::sc**  
Ratio of surface to atmosphere temperature.
- real(kind=8) **params::sb**  
Stefan–Boltzmann constant.
- real(kind=8) **params::betp**  
 $\beta'$  - Non-dimensional beta parameter
- real(kind=8) **params::nua** =0.D0  
Dissipation in the atmosphere.
- real(kind=8) **params::nuo** =0.D0  
Dissipation in the ocean.
- real(kind=8) **params::nuap**  
Non-dimensional dissipation in the atmosphere.
- real(kind=8) **params::nuop**  
Non-dimensional dissipation in the ocean.
- real(kind=8) **params::t\_trans**  
Transient time period.
- real(kind=8) **params::t\_run**  
Effective intergration time (length of the generated trajectory)
- real(kind=8) **params::dt**  
Integration time step.
- real(kind=8) **params::tw**  
Write all variables every tw time units.
- logical **params::writeout**  
Write to file boolean.
- integer **params::nboc**  
Number of atmospheric blocks.
- integer **params::nbatm**  
Number of oceanic blocks.
- integer **params::natm** =0  
Number of atmospheric basis functions.
- integer **params::noc** =0  
Number of oceanic basis functions.
- integer **params::ndim**  
Number of variables (dimension of the model)

- integer, dimension(:, :), allocatable `params::oms`  
*Ocean mode selection array.*
- integer, dimension(:, :), allocatable `params::ams`  
*Atmospheric mode selection array.*

## 10.23 rk2\_integrator.f90 File Reference

### Modules

- module `integrator`  
*Module with the integration routines.*

### Functions/Subroutines

- subroutine, public `integrator::init_integrator`  
*Routine to initialise the integration buffers.*
- subroutine `integrator::tendencies` (`t, y, res`)  
*Routine computing the tendencies of the model.*
- subroutine, public `integrator::step` (`y, t, dt, res`)  
*Routine to perform an integration step (Heun algorithm). The incremented time is returned.*

### Variables

- real(kind=8), dimension(:, :), allocatable `integrator::buf_y1`  
*Buffer to hold the intermediate position (Heun algorithm)*
- real(kind=8), dimension(:, :), allocatable `integrator::buf_f0`  
*Buffer to hold tendencies at the initial position.*
- real(kind=8), dimension(:, :), allocatable `integrator::buf_f1`  
*Buffer to hold tendencies at the intermediate position.*

## 10.24 rk2\_MTV\_integrator.f90 File Reference

### Modules

- module `rk2_mtv_integrator`  
*Module with the MTV rk2 integration routines.*

### Functions/Subroutines

- subroutine, public `rk2_mtv_integrator::init_integrator`  
*Subroutine to initialize the MTV rk2 integrator.*
- subroutine `rk2_mtv_integrator::init_noise`  
*Routine to initialize the noise vectors and buffers.*
- subroutine `rk2_mtv_integrator::init_g`  
*Routine to initialize the G term.*
- subroutine `rk2_mtv_integrator::compg` (`y`)  
*Routine to actualize the G term based on the state `y` of the MTV system.*
- subroutine, public `rk2_mtv_integrator::step` (`y, t, dt, dtn, res, tend`)  
*Routine to perform an integration step (Heun algorithm) of the MTV system. The incremented time is returned.*
- subroutine, public `rk2_mtv_integrator::full_step` (`y, t, dt, dtn, res`)  
*Routine to perform an integration step (Heun algorithm) of the full stochastic system. The incremented time is returned.*

## Variables

- real(kind=8), dimension(:), allocatable `rk2_mtv_integrator::buf_y1`
- real(kind=8), dimension(:), allocatable `rk2_mtv_integrator::buf_f0`
- real(kind=8), dimension(:), allocatable `rk2_mtv_integrator::buf_f1`  
*Integration buffers.*
- real(kind=8), dimension(:), allocatable `rk2_mtv_integrator::dw`
- real(kind=8), dimension(:), allocatable `rk2_mtv_integrator::dwmult`  
*Standard gaussian noise buffers.*
- real(kind=8), dimension(:), allocatable `rk2_mtv_integrator::dwar`
- real(kind=8), dimension(:), allocatable `rk2_mtv_integrator::dwau`
- real(kind=8), dimension(:), allocatable `rk2_mtv_integrator::dwor`
- real(kind=8), dimension(:), allocatable `rk2_mtv_integrator::dwou`  
*Standard gaussian noise buffers.*
- real(kind=8), dimension(:), allocatable `rk2_mtv_integrator::anoise`
- real(kind=8), dimension(:), allocatable `rk2_mtv_integrator::noise`  
*Additive noise term.*
- real(kind=8), dimension(:), allocatable `rk2_mtv_integrator::noisemult`  
*Multiplicative noise term.*
- real(kind=8), dimension(:), allocatable `rk2_mtv_integrator::g`  
*G term of the MTV tendencies.*
- real(kind=8), dimension(:), allocatable `rk2_mtv_integrator::buf_g`  
*Buffer for the G term computation.*
- logical `rk2_mtv_integrator::mult`  
*Logical indicating if the sigma1 matrix must be computed for every state change.*
- logical `rk2_mtv_integrator::q1fill`  
*Logical indicating if the matrix Q1 is non-zero.*
- logical `rk2_mtv_integrator::compute_mult`  
*Logical indicating if the Gaussian noise for the multiplicative noise must be computed.*
- real(kind=8), parameter `rk2_mtv_integrator::sq2 = sqrt(2.D0)`  
*Hard coded square root of 2.*

## 10.25 rk2\_ss\_integrator.f90 File Reference

### Modules

- module `rk2_ss_integrator`  
*Module with the stochastic uncoupled resolved nonlinear and tangent linear rk2 dynamics integration routines.*

### Functions/Subroutines

- subroutine, public `rk2_ss_integrator::init_ss_integrator`  
*Subroutine to initialize the uncoupled resolved rk2 integrator.*
- subroutine, public `rk2_ss_integrator::tendencies (t, y, res)`  
*Routine computing the tendencies of the uncoupled resolved model.*
- subroutine, public `rk2_ss_integrator::tl_tendencies (t, y, ys, res)`  
*Tendencies for the tangent linear model of the uncoupled resolved dynamics in point ystar for perturbation deltatay.*
- subroutine, public `rk2_ss_integrator::ss_step (y, ys, t, dt, dtn, res)`  
*Routine to perform a stochastic integration step of the unresolved uncoupled dynamics (Heun algorithm). The incremented time is returned.*
- subroutine, public `rk2_ss_integrator::ss_tl_step (y, ys, t, dt, dtn, res)`  
*Routine to perform a stochastic integration step of the unresolved uncoupled tangent linear dynamics (Heun algorithm). The incremented time is returned.*

## Variables

- real(kind=8), dimension(:), allocatable `rk2_ss_integrator::dwar`
- real(kind=8), dimension(:), allocatable `rk2_ss_integrator::dwor`  
*Standard gaussian noise buffers.*
- real(kind=8), dimension(:), allocatable `rk2_ss_integrator::anoise`  
*Additive noise term.*
- real(kind=8), dimension(:), allocatable `rk2_ss_integrator::buf_y1`
- real(kind=8), dimension(:), allocatable `rk2_ss_integrator::buf_f0`
- real(kind=8), dimension(:), allocatable `rk2_ss_integrator::buf_f1`  
*Integration buffers.*

## 10.26 rk2\_stoch\_integrator.f90 File Reference

## Modules

- module `rk2_stoch_integrator`  
*Module with the stochastic rk2 integration routines.*

## Functions/Subroutines

- subroutine, public `rk2_stoch_integrator::init_integrator` (`force`)  
*Subroutine to initialize the integrator.*
- subroutine `rk2_stoch_integrator::tendencies` (`t, y, res`)  
*Routine computing the tendencies of the selected model.*
- subroutine, public `rk2_stoch_integrator::step` (`y, t, dt, dtn, res, tend`)  
*Routine to perform a stochastic step of the selected dynamics (Heun algorithm). The incremented time is returned.*

## Variables

- real(kind=8), dimension(:), allocatable `rk2_stoch_integrator::dwar`
- real(kind=8), dimension(:), allocatable `rk2_stoch_integrator::dwau`
- real(kind=8), dimension(:), allocatable `rk2_stoch_integrator::dwor`
- real(kind=8), dimension(:), allocatable `rk2_stoch_integrator::dwou`  
*Standard gaussian noise buffers.*
- real(kind=8), dimension(:), allocatable `rk2_stoch_integrator::buf_y1`
- real(kind=8), dimension(:), allocatable `rk2_stoch_integrator::buf_f0`
- real(kind=8), dimension(:), allocatable `rk2_stoch_integrator::buf_f1`  
*Integration buffers.*
- real(kind=8), dimension(:), allocatable `rk2_stoch_integrator::anoise`  
*Additive noise term.*
- type(coolist), dimension(:), allocatable `rk2_stoch_integrator::int_tensor`  
*Dummy tensor that will hold the tendencies tensor.*

## 10.27 rk2\_tl\_ad\_integrator.f90 File Reference

### Modules

- module [tl\\_ad\\_integrator](#)

*Tangent Linear (TL) and Adjoint (AD) model versions of MAOOAM. Integrators module.*

### Functions/Subroutines

- subroutine, public [tl\\_ad\\_integrator::init\\_tl\\_ad\\_integrator](#)

*Routine to initialise the integration buffers.*

- subroutine, public [tl\\_ad\\_integrator::ad\\_step](#) (y, ystar, t, dt, res)

*Routine to perform an integration step (Heun algorithm) of the adjoint model. The incremented time is returned.*

- subroutine, public [tl\\_ad\\_integrator::tl\\_step](#) (y, ystar, t, dt, res)

*Routine to perform an integration step (Heun algorithm) of the tangent linear model. The incremented time is returned.*

### Variables

- real(kind=8), dimension(:), allocatable [tl\\_ad\\_integrator::buf\\_y1](#)

*Buffer to hold the intermediate position (Heun algorithm) of the tangent linear model.*

- real(kind=8), dimension(:), allocatable [tl\\_ad\\_integrator::buf\\_f0](#)

*Buffer to hold tendencies at the initial position of the tangent linear model.*

- real(kind=8), dimension(:), allocatable [tl\\_ad\\_integrator::buf\\_f1](#)

*Buffer to hold tendencies at the intermediate position of the tangent linear model.*

## 10.28 rk2\_WL\_integrator.f90 File Reference

### Modules

- module [rk2\\_wl\\_integrator](#)

*Module with the WL rk2 integration routines.*

### Functions/Subroutines

- subroutine, public [rk2\\_wl\\_integrator::init\\_integrator](#)

*Subroutine that initialize the MARs, the memory unit and the integration buffers.*

- subroutine [rk2\\_wl\\_integrator::compute\\_m1](#) (y)

*Routine to compute the  $M_1$  term.*

- subroutine [rk2\\_wl\\_integrator::compute\\_m2](#) (y)

*Routine to compute the  $M_2$  term.*

- subroutine, public [rk2\\_wl\\_integrator::step](#) (y, t, dt, dtn, res, tend)

*Routine to perform an integration step (Heun algorithm) of the WL system. The incremented time is returned.*

- subroutine, public [rk2\\_wl\\_integrator::full\\_step](#) (y, t, dt, dtn, res)

*Routine to perform an integration step (Heun algorithm) of the full stochastic system. The incremented time is returned.*

## Variables

- real(kind=8), dimension(:), allocatable `rk2_wl_integrator::buf_y1`
- real(kind=8), dimension(:), allocatable `rk2_wl_integrator::buf_f0`
- real(kind=8), dimension(:), allocatable `rk2_wl_integrator::buf_f1`  
*Integration buffers.*
- real(kind=8), dimension(:), allocatable `rk2_wl_integrator::buf_m2`
- real(kind=8), dimension(:), allocatable `rk2_wl_integrator::buf_m1`
- real(kind=8), dimension(:), allocatable `rk2_wl_integrator::buf_m3`
- real(kind=8), dimension(:), allocatable `rk2_wl_integrator::buf_m`
- real(kind=8), dimension(:), allocatable `rk2_wl_integrator::buf_m3s`  
*Dummy buffers holding the terms /\$M\_i.*
- real(kind=8), dimension(:), allocatable `rk2_wl_integrator::anoise`  
*Additive noise term.*
- real(kind=8), dimension(:), allocatable `rk2_wl_integrator::dwar`
- real(kind=8), dimension(:), allocatable `rk2_wl_integrator::dwau`
- real(kind=8), dimension(:), allocatable `rk2_wl_integrator::dwor`
- real(kind=8), dimension(:), allocatable `rk2_wl_integrator::dwou`  
*Standard gaussian noise buffers.*
- real(kind=8), dimension(:, :), allocatable `rk2_wl_integrator::x1`  
*Buffer holding the subsequent states of the first MAR.*
- real(kind=8), dimension(:, :), allocatable `rk2_wl_integrator::x2`  
*Buffer holding the subsequent states of the second MAR.*

## 10.29 rk4\_integrator.f90 File Reference

### Modules

- module `integrator`  
*Module with the integration routines.*

### Functions/Subroutines

- subroutine, public `integrator::init_integrator`  
*Routine to initialise the integration buffers.*
- subroutine `integrator::tendencies` (`t, y, res`)  
*Routine computing the tendencies of the model.*
- subroutine, public `integrator::step` (`y, t, dt, res`)  
*Routine to perform an integration step (Heun algorithm). The incremented time is returned.*

## Variables

- real(kind=8), dimension(:), allocatable `integrator::buf_ka`  
*Buffer A to hold tendencies.*
- real(kind=8), dimension(:), allocatable `integrator::buf_kb`  
*Buffer B to hold tendencies.*

## 10.30 rk4\_tl\_ad\_integrator.f90 File Reference

### Modules

- module [tl\\_ad\\_integrator](#)

*Tangent Linear (TL) and Adjoint (AD) model versions of MAOOAM. Integrators module.*

### Functions/Subroutines

- subroutine, public [tl\\_ad\\_integrator::init\\_tl\\_ad\\_integrator](#)

*Routine to initialise the integration buffers.*

- subroutine, public [tl\\_ad\\_integrator::ad\\_step](#) (y, ystar, t, dt, res)

*Routine to perform an integration step (Heun algorithm) of the adjoint model. The incremented time is returned.*

- subroutine, public [tl\\_ad\\_integrator::tl\\_step](#) (y, ystar, t, dt, res)

*Routine to perform an integration step (Heun algorithm) of the tangent linear model. The incremented time is returned.*

### Variables

- real(kind=8), dimension(:), allocatable [tl\\_ad\\_integrator::buf\\_ka](#)

*Buffer to hold tendencies in the RK4 scheme for the tangent linear model.*

- real(kind=8), dimension(:), allocatable [tl\\_ad\\_integrator::buf\\_kb](#)

*Buffer to hold tendencies in the RK4 scheme for the tangent linear model.*

## 10.31 sf\_def.f90 File Reference

### Modules

- module [sf\\_def](#)

*Module to select the resolved-unresolved components.*

### Functions/Subroutines

- subroutine, public [sf\\_def::load\\_sf](#)

*Subroutine to load the unresolved variable definition vector SF from SF.nml if it exists. If it does not, then write SF.nml with no unresolved variables specified (null vector).*

## Variables

- logical `sf_def::exists`  
*Boolean to test for file existence.*
- integer, dimension(:), allocatable, public `sf_def::sf`  
*Unresolved variable definition vector.*
- integer, dimension(:), allocatable, public `sf_def::ind`
- integer, dimension(:), allocatable, public `sf_def::rind`  
*Unresolved reduction indices.*
- integer, dimension(:), allocatable, public `sf_def::sl_ind`
- integer, dimension(:), allocatable, public `sf_def::sl_rind`  
*Resolved reduction indices.*
- integer, public `sf_def::n_unres`  
*Number of unresolved variables.*
- integer, public `sf_def::n_res`  
*Number of resolved variables.*
- integer, dimension(:, :, :), allocatable, public `sf_def::bar`
- integer, dimension(:, :, :), allocatable, public `sf_def::bau`
- integer, dimension(:, :, :), allocatable, public `sf_def::bor`
- integer, dimension(:, :, :), allocatable, public `sf_def::bou`  
*Filter matrices.*

## 10.32 sqrt\_mod.f90 File Reference

### Modules

- module `sqr_mod`  
*Utility module with various routine to compute matrix square root.*

### Functions/Subroutines

- subroutine, public `sqr_mod::init_sqrt`
- subroutine, public `sqr_mod::sqrtm` (A, sqA, info, info\_triu, bs)  
*Routine to compute a real square-root of a matrix.*
- logical function `sqr_mod::selectev` (a, b)
- subroutine `sqr_mod::sqrtm_triu` (A, sqA, info, bs)
- subroutine `sqr_mod::csqrtm_triu` (A, sqA, info, bs)
- subroutine `sqr_mod::rsf2csf` (T, Z, Tz, Zz)
- subroutine, public `sqr_mod::chol` (A, sqA, info)  
*Routine to perform a Cholesky decomposition.*
- subroutine, public `sqr_mod::sqrtm_svd` (A, sqA, info, info\_triu, bs)  
*Routine to compute a real square-root of a matrix via a SVD decomposition.*

### Variables

- real(kind=8), dimension(:), allocatable `sqr_mod::work`
- integer `sqr_mod::lwork`
- real(kind=8), parameter `sqr_mod::real_eps` = 2.2204460492503131e-16

## 10.33 stat.f90 File Reference

### Modules

- module **stat**  
*Statistics accumulators.*

### Functions/Subroutines

- subroutine, public **stat::init\_stat**  
*Initialise the accumulators.*
- subroutine, public **stat::acc** (x)  
*Accumulate one state.*
- real(kind=8) function, dimension(0:ndim), public **stat::mean** ()  
*Function returning the mean.*
- real(kind=8) function, dimension(0:ndim), public **stat::var** ()  
*Function returning the variance.*
- integer function, public **stat::iter** ()  
*Function returning the number of data accumulated.*
- subroutine, public **stat::reset**  
*Routine resetting the accumulators.*

### Variables

- integer **stat::i** =0  
*Number of stats accumulated.*
- real(kind=8), dimension(:), allocatable **stat::m**  
*Vector storing the inline mean.*
- real(kind=8), dimension(:), allocatable **stat::mprev**  
*Previous mean vector.*
- real(kind=8), dimension(:), allocatable **stat::v**  
*Vector storing the inline variance.*
- real(kind=8), dimension(:), allocatable **stat::mtmp**

## 10.34 stoch\_mod.f90 File Reference

### Modules

- module **stoch\_mod**  
*Utility module containing the stochastic related routines.*

## Functions/Subroutines

- real(kind=8) function, public **stoch\_mod::gasdev** ()
  - subroutine, public **stoch\_mod::stoch\_vec** (dW)
 

*Routine to fill a vector with standard Gaussian noise process values.*
- subroutine, public **stoch\_mod::stoch\_atm\_vec** (dW)
 

*routine to fill the atmospheric component of a vector with standard gaussian noise process values*
- subroutine, public **stoch\_mod::stoch\_atm\_res\_vec** (dW)
 

*routine to fill the resolved atmospheric component of a vector with standard gaussian noise process values*
- subroutine, public **stoch\_mod::stoch\_atm\_unres\_vec** (dW)
 

*routine to fill the unresolved atmospheric component of a vector with standard gaussian noise process values*
- subroutine, public **stoch\_mod::stoch\_oc\_vec** (dW)
 

*routine to fill the oceanic component of a vector with standard gaussian noise process values*
- subroutine, public **stoch\_mod::stoch\_oc\_res\_vec** (dW)
 

*routine to fill the resolved oceanic component of a vector with standard gaussian noise process values*
- subroutine, public **stoch\_mod::stoch\_oc\_unres\_vec** (dW)
 

*routine to fill the unresolved oceanic component of a vector with standard gaussian noise process values*

## Variables

- integer **stoch\_mod::iset** =0
- real(kind=8) **stoch\_mod::gset**

## 10.35 stoch\_params.f90 File Reference

### Modules

- module **stoch\_params**

*The stochastic models parameters module.*

## Functions/Subroutines

- subroutine **stoch\_params::init\_stoch\_params**

*Stochastic parameters initialization routine.*

## Variables

- real(kind=8) **stoch\_params::mnuti**

*Multiplicative noise update time interval.*
- real(kind=8) **stoch\_params::t\_trans\_stoch**

*Transient time period of the stochastic model evolution.*
- real(kind=8) **stoch\_params::q\_ar**

*Atmospheric resolved component noise amplitude.*
- real(kind=8) **stoch\_params::q\_au**

*Atmospheric unresolved component noise amplitude.*
- real(kind=8) **stoch\_params::q\_or**

*Oceanic resolved component noise amplitude.*

- real(kind=8) `stoch_params::q_ou`  
*Oceanic unresolved component noise amplitude.*
- real(kind=8) `stoch_params::dtn`  
*Square root of the timestep.*
- real(kind=8) `stoch_params::eps_pert`  
*Perturbation parameter for the coupling.*
- real(kind=8) `stoch_params::tdelta`  
*Time separation parameter.*
- real(kind=8) `stoch_params::muti`  
*Memory update time interval.*
- real(kind=8) `stoch_params::meml`  
*Time over which the memory kernel is integrated.*
- real(kind=8) `stoch_params::t_trans_mem`  
*Transient time period to initialize the memory term.*
- character(len=4) `stoch_params::x_int_mode`  
*Integration mode for the resolved component.*
- real(kind=8) `stoch_params::dts`  
*Intrinsic resolved dynamics time step.*
- integer `stoch_params::mems`  
*Number of steps in the memory kernel integral.*
- real(kind=8) `stoch_params::dtsn`  
*Square root of the intrinsic resolved dynamics time step.*
- real(kind=8) `stoch_params::maxint`  
*Upper integration limit of the correlations.*
- character(len=4) `stoch_params::load_mode`  
*Loading mode for the correlations.*
- character(len=4) `stoch_params::int_corr_mode`  
*Correlation integration mode.*
- character(len=4) `stoch_params::mode`  
*Stochastic mode parameter.*

## 10.36 tensor.f90 File Reference

### Data Types

- type `tensor::coolist_elem`  
*Coordinate list element type. Elementary elements of the sparse tensors.*
- type `tensor::coolist_elem4`  
*4d coordinate list element type. Elementary elements of the 4d sparse tensors.*
- type `tensor::coolist`  
*Coordinate list. Type used to represent the sparse tensor.*
- type `tensor::coolist4`  
*4d coordinate list. Type used to represent the rank-4 sparse tensor.*

### Modules

- module `tensor`  
*Tensor utility module.*

## Functions/Subroutines

- subroutine, public [tensor::copy\\_coo](#) (src, dst)  
*Routine to copy a coolist.*
- subroutine, public [tensor::mat\\_to\\_coo](#) (src, dst)  
*Routine to convert a matrix to a tensor.*
- subroutine, public [tensor::sparse\\_mul3](#) (coolist\_ijk, arr\_j, arr\_k, res)  
*Sparse multiplication of a tensor with two vectors:*  $\sum_{j,k=0}^{ndim} \mathcal{T}_{i,j,k} a_j b_k.$
- subroutine, public [tensor::jsparse\\_mul](#) (coolist\_ijk, arr\_j, jcoo\_ij)  
*Sparse multiplication of two tensors to determine the Jacobian:*

$$J_{i,j} = \sum_{k=0}^{ndim} (\mathcal{T}_{i,j,k} + \mathcal{T}_{i,k,j}) a_k.$$

*It's implemented slightly differently: for every  $\mathcal{T}_{i,j,k}$ , we add to  $J_{i,j}$  as follows:*

$$J_{i,j} = J_{i,j} + \mathcal{T}_{i,j,k} a_k J_{i,k} = J_{i,k} + \mathcal{T}_{i,j,k} a_j$$

*This version return a coolist (sparse tensor).*

- subroutine, public [tensor::jsparse\\_mul\\_mat](#) (coolist\_ijk, arr\_j, jcoo\_ij)

*Sparse multiplication of two tensors to determine the Jacobian:*

$$J_{i,j} = \sum_{k=0}^{ndim} (\mathcal{T}_{i,j,k} + \mathcal{T}_{i,k,j}) a_k.$$

*It's implemented slightly differently: for every  $\mathcal{T}_{i,j,k}$ , we add to  $J_{i,j}$  as follows:*

$$J_{i,j} = J_{i,j} + \mathcal{T}_{i,j,k} a_k J_{i,k} = J_{i,k} + \mathcal{T}_{i,j,k} a_j$$

*This version return a matrix.*

- subroutine, public [tensor::sparse\\_mul2](#) (coolist\_ij, arr\_j, res)

*Sparse multiplication of a 2d sparse tensor with a vector:*  $\sum_{j=0}^{ndim} \mathcal{T}_{i,j,k} a_j.$

- subroutine, public [tensor::simplify](#) (tensor)

*Routine to simplify a coolist (sparse tensor). For each index i, it upper triangularize the matrix*

$$\mathcal{T}_{i,j,k} \quad 0 \leq j, k \leq ndim.$$

- subroutine, public [tensor::add\\_elem](#) (t, i, j, k, v)

*Subroutine to add element to a coolist.*

- subroutine, public [tensor::add\\_check](#) (t, i, j, k, v, dst)

*Subroutine to add element to a coolist and check for overflow. Once the t buffer tensor is full, add it to the destination buffer.*

- subroutine, public [tensor::add\\_to\\_tensor](#) (src, dst)

*Routine to add a rank-3 tensor to another one.*

- subroutine, public [tensor::print\\_tensor](#) (t, s)

*Routine to print a rank 3 tensor coolist.*

- subroutine, public [tensor::write\\_tensor\\_to\\_file](#) (s, t)

*Load a rank-4 tensor coolist from a file definition.*

- subroutine, public [tensor::load\\_tensor\\_from\\_file](#) (s, t)

*Load a rank-4 tensor coolist from a file definition.*

- subroutine, public [tensor::add\\_matc\\_to\\_tensor](#) (i, src, dst)

*Routine to add a matrix to a rank-3 tensor.*

- subroutine, public [tensor::add\\_matc\\_to\\_tensor4](#) (i, j, src, dst)

*Routine to add a matrix to a rank-4 tensor.*

- subroutine, public `tensor::add_vec_jk_to_tensor` (j, k, src, dst)
 

*Routine to add a vector to a rank-3 tensor.*
- subroutine, public `tensor::add_vec_ikl_to_tensor4_perm` (i, k, l, src, dst)
 

*Routine to add a vector to a rank-4 tensor plus permutation.*
- subroutine, public `tensor::add_vec_ikl_to_tensor4` (i, k, l, src, dst)
 

*Routine to add a vector to a rank-4 tensor.*
- subroutine, public `tensor::add_vec_ijk_to_tensor4` (i, j, k, src, dst)
 

*Routine to add a vector to a rank-4 tensor.*
- subroutine, public `tensor::tensor_to_coo` (src, dst)
 

*Routine to convert a rank-3 tensor from matrix to coolist representation.*
- subroutine, public `tensor::tensor4_to_coo4` (src, dst)
 

*Routine to convert a rank-4 tensor from matrix to coolist representation.*
- subroutine, public `tensor::print_tensor4` (t)
 

*Routine to print a rank-4 tensor coolist.*
- subroutine, public `tensor::sparse_mul3_mat` (coolist\_ijk, arr\_k, res)
 

*Sparse multiplication of a rank-3 tensor coolist with a vector:  $\sum_{k=0}^{ndim} \mathcal{T}_{i,j,k} b_k$ . Its output is a matrix.*
- subroutine, public `tensor::sparse_mul4` (coolist\_ijkl, arr\_j, arr\_k, arr\_l, res)
 

*Sparse multiplication of a rank-4 tensor coolist with three vectors:  $\sum_{j,k,l=0}^{ndim} \mathcal{T}_{i,j,k,l} a_j b_k c_l$ .*
- subroutine, public `tensor::sparse_mul4_mat` (coolist\_ijkl, arr\_k, arr\_l, res)
 

*Sparse multiplication of a tensor with two vectors:  $\sum_{k,l=0}^{ndim} \mathcal{T}_{i,j,k,l} b_k c_l$ .*
- subroutine, public `tensor::sparse_mul2_j` (coolist\_ijk, arr\_j, res)
 

*Sparse multiplication of a 3d sparse tensor with a vectors:  $\sum_{j=0}^{ndim} \mathcal{T}_{i,j,k} a_j$ .*
- subroutine, public `tensor::sparse_mul2_k` (coolist\_ijk, arr\_k, res)
 

*Sparse multiplication of a rank-3 sparse tensor coolist with a vector:  $\sum_{k=0}^{ndim} \mathcal{T}_{i,j,k} a_k$ .*
- subroutine, public `tensor::coo_to_mat_ik` (src, dst)
 

*Routine to convert a rank-3 tensor coolist component into a matrix with i and k indices.*
- subroutine, public `tensor::coo_to_mat_ij` (src, dst)
 

*Routine to convert a rank-3 tensor coolist component into a matrix with i and j indices.*
- subroutine, public `tensor::coo_to_mat_i` (i, src, dst)
 

*Routine to convert a rank-3 tensor coolist component into a matrix.*
- subroutine, public `tensor::coo_to_vec_jk` (j, k, src, dst)
 

*Routine to convert a rank-3 tensor coolist component into a vector.*
- subroutine, public `tensor::coo_to_mat_j` (j, src, dst)
 

*Routine to convert a rank-3 tensor coolist component into a matrix.*
- subroutine, public `tensor::sparse_mul4_with_mat_jl` (coolist\_ijkl, mat\_jl, res)
 

*Sparse multiplication of a rank-4 tensor coolist with a matrix :  $\sum_{j,l=0}^{ndim} \mathcal{T}_{i,j,k,l} m_{j,l}$ .*
- subroutine, public `tensor::sparse_mul4_with_mat_kl` (coolist\_ijkl, mat\_kl, res)
 

*Sparse multiplication of a rank-4 tensor coolist with a matrix :  $\sum_{j,l=0}^{ndim} \mathcal{T}_{i,j,k,l} m_{k,l}$ .*
- subroutine, public `tensor::sparse_mul3_with_mat` (coolist\_ijk, mat\_jk, res)
 

*Sparse multiplication of a rank-3 tensor coolist with a matrix:  $\sum_{j,k=0}^{ndim} \mathcal{T}_{i,j,k} m_{j,k}$ .*
- subroutine, public `tensor::matc_to_coo` (src, dst)

- Routine to convert a matrix to a rank-3 tensor.*
- subroutine, public [tensor::scal\\_mul\\_coo](#) (s, t)  
*Routine to multiply a rank-3 tensor by a scalar.*
  - logical function, public [tensor::tensor\\_empty](#) (t)  
*Test if a rank-3 tensor coolist is empty.*
  - logical function, public [tensor::tensor4\\_empty](#) (t)  
*Test if a rank-4 tensor coolist is empty.*
  - subroutine, public [tensor::load\\_tensor4\\_from\\_file](#) (s, t)  
*Load a rank-4 tensor coolist from a file definition.*
  - subroutine, public [tensor::write\\_tensor4\\_to\\_file](#) (s, t)  
*Load a rank-4 tensor coolist from a file definition.*

## Variables

- real(kind=8), parameter [tensor::real\\_eps](#) = 2.2204460492503131e-16  
*Parameter to test the equality with zero.*

## 10.37 test\_aotensor.f90 File Reference

### Functions/Subroutines

- program [test\\_aotensor](#)  
*Small program to print the inner products.*

#### 10.37.1 Function/Subroutine Documentation

##### 10.37.1.1 program test\_aotensor ( )

Small program to print the inner products.

#### Copyright

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Definition at line 13 of file test\_aotensor.f90.

## 10.38 test\_corr.f90 File Reference

### Functions/Subroutines

- program [test\\_corr](#)  
*Small program to print the correlation and covariance matrices.*

### 10.38.1 Function/Subroutine Documentation

#### 10.38.1.1 program test\_corr( )

Small program to print the correlation and covariance matrices.

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Definition at line 12 of file `test_corr.f90`.

## 10.39 test\_corr\_tensor.f90 File Reference

### Functions/Subroutines

- program [test\\_corr\\_tensor](#)  
*Small program to print the time correlations tensors.*

#### 10.39.1 Function/Subroutine Documentation

##### 10.39.1.1 program test\_corr\_tensor( )

Small program to print the time correlations tensors.

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Definition at line 12 of file `test_corr_tensor.f90`.

## 10.40 test\_dec\_tensor.f90 File Reference

### Functions/Subroutines

- program [test\\_dec\\_tensor](#)  
*Small program to print the decomposed tensors.*

#### 10.40.1 Function/Subroutine Documentation

##### 10.40.1.1 program test\_dec\_tensor( )

Small program to print the decomposed tensors.

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Definition at line 12 of file `test_dec_tensor.f90`.

## 10.41 test\_inprod\_analytic.f90 File Reference

### Functions/Subroutines

- program [inprod\\_analytic\\_test](#)

*Small program to print the inner products.*

#### 10.41.1 Function/Subroutine Documentation

##### 10.41.1.1 program inprod\_analytic\_test( )

Small program to print the inner products.

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#### Remarks

Print in the same order as test\_inprod.lua

Definition at line 18 of file test\_inprod\_analytic.f90.

## 10.42 test\_MAR.f90 File Reference

### Functions/Subroutines

- program [test\\_mar](#)

*Small program to test the Multivariate AutoRegressive model.*

#### 10.42.1 Function/Subroutine Documentation

##### 10.42.1.1 program test\_mar( )

Small program to test the Multivariate AutoRegressive model.

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Definition at line 12 of file test\_MAR.f90.

## 10.43 test\_memory.f90 File Reference

### Functions/Subroutines

- program [test\\_memory](#)  
*Small program to test the WL memory module.*

#### 10.43.1 Function/Subroutine Documentation

##### 10.43.1.1 program test\_memory( )

Small program to test the WL memory module.

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Definition at line 12 of file test\_memory.f90.

## 10.44 test\_MTV\_int\_tensor.f90 File Reference

### Functions/Subroutines

- program [test\\_mtv\\_int\\_tensor](#)  
*Small program to print the MTV integrated tensors.*

#### 10.44.1 Function/Subroutine Documentation

##### 10.44.1.1 program test\_mtv\_int\_tensor( )

Small program to print the MTV integrated tensors.

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Definition at line 12 of file test\_MTV\_int\_tensor.f90.

## 10.45 test\_MTV\_sigma\_tensor.f90 File Reference

### Functions/Subroutines

- program [test\\_sigma](#)  
*Small program to test the MTV noise sigma matrices.*

### 10.45.1 Function/Subroutine Documentation

#### 10.45.1.1 program test\_sigma( )

Small program to test the MTV noise sigma matrices.

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Definition at line 12 of file test\_MTV\_sigma\_tensor.f90.

## 10.46 test\_sqrtm.f90 File Reference

### Functions/Subroutines

- program [test\\_sqrtm](#)  
*Small program to test the matrix square-root module.*

### 10.46.1 Function/Subroutine Documentation

#### 10.46.1.1 program test\_sqrtm( )

Small program to test the matrix square-root module.

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Definition at line 12 of file test\_sqrtm.f90.

## 10.47 test\_tl\_ad.f90 File Reference

### Functions/Subroutines

- program [test\\_tl\\_ad](#)  
*Tests for the Tangent Linear (TL) and Adjoint (AD) model versions of MAOOAM.*

### 10.47.1 Function/Subroutine Documentation

#### 10.47.1.1 program test\_tl\_ad( )

Tests for the Tangent Linear (TL) and Adjoint (AD) model versions of MAOOAM.

##### Copyright

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Definition at line 14 of file test\_tl\_ad.f90.

## 10.48 test\_WL\_tensor.f90 File Reference

### Functions/Subroutines

- program [test\\_wl\\_tensor](#)  
*Small program to print the WL tensors.*

#### 10.48.1 Function/Subroutine Documentation

##### 10.48.1.1 program test\_wl\_tensor ( )

Small program to print the WL tensors.

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Definition at line 11 of file test\_WL\_tensor.f90.

## 10.49 tl\_ad\_tensor.f90 File Reference

### Modules

- module [tl\\_ad\\_tensor](#)  
*Tangent Linear (TL) and Adjoint (AD) model versions of MAOOAM. Tensors definition module.*

### Functions/Subroutines

- type(coolist) function, dimension(ndim) [tl\\_ad\\_tensor::jacobian](#) (ystar)  
*Compute the Jacobian of MAOOAM in point ystar.*
- real(kind=8) function, dimension(ndim, ndim), public [tl\\_ad\\_tensor::jacobian\\_mat](#) (ystar)  
*Compute the Jacobian of MAOOAM in point ystar.*
- subroutine, public [tl\\_ad\\_tensor::init\\_tltensor](#)  
*Routine to initialize the TL tensor.*
- subroutine [tl\\_ad\\_tensor::compute\\_tltensor](#) (func)  
*Routine to compute the TL tensor from the original MAOOAM one.*
- subroutine [tl\\_ad\\_tensor::tl\\_add\\_count](#) (i, j, k, v)  
*Subroutine used to count the number of TL tensor entries.*
- subroutine [tl\\_ad\\_tensor::tl\\_coeff](#) (i, j, k, v)  
*Subroutine used to compute the TL tensor entries.*
- subroutine, public [tl\\_ad\\_tensor::init\\_adtensor](#)  
*Routine to initialize the AD tensor.*
- subroutine [tl\\_ad\\_tensor::compute\\_adtensor](#) (func)  
*Subroutine to compute the AD tensor from the original MAOOAM one.*
- subroutine [tl\\_ad\\_tensor::ad\\_add\\_count](#) (i, j, k, v)  
*Subroutine used to count the number of AD tensor entries.*
- subroutine [tl\\_ad\\_tensor::ad\\_coeff](#) (i, j, k, v)

- subroutine, public `tl_ad_tensor::init_adtensor_ref`  
*Alternate method to initialize the AD tensor from the TL tensor.*
- subroutine `tl_ad_tensor::compute_adtensor_ref` (`func`)  
*Alternate subroutine to compute the AD tensor from the TL one.*
- subroutine `tl_ad_tensor::ad_add_count_ref` (`i, j, k, v`)  
*Alternate subroutine used to count the number of AD tensor entries from the TL tensor.*
- subroutine `tl_ad_tensor::ad_coeff_ref` (`i, j, k, v`)  
*Alternate subroutine used to compute the AD tensor entries from the TL tensor.*
- subroutine, public `tl_ad_tensor::ad` (`t, ystar, deltay, buf`)  
*Tendencies for the AD of MAOOAM in point `ystar` for perturbation `deltay`.*
- subroutine, public `tl_ad_tensor::tl` (`t, ystar, deltay, buf`)  
*Tendencies for the TL of MAOOAM in point `ystar` for perturbation `deltay`.*

## Variables

- real(kind=8), parameter `tl_ad_tensor::real_eps` = `2.2204460492503131e-16`  
*Epsilon to test equality with 0.*
- integer, dimension(:), allocatable `tl_ad_tensor::count_elems`  
*Vector used to count the tensor elements.*
- type(coolist), dimension(:), allocatable, public `tl_ad_tensor::tltensor`  
*Tensor representation of the Tangent Linear tendencies.*
- type(coolist), dimension(:), allocatable, public `tl_ad_tensor::adtensor`  
*Tensor representation of the Adjoint tendencies.*

## 10.50 util.f90 File Reference

### Modules

- module `util`  
*Utility module.*

### Functions/Subroutines

- character(len=20) function, public `util::str` (`k`)  
*Convert an integer to string.*
- character(len=40) function, public `util::rstr` (`x, fm`)  
*Convert a real to string with a given format.*
- integer function, dimension(size(`s`)), public `util::isin` (`c, s`)  
*Determine if a character is in a string and where.*
- subroutine, public `util::init_random_seed` ()  
*Random generator initialization routine.*
- integer function `lcg` (`s`)
- subroutine, public `util::piksrt` (`k, arr, par`)  
*Simple card player sorting function.*
- subroutine, public `util::init_one` (`A`)  
*Initialize a square matrix A as a unit matrix.*
- real(kind=8) function, public `util::mat_trace` (`A`)
- real(kind=8) function, public `util::mat_contract` (`A, B`)

- subroutine, public [util::choldc](#) (a, p)
- subroutine, public [util::printmat](#) (A)
- subroutine, public [util::cprintmat](#) (A)
- real(kind=8) function, dimension(size(a, 1), size(a, 2)), public [util::invmat](#) (A)
- subroutine, public [util::triu](#) (A, T)
- subroutine, public [util::diag](#) (A, d)
- subroutine, public [util::cdiag](#) (A, d)
- integer function, public [util::floordiv](#) (i, j)
- subroutine, public [util::reduce](#) (A, Ared, n, ind, rind)
- subroutine, public [util::ireduce](#) (A, Ared, n, ind, rind)
- subroutine, public [util::vector\\_outer](#) (u, v, A)

## 10.50.1 Function/Subroutine Documentation

### 10.50.1.1 integer function init\_random\_seed::lcg ( integer(int64) s )

Definition at line 104 of file util.f90.

```

104      integer :: lcg
105      integer(int64) :: s
106      IF (s == 0) THEN
107          s = 104729
108      ELSE
109          s = mod(s, 4294967296_int64)
110      END IF
111      s = mod(s * 279470273_int64, 4294967291_int64)
112      lcg = int(mod(s, int(huge(0), int64)), kind(0))
113  END FUNCTION lcg

```

## 10.51 WL\_tensor.f90 File Reference

### Modules

- module [wl\\_tensor](#)

*The WL tensors used to integrate the model.*

### Functions/Subroutines

- subroutine, public [wl\\_tensor::init\\_wl\\_tensor](#)

*Subroutine to initialise the WL tensor.*

## Variables

- real(kind=8), dimension(:), allocatable, public `wl_tensor::m11`  
*First component of the M1 term.*
- type(coolist), dimension(:), allocatable, public `wl_tensor::m12`  
*Second component of the M1 term.*
- real(kind=8), dimension(:), allocatable, public `wl_tensor::m13`  
*Third component of the M1 term.*
- real(kind=8), dimension(:), allocatable, public `wl_tensor::m1tot`  
*Total M<sub>1</sub> vector.*
- type(coolist), dimension(:), allocatable, public `wl_tensor::m21`  
*First tensor of the M2 term.*
- type(coolist), dimension(:), allocatable, public `wl_tensor::m22`  
*Second tensor of the M2 term.*
- type(coolist), dimension(:, :), allocatable, public `wl_tensor::l1`  
*First linear tensor.*
- type(coolist), dimension(:, :), allocatable, public `wl_tensor::l2`  
*Second linear tensor.*
- type(coolist), dimension(:, :), allocatable, public `wl_tensor::l4`  
*Fourth linear tensor.*
- type(coolist), dimension(:, :), allocatable, public `wl_tensor::l5`  
*Fifth linear tensor.*
- type(coolist), dimension(:, :), allocatable, public `wl_tensor::ltot`  
*Total linear tensor.*
- type(coolist), dimension(:, :), allocatable, public `wl_tensor::b1`  
*First quadratic tensor.*
- type(coolist), dimension(:, :), allocatable, public `wl_tensor::b2`  
*Second quadratic tensor.*
- type(coolist), dimension(:, :), allocatable, public `wl_tensor::b3`  
*Third quadratic tensor.*
- type(coolist), dimension(:, :), allocatable, public `wl_tensor::b4`  
*Fourth quadratic tensor.*
- type(coolist), dimension(:, :), allocatable, public `wl_tensor::b14`  
*Joint 1st and 4th tensors.*
- type(coolist), dimension(:, :), allocatable, public `wl_tensor::b23`  
*Joint 2nd and 3rd tensors.*
- type(coolist4), dimension(:, :), allocatable, public `wl_tensor::mtot`  
*Tensor for the cubic terms.*
- real(kind=8), dimension(:), allocatable `wl_tensor::dumb_vec`  
*Dummy vector.*
- real(kind=8), dimension(:, :), allocatable `wl_tensor::dumb_mat1`  
*Dummy matrix.*
- real(kind=8), dimension(:, :), allocatable `wl_tensor::dumb_mat2`  
*Dummy matrix.*
- real(kind=8), dimension(:, :), allocatable `wl_tensor::dumb_mat3`  
*Dummy matrix.*
- real(kind=8), dimension(:, :), allocatable `wl_tensor::dumb_mat4`  
*Dummy matrix.*
- logical, public `wl_tensor::m12def`
- logical, public `wl_tensor::m21def`
- logical, public `wl_tensor::m22def`

- logical, public `wl_tensor::ldef`
- logical, public `wl_tensor::b14def`
- logical, public `wl_tensor::b23def`
- logical, public `wl_tensor::mdef`

*Boolean to (de)activate the computation of the terms.*

# Index

a  
    aotensor\_def, 26  
    inprod\_analytic::atm\_tensors, 221  
acc  
    stat, 150  
ad  
    tl\_ad\_tensor, 196  
ad\_add\_count  
    tl\_ad\_tensor, 196  
ad\_add\_count\_ref  
    tl\_ad\_tensor, 197  
ad\_coeff  
    tl\_ad\_tensor, 197  
ad\_coeff\_ref  
    tl\_ad\_tensor, 198  
ad\_step  
    tl\_ad\_integrator, 192  
add\_check  
    tensor, 165  
add\_count  
    aotensor\_def, 26  
add\_elem  
    tensor, 165  
add\_matc\_to\_tensor  
    tensor, 166  
add\_matc\_to\_tensor4  
    tensor, 167  
add\_to\_tensor  
    tensor, 168  
add\_vec\_ijk\_to\_tensor4  
    tensor, 169  
add\_vec\_ikl\_to\_tensor4  
    tensor, 169  
add\_vec\_ikl\_to\_tensor4\_perm  
    tensor, 170  
add\_vec\_jk\_to\_tensor  
    tensor, 171  
adtensor  
    tl\_ad\_tensor, 203  
ams  
    params, 101  
anoise  
    rk2\_mtv\_integrator, 117  
    rk2\_ss\_integrator, 123  
    rk2\_stoch\_integrator, 127  
    rk2\_wl\_integrator, 132  
aotensor  
    aotensor\_def, 29  
aotensor\_def, 25  
a, 26  
add\_count, 26  
aotensor, 29  
coeff, 26  
compute\_aotensor, 27  
count\_elems, 29  
init\_aotensor, 27  
kdelta, 28  
psi, 28  
real\_eps, 29  
t, 28  
theta, 28  
aotensor\_def.f90, 231  
atmos  
    inprod\_analytic, 67  
awavenum  
    inprod\_analytic, 67  
b  
    inprod\_analytic::atm\_tensors, 221  
b1  
    inprod\_analytic, 59  
    mtv\_int\_tensor, 92  
    wl\_tensor, 214  
b14  
    wl\_tensor, 214  
b14def  
    wl\_tensor, 214  
b2  
    inprod\_analytic, 59  
    mtv\_int\_tensor, 93  
    wl\_tensor, 214  
b23  
    wl\_tensor, 215  
b23def  
    wl\_tensor, 215  
b3  
    wl\_tensor, 215  
b4  
    wl\_tensor, 215  
bar  
    sf\_def, 137  
bau  
    sf\_def, 137  
betp  
    params, 101  
bor  
    sf\_def, 137  
bou  
    sf\_def, 137

btot  
     mtv\_int\_tensor, 93  
 buf\_f0  
     integrator, 79  
     rk2\_mtv\_integrator, 117  
     rk2\_ss\_integrator, 123  
     rk2\_stoch\_integrator, 127  
     rk2\_wl\_integrator, 132  
     tl\_ad\_integrator, 194  
 buf\_f1  
     integrator, 79  
     rk2\_mtv\_integrator, 117  
     rk2\_ss\_integrator, 123  
     rk2\_stoch\_integrator, 127  
     rk2\_wl\_integrator, 133  
     tl\_ad\_integrator, 194  
 buf\_g  
     rk2\_mtv\_integrator, 117  
 buf\_ka  
     integrator, 79  
     tl\_ad\_integrator, 194  
 buf\_kb  
     integrator, 79  
     tl\_ad\_integrator, 194  
 buf\_m  
     memory, 87  
     rk2\_wl\_integrator, 133  
 buf\_m1  
     rk2\_wl\_integrator, 133  
 buf\_m2  
     rk2\_wl\_integrator, 133  
 buf\_m3  
     memory, 87  
     rk2\_wl\_integrator, 133  
 buf\_m3s  
     rk2\_wl\_integrator, 133  
 buf\_y  
     mar, 82  
 buf\_y1  
     integrator, 79  
     rk2\_mtv\_integrator, 117  
     rk2\_ss\_integrator, 123  
     rk2\_stoch\_integrator, 127  
     rk2\_wl\_integrator, 133  
     tl\_ad\_integrator, 194  
 bxxx  
     dec\_tensor, 51  
 bxyy  
     dec\_tensor, 51  
 byxx  
     dec\_tensor, 51  
 byyx  
     dec\_tensor, 52  
 byyy  
     dec\_tensor, 52  
 c  
     inprod\_analytic::atm\_tensors, 221  
     inprod\_analytic::ocean\_tensors, 228  
 CLAIM  
     LICENSE.txt, 240  
 CONTRACT  
     LICENSE.txt, 240  
 ca  
     params, 101  
 calculate\_a  
     inprod\_analytic, 59  
 calculate\_b  
     inprod\_analytic, 60  
 calculate\_c\_atm  
     inprod\_analytic, 60  
 calculate\_c\_oc  
     inprod\_analytic, 60  
 calculate\_d  
     inprod\_analytic, 61  
 calculate\_g  
     inprod\_analytic, 61  
 calculate\_k  
     inprod\_analytic, 62  
 calculate\_m  
     inprod\_analytic, 62  
 calculate\_n  
     inprod\_analytic, 63  
 calculate\_o  
     inprod\_analytic, 63  
 calculate\_s  
     inprod\_analytic, 63  
 calculate\_w  
     inprod\_analytic, 64  
 cdiaq  
     util, 204  
 charge  
     LICENSE.txt, 240  
 chol  
     sqrt\_mod, 143  
 choldc  
     util, 204  
 co  
     params, 101  
 coeff  
     aotensor\_def, 26  
 comp\_corrint  
     int\_corr, 73  
 compg  
     rk2\_mtv\_integrator, 114  
 compute\_adtensor  
     tl\_ad\_tensor, 198  
 compute\_adtensor\_ref  
     tl\_ad\_tensor, 198  
 compute\_aotensor  
     aotensor\_def, 27  
 compute\_m1  
     rk2\_wl\_integrator, 129  
 compute\_m2  
     rk2\_wl\_integrator, 130

compute\_m3  
    memory, 85  
compute\_mult  
    rk2\_mtv\_integrator, 117  
compute\_mult\_sigma  
    sigma, 139  
compute\_tltensor  
    tl\_ad\_tensor, 199  
conditions  
    LICENSE.txt, 240  
coo\_to\_mat\_i  
    tensor, 172  
coo\_to\_mat\_ij  
    tensor, 173  
coo\_to\_mat\_ik  
    tensor, 173  
coo\_to\_mat\_j  
    tensor, 173  
coo\_to\_vec\_jk  
    tensor, 174  
copy  
    LICENSE.txt, 240  
copy\_coo  
    tensor, 174  
corr2int  
    int\_corr, 75  
corr\_i  
    corrmod, 41  
corr\_i\_full  
    corrmod, 41  
corr\_ij  
    corrmod, 41  
corr\_tensor, 29  
    dumb\_mat1, 31  
    dumb\_mat2, 31  
    dumb\_vec, 31  
    dy, 32  
    ddy, 32  
    exprm, 32  
    init\_corr\_tensor, 30  
    ydy, 32  
    yddy, 32  
    yy, 33  
corr\_tensor.f90, 232  
corrcmp  
    corrmod, 41  
corrcmp\_from\_def  
    corrmod, 34  
corrcmp\_from\_fit  
    corrmod, 38  
corrcmp\_from\_spline  
    corrmod, 38  
corrint  
    int\_corr, 75  
corrmod, 33  
    corr\_i, 41  
    corr\_i\_full, 41  
    corr\_ij, 41  
    corrcomp, 41  
    corrcomp\_from\_def, 34  
    corrcomp\_from\_fit, 38  
    corrcomp\_from\_spline, 38  
    fs, 39  
    init\_corr, 39  
    inv\_corr\_i, 42  
    inv\_corr\_i\_full, 42  
    khi, 42  
    klo, 42  
    mean, 42  
    mean\_full, 42  
    nspl, 43  
    splint, 40  
    xa, 43  
    y2, 43  
    ya, 43  
corrmod.f90, 232  
count\_elems  
    aotensor\_def, 29  
    tl\_ad\_tensor, 203  
cpa  
    params, 102  
cpo  
    params, 102  
cprintmat  
    util, 205  
csqrtn\_triu  
    sqrt\_mod, 144  
  
d  
    inprod\_analytic::atm\_tensors, 222  
    params, 102  
dec\_tensor, 44  
    bxxx, 51  
    bxxy, 51  
    bxyy, 51  
    byxx, 52  
    byxy, 52  
    byyy, 52  
    dumb, 52  
    ff\_tensor, 52  
    fs\_tensor, 53  
    hx, 53  
    hy, 53  
    init\_dec\_tensor, 45  
    init\_sub\_tensor, 49  
    lxx, 53  
    lxy, 53  
    lyx, 54  
    lyy, 54  
    reorder, 49  
    sf\_tensor, 54  
    ss\_tensor, 54  
    ss\_tl\_tensor, 54  
    suppress\_and, 50  
    suppress\_or, 50  
dec\_tensor.f90, 233  
delta

inprod\_analytic, 64  
 diag  
 util, 205  
 distribute  
 LICENSE.txt, 240  
 doc/def\_doc.md, 235  
 doc/gen\_doc.md, 235  
 doc/sto\_doc.md, 235  
 doc/tl\_ad\_doc.md, 235  
 dp  
 params, 102  
 dt  
 params, 102  
 dtn  
 stoch\_params, 158  
 dts  
 stoch\_params, 158  
 dtsn  
 stoch\_params, 158  
 dumb  
 dec\_tensor, 52  
 dumb\_mat1  
 corr\_tensor, 31  
 mtv\_int\_tensor, 93  
 sigma, 141  
 wl\_tensor, 215  
 dumb\_mat2  
 corr\_tensor, 31  
 mtv\_int\_tensor, 93  
 sigma, 141  
 wl\_tensor, 215  
 dumb\_mat3  
 mtv\_int\_tensor, 93  
 sigma, 141  
 wl\_tensor, 216  
 dumb\_mat4  
 mtv\_int\_tensor, 94  
 sigma, 141  
 wl\_tensor, 216  
 dumb\_vec  
 corr\_tensor, 31  
 mtv\_int\_tensor, 94  
 wl\_tensor, 216  
 dw  
 mar, 82  
 rk2\_mtv\_integrator, 118  
 dwar  
 rk2\_mtv\_integrator, 118  
 rk2\_ss\_integrator, 123  
 rk2\_stoch\_integrator, 127  
 rk2\_wl\_integrator, 133  
 dwau  
 rk2\_mtv\_integrator, 118  
 rk2\_stoch\_integrator, 127  
 rk2\_wl\_integrator, 134  
 dwmult  
 rk2\_mtv\_integrator, 118  
 dwor

rk2\_mtv\_integrator, 118  
 rk2\_ss\_integrator, 124  
 rk2\_stoch\_integrator, 128  
 rk2\_wl\_integrator, 134  
 dwou  
 rk2\_mtv\_integrator, 118  
 rk2\_stoch\_integrator, 128  
 rk2\_wl\_integrator, 134  
 dy  
 corr\_tensor, 32  
 dyy  
 corr\_tensor, 32  
 elems  
 tensor::coolist, 224  
 tensor::coolist4, 225  
 eps\_pert  
 stoch\_params, 158  
 epsa  
 params, 103  
 exists  
 ic\_def, 57  
 sf\_def, 137  
 expm  
 corr\_tensor, 32  
 f0  
 params, 103  
 FROM  
 LICENSE.txt, 240  
 ff\_tensor  
 dec\_tensor, 52  
 files  
 LICENSE.txt, 239  
 flambda  
 inprod\_analytic, 64  
 floordiv  
 util, 205  
 fs  
 cormod, 39  
 fs\_tensor  
 dec\_tensor, 53  
 full\_step  
 rk2\_mtv\_integrator, 114  
 rk2\_wl\_integrator, 130  
 func\_ij  
 int\_corr, 74  
 func\_ijkl  
 int\_corr, 75  
 g  
 inprod\_analytic::atm\_tensors, 222  
 params, 103  
 rk2\_mtv\_integrator, 118  
 ga  
 params, 103  
 gasdev  
 stoch\_mod, 153  
 go

params, 103  
gp  
    params, 104  
gset  
    stoch\_mod, 156

h  
    inprod\_analytic::atm\_wavenum, 223  
    inprod\_analytic::ocean\_wavenum, 229  
    params, 104

h1  
    mtv\_int\_tensor, 94

h2  
    mtv\_int\_tensor, 94

h3  
    mtv\_int\_tensor, 94

htot  
    mtv\_int\_tensor, 95

hx  
    dec\_tensor, 53

hy  
    dec\_tensor, 53

i  
    stat, 152

IMPLIED  
    LICENSE.txt, 241

ic  
    ic\_def, 57

ic\_def, 55  
    exists, 57  
    ic, 57  
    load\_ic, 55

ic\_def.f90, 235

ind  
    sf\_def, 137

ind1  
    sigma, 141

ind2  
    sigma, 141

init\_adtensor  
    tl\_ad\_tensor, 199

init\_adtensor\_ref  
    tl\_ad\_tensor, 199

init\_aotensor  
    aotensor\_def, 27

init\_corr  
    corrmod, 39

init\_corr\_tensor  
    corr\_tensor, 30

init\_corrint  
    int\_corr, 75

init\_dec\_tensor  
    dec\_tensor, 45

init\_g  
    rk2\_mtv\_integrator, 115

init\_inprod  
    inprod\_analytic, 65

init\_integrator

    integrator, 78  
    rk2\_mtv\_integrator, 115  
    rk2\_stoch\_integrator, 125  
    rk2\_wl\_integrator, 130

init\_mar  
    mar, 81

init\_memory  
    memory, 86

init\_mtv\_int\_tensor  
    mtv\_int\_tensor, 90

init\_nml  
    params, 100

init\_noise  
    rk2\_mtv\_integrator, 115

init\_one  
    util, 205

init\_params  
    params, 100

init\_random\_seed  
    util, 206

init\_sigma  
    sigma, 140

init\_sqrt  
    sqrt\_mod, 145

init\_ss\_integrator  
    rk2\_ss\_integrator, 121

init\_stat  
    stat, 150

init\_stoch\_params  
    stoch\_params, 158

init\_sub\_tensor  
    dec\_tensor, 49

init\_tl\_ad\_integrator  
    tl\_ad\_integrator, 193

init\_tltensor  
    tl\_ad\_tensor, 200

init\_wl\_tensor  
    wl\_tensor, 211

inprod\_analytic, 57  
    atmos, 67  
    awavenum, 67  
    b1, 59  
    b2, 59  
    calculate\_a, 59  
    calculate\_b, 60  
    calculate\_c\_atm, 60  
    calculate\_c\_oc, 60  
    calculate\_d, 61  
    calculate\_g, 61  
    calculate\_k, 62  
    calculate\_m, 62  
    calculate\_n, 63  
    calculate\_o, 63  
    calculate\_s, 63  
    calculate\_w, 64  
    delta, 64  
    flambda, 64  
    init\_inprod, 65

ocean, 67  
 owavenum, 67  
 s1, 66  
 s2, 66  
 s3, 66  
 s4, 66  
 inprod\_analytic.f90, 235  
 inprod\_analytic::atm\_tensors, 221  
   a, 221  
   b, 221  
   c, 221  
   d, 222  
   g, 222  
   s, 222  
 inprod\_analytic::atm\_wavenum, 222  
   h, 223  
   m, 223  
   nx, 223  
   ny, 223  
   p, 223  
   typ, 223  
 inprod\_analytic::ocean\_tensors, 227  
   c, 228  
   k, 228  
   m, 228  
   n, 228  
   o, 228  
   w, 229  
 inprod\_analytic::ocean\_wavenum, 229  
   h, 229  
   nx, 229  
   ny, 229  
   p, 230  
 inprod\_analytic\_test  
   test\_inprod\_analytic.f90, 265  
 int\_comp, 68  
   integrate, 68  
   midexp, 69  
   midpnt, 69  
   polint, 70  
   qromb, 70  
   qromo, 71  
   trapzd, 71  
 int\_comp.f90, 237  
 int\_corr, 72  
   comp\_corrint, 73  
   corr2int, 75  
   corrint, 75  
   func\_ij, 74  
   func\_ijkl, 75  
   init\_corrint, 75  
   oi, 76  
   oj, 76  
   ok, 76  
   ol, 76  
   real\_eps, 76  
 int\_corr.f90, 237  
 int\_corr\_mode

stoch\_params, 159  
 int\_tensor  
   rk2\_stoch\_integrator, 128  
 integrate  
   int\_comp, 68  
 integrator, 76  
   buf\_f0, 79  
   buf\_f1, 79  
   buf\_ka, 79  
   buf\_kb, 79  
   buf\_y1, 79  
   init\_integrator, 78  
   step, 78  
   tendencies, 78  
 inv\_corr\_i  
   corrmod, 42  
 inv\_corr\_i\_full  
   corrmod, 42  
 invmat  
   util, 206  
 ireduce  
   util, 206  
 iset  
   stoch\_mod, 156  
 isin  
   util, 206  
 iter  
   stat, 151

j  
   tensor::coolist\_elem, 226  
   tensor::coolist\_elem4, 227

jacobian  
   tl\_ad\_tensor, 200

jacobian\_mat  
   tl\_ad\_tensor, 201

jsparse\_mul  
   tensor, 175

jsparse\_mul\_mat  
   tensor, 176

k  
   inprod\_analytic::ocean\_tensors, 228  
   params, 104  
   tensor::coolist\_elem, 226  
   tensor::coolist\_elem4, 227

KIND  
   LICENSE.txt, 241

kd  
   params, 104

kdelta  
   aotensor\_def, 28

kdp  
   params, 104

khi  
   corrmod, 42

klo  
   corrmod, 42

kp

params, 105

I  
  params, 105  
  tensor::coolist\_elem4, 227

I1  
  mtv\_int\_tensor, 95  
  wl\_tensor, 216

I2  
  mtv\_int\_tensor, 95  
  wl\_tensor, 216

I3  
  mtv\_int\_tensor, 95

I4  
  wl\_tensor, 217

I5  
  wl\_tensor, 217

LIABILITY  
  LICENSE.txt, 241

LICENSE.txt, 238  
  CLAIM, 240  
  CONTRACT, 240  
  charge, 240  
  conditions, 240  
  copy, 240  
  distribute, 240  
  FROM, 240  
  files, 239  
   IMPLIED, 241  
  KIND, 241  
  LIABILITY, 241  
  License, 240  
  MERCHANTABILITY, 241  
  merge, 241  
  modify, 241  
  OTHERWISE, 241  
  publish, 242  
  restriction, 242  
  so, 242  
  Software, 242  
  sublicense, 242  
  use, 242

lambda  
  params, 105

lcg  
  util.f90, 270

ldef  
  wl\_tensor, 217

License  
  LICENSE.txt, 240

load\_ic  
  ic\_def, 55

load\_mode  
  stoch\_params, 159

load\_sf  
  sf\_def, 135

load\_tensor4\_from\_file  
  tensor, 176

load\_tensor\_from\_file

  tensor, 178

lpa  
  params, 105

lpo  
  params, 105

lr  
  params, 106

lsbpa  
  params, 106

lsbpo  
  params, 106

ltot  
  mtv\_int\_tensor, 95  
  wl\_tensor, 217

lwork  
  sqrt\_mod, 149

lxz  
  dec\_tensor, 53

lxy  
  dec\_tensor, 53

lyx  
  dec\_tensor, 54

lyy  
  dec\_tensor, 54

m  
  inprod\_analytic::atm\_wavenum, 223  
  inprod\_analytic::ocean\_tensors, 228  
  stat, 152

m11  
  wl\_tensor, 217

m12  
  wl\_tensor, 217

m12def  
  wl\_tensor, 218

m13  
  wl\_tensor, 218

m1tot  
  wl\_tensor, 218

m21  
  wl\_tensor, 218

m21def  
  wl\_tensor, 218

m22  
  wl\_tensor, 218

m22def  
  wl\_tensor, 219

MAR.f90, 244

MERCHANTABILITY  
  LICENSE.txt, 241

MTV\_int\_tensor.f90, 246

MTV\_sigma\_tensor.f90, 247

maooam  
  maooam.f90, 243

maooam.f90, 242  
  maooam, 243

maooam\_MTV.f90, 243  
  maooam\_mtv, 243

maooam\_WL.f90, 244

maooom\_wl, 244  
 maooom\_mtv  
     maooom\_MTV.f90, 243  
 maooom\_stoch  
     maooom\_stoch.f90, 244  
 maooom\_stoch.f90, 243  
     maooom\_stoch, 244  
 maooom\_wl  
     maooom\_WL.f90, 244  
 mar, 80  
     buf\_y, 82  
     dw, 82  
     init\_mar, 81  
     mar\_step, 81  
     mar\_step\_red, 82  
     ms, 83  
     q, 83  
     qred, 83  
     rred, 83  
     stoch\_vec, 82  
     w, 83  
     wred, 83  
 mar\_step  
     mar, 81  
 mar\_step\_red  
     mar, 82  
 mat\_contract  
     util, 207  
 mat\_to\_coo  
     tensor, 179  
 mat\_trace  
     util, 207  
 matc\_to\_coo  
     tensor, 179  
 maxint  
     stoch\_params, 159  
 mdef  
     wl\_tensor, 219  
 mean  
     corrmod, 42  
     stat, 151  
 mean\_full  
     corrmod, 42  
 meml  
     stoch\_params, 159  
 memory, 84  
     buf\_m, 87  
     buf\_m3, 87  
     compute\_m3, 85  
     init\_memory, 86  
     step, 87  
     t\_index, 87  
     test\_m3, 86  
     x, 87  
     xs, 88  
     zs, 88  
 memory.f90, 245  
 mems

stoch\_params, 159  
 merge  
     LICENSE.txt, 241  
 midexp  
     int\_comp, 69  
 midpnt  
     int\_comp, 69  
 mnuti  
     stoch\_params, 160  
 mode  
     stoch\_params, 160  
 modify  
     LICENSE.txt, 241  
 mprev  
     stat, 152  
 ms  
     mar, 83  
 mtimp  
     stat, 152  
 mtot  
     mtv\_int\_tensor, 96  
     wl\_tensor, 219  
 mtv\_int\_tensor, 88  
     b1, 92  
     b2, 93  
     btot, 93  
     dumb\_mat1, 93  
     dumb\_mat2, 93  
     dumb\_mat3, 93  
     dumb\_mat4, 94  
     dumb\_vec, 94  
     h1, 94  
     h2, 94  
     h3, 94  
     htot, 95  
     init\_mtv\_int\_tensor, 90  
     l1, 95  
     l2, 95  
     l3, 95  
     ltot, 95  
     mtot, 96  
     q1, 96  
     q2, 96  
     utot, 96  
     vtot, 96  
 mult  
     rk2\_mtv\_integrator, 119  
 muti  
     stoch\_params, 160  
 n  
     inprod\_analytic::oceantensors, 228  
     params, 106  
 n1  
     sigma, 141  
 n2  
     sigma, 142  
 n\_res  
     sf\_def, 137

n\_unres  
    sf\_def, 138  
natm  
    params, 106  
nbatm  
    params, 107  
nboc  
    params, 107  
ndim  
    params, 107  
nelems  
    tensor::coolist, 224  
    tensor::coolist4, 225  
noc  
    params, 107  
noise  
    rk2\_mtv\_integrator, 119  
noisemult  
    rk2\_mtv\_integrator, 119  
nspl  
    corrmod, 43  
nua  
    params, 107  
nuap  
    params, 108  
nuo  
    params, 108  
nuop  
    params, 108  
nx  
    inprod\_analytic::atm\_wavenum, 223  
    inprod\_analytic::ocean\_wavenum, 229  
ny  
    inprod\_analytic::atm\_wavenum, 223  
    inprod\_analytic::ocean\_wavenum, 229  
  
o  
    inprod\_analytic::ocean\_tensors, 228  
OTHERWISE  
    LICENSE.txt, 241  
ocean  
    inprod\_analytic, 67  
oi  
    int\_corr, 76  
oj  
    int\_corr, 76  
ok  
    int\_corr, 76  
ol  
    int\_corr, 76  
oms  
    params, 108  
owavenum  
    inprod\_analytic, 67  
  
p  
    inprod\_analytic::atm\_wavenum, 223  
    inprod\_analytic::ocean\_wavenum, 230  
params, 97  
ams, 101  
betp, 101  
ca, 101  
co, 101  
cpa, 102  
cpo, 102  
d, 102  
dp, 102  
dt, 102  
epsa, 103  
f0, 103  
g, 103  
ga, 103  
go, 103  
gp, 104  
h, 104  
init\_nml, 100  
init\_params, 100  
k, 104  
kd, 104  
kdp, 104  
kp, 105  
l, 105  
lambda, 105  
lpa, 105  
lpo, 105  
lr, 106  
lsbpa, 106  
lsbpo, 106  
n, 106  
natm, 106  
nbatm, 107  
nboc, 107  
ndim, 107  
noc, 107  
nua, 107  
nuap, 108  
nuo, 108  
nuop, 108  
oms, 108  
phi0, 108  
phi0\_npi, 109  
pi, 109  
r, 109  
rp, 109  
rr, 109  
rra, 110  
sb, 110  
sbpa, 110  
sbpo, 110  
sc, 110  
scale, 111  
sig0, 111  
t\_run, 111  
t\_trans, 111  
ta0, 111  
to0, 112  
tw, 112

writeout, 112  
 params.f90, 248  
 phi0  
     params, 108  
 phi0\_npi  
     params, 109  
 pi  
     params, 109  
 piksrt  
     util, 207  
 polint  
     int\_comp, 70  
 print\_tensor  
     tensor, 180  
 print\_tensor4  
     tensor, 180  
 printmat  
     util, 208  
 psi  
     aotensor\_def, 28  
 publish  
     LICENSE.txt, 242  
  
 q  
     mar, 83  
 q1  
     mtv\_int\_tensor, 96  
 q1fill  
     rk2\_mtv\_integrator, 119  
 q2  
     mtv\_int\_tensor, 96  
 q\_ar  
     stoch\_params, 160  
 q\_au  
     stoch\_params, 160  
 q\_or  
     stoch\_params, 161  
 q\_ou  
     stoch\_params, 161  
 qred  
     mar, 83  
 qromb  
     int\_comp, 70  
 qromo  
     int\_comp, 71  
  
 r  
     params, 109  
 real\_eps  
     aotensor\_def, 29  
     int\_corr, 76  
     sqrt\_mod, 149  
     tensor, 191  
     tl\_ad\_tensor, 203  
 reduce  
     util, 208  
 reorder  
     dec\_tensor, 49  
 reset

stat, 151  
 restriction  
     LICENSE.txt, 242  
 rind  
     sf\_def, 138  
 rind1  
     sigma, 142  
 rind2  
     sigma, 142  
 rk2\_MTV\_integrator.f90, 251  
 rk2\_WL\_integrator.f90, 254  
 rk2\_integrator.f90, 251  
 rk2\_mtv\_integrator, 112  
     anoise, 117  
     buf\_f0, 117  
     buf\_f1, 117  
     buf\_g, 117  
     buf\_y1, 117  
     compg, 114  
     compute\_mult, 117  
     dw, 118  
     dwar, 118  
     dwau, 118  
     dwmult, 118  
     dwor, 118  
     dwou, 118  
     full\_step, 114  
     g, 118  
     init\_g, 115  
     init\_integrator, 115  
     init\_noise, 115  
     mult, 119  
     noise, 119  
     noisemult, 119  
     q1fill, 119  
     sq2, 119  
     step, 116  
 rk2\_ss\_integrator, 120  
     anoise, 123  
     buf\_f0, 123  
     buf\_f1, 123  
     buf\_y1, 123  
     dwar, 123  
     dwor, 124  
     init\_ss\_integrator, 121  
     ss\_step, 121  
     ss\_tl\_step, 121  
     tendencies, 122  
     tl\_tendencies, 122  
 rk2\_ss\_integrator.f90, 252  
 rk2\_stoch\_integrator, 124  
     anoise, 127  
     buf\_f0, 127  
     buf\_f1, 127  
     buf\_y1, 127  
     dwar, 127  
     dwau, 127  
     dwor, 128

dwou, 128  
init\_integrator, 125  
int\_tensor, 128  
step, 126  
tendencies, 126  
rk2\_stoch\_integrator.f90, 253  
rk2\_tl\_ad\_integrator.f90, 254  
rk2\_wl\_integrator, 128  
anoise, 132  
buf\_f0, 132  
buf\_f1, 133  
buf\_m, 133  
buf\_m1, 133  
buf\_m2, 133  
buf\_m3, 133  
buf\_m3s, 133  
buf\_y1, 133  
compute\_m1, 129  
compute\_m2, 130  
dwar, 133  
dwau, 134  
dwor, 134  
dwou, 134  
full\_step, 130  
init\_integrator, 130  
step, 131  
x1, 134  
x2, 134  
rk4\_integrator.f90, 255  
rk4\_tl\_ad\_integrator.f90, 256  
rp  
    params, 109  
rr  
    params, 109  
rra  
    params, 110  
rred  
    mar, 83  
rsf2csf  
    sqrt\_mod, 145  
rstr  
    util, 208  
  
s  
    inprod\_analytic::atm\_tensors, 222  
s1  
    inprod\_analytic, 66  
s2  
    inprod\_analytic, 66  
s3  
    inprod\_analytic, 66  
s4  
    inprod\_analytic, 66  
sb  
    params, 110  
sbpa  
    params, 110  
sbpo  
    params, 110  
  
sc  
    params, 110  
scal\_mul\_coo  
    tensor, 181  
scale  
    params, 111  
selectev  
    sqrt\_mod, 146  
sf  
    sf\_def, 138  
sf\_def, 134  
    bar, 137  
    bau, 137  
    bor, 137  
    bou, 137  
    exists, 137  
    ind, 137  
    load\_sf, 135  
    n\_res, 137  
    n\_unres, 138  
    rind, 138  
    sf, 138  
    sl\_ind, 138  
    sl\_rind, 138  
sf\_def.f90, 256  
sf\_tensor  
    dec\_tensor, 54  
sig0  
    params, 111  
sig1  
    sigma, 142  
sig1r  
    sigma, 142  
sig2  
    sigma, 142  
sigma, 139  
    compute\_mult\_sigma, 139  
    dumb\_mat1, 141  
    dumb\_mat2, 141  
    dumb\_mat3, 141  
    dumb\_mat4, 141  
    ind1, 141  
    ind2, 141  
    init\_sigma, 140  
    n1, 141  
    n2, 142  
    rind1, 142  
    rind2, 142  
    sig1, 142  
    sig1r, 142  
    sig2, 142  
simplify  
    tensor, 181  
sl\_ind  
    sf\_def, 138  
sl\_rind  
    sf\_def, 138  
so

LICENSE.txt, 242  
 Software  
   LICENSE.txt, 242  
 sparse\_mul2  
   tensor, 182  
 sparse\_mul2\_j  
   tensor, 183  
 sparse\_mul2\_k  
   tensor, 183  
 sparse\_mul3  
   tensor, 184  
 sparse\_mul3\_mat  
   tensor, 184  
 sparse\_mul3\_with\_mat  
   tensor, 185  
 sparse\_mul4  
   tensor, 186  
 sparse\_mul4\_mat  
   tensor, 186  
 sparse\_mul4\_with\_mat\_jl  
   tensor, 187  
 sparse\_mul4\_with\_mat\_kl  
   tensor, 188  
 splint  
   corrmod, 40  
 sq2  
   rk2\_mtv\_integrator, 119  
 sqrt\_mod, 143  
   chol, 143  
   csqrtm\_triu, 144  
   init\_sqrt, 145  
   lwork, 149  
   real\_eps, 149  
   rsf2csf, 145  
   selectev, 146  
   sqrtm, 146  
   sqrtm\_svd, 147  
   sqrtm\_triu, 148  
   work, 149  
 sqrt\_mod.f90, 257  
 sqrtm  
   sqrt\_mod, 146  
 sqrtm\_svd  
   sqrt\_mod, 147  
 sqrtm\_triu  
   sqrt\_mod, 148  
 ss\_step  
   rk2\_ss\_integrator, 121  
 ss\_tensor  
   dec\_tensor, 54  
 ss\_tl\_step  
   rk2\_ss\_integrator, 121  
 ss\_tl\_tensor  
   dec\_tensor, 54  
 stat, 150  
   acc, 150  
   i, 152  
   init\_stat, 150  
     iter, 151  
     m, 152  
     mean, 151  
     mprev, 152  
     mtmp, 152  
     reset, 151  
     v, 152  
     var, 151  
 stat.f90, 258  
 step  
   integrator, 78  
   memory, 87  
   rk2\_mtv\_integrator, 116  
   rk2\_stoch\_integrator, 126  
   rk2\_wl\_integrator, 131  
   stoch\_atm\_res\_vec  
    stoch\_mod, 153  
   stoch\_atm\_unres\_vec  
    stoch\_mod, 154  
   stoch\_atm\_vec  
    stoch\_mod, 154  
   stoch\_mod, 152  
    gasdev, 153  
    gset, 156  
    iset, 156  
    stoch\_atm\_res\_vec, 153  
    stoch\_atm\_unres\_vec, 154  
    stoch\_atm\_vec, 154  
    stoch\_oc\_res\_vec, 154  
    stoch\_oc\_unres\_vec, 155  
    stoch\_oc\_vec, 155  
    stoch\_vec, 155  
   stoch\_mod.f90, 258  
   stoch\_oc\_res\_vec  
    stoch\_mod, 154  
   stoch\_oc\_unres\_vec  
    stoch\_mod, 155  
   stoch\_oc\_vec  
    stoch\_mod, 155  
   stoch\_params, 156  
    dtn, 158  
    dts, 158  
    dtss, 158  
    eps\_pert, 158  
    init\_stoch\_params, 158  
    int\_corr\_mode, 159  
    load\_mode, 159  
    maxint, 159  
    meml, 159  
    mems, 159  
    mnuti, 160  
    mode, 160  
    muti, 160  
    q\_ar, 160  
    q\_au, 160  
    q\_or, 161  
    q\_ou, 161  
    t\_trans\_mem, 161

t\_trans\_stoch, 161  
tdelta, 161  
x\_int\_mode, 162  
stoch\_params.f90, 259  
stoch\_vec  
    mar, 82  
    stoch\_mod, 155  
str  
    util, 209  
sublicense  
    LICENSE.txt, 242  
suppress\_and  
    dec\_tensor, 50  
suppress\_or  
    dec\_tensor, 50  
  
t  
    aotensor\_def, 28  
t\_index  
    memory, 87  
t\_run  
    params, 111  
t\_trans  
    params, 111  
t\_trans\_mem  
    stoch\_params, 161  
t\_trans\_stoch  
    stoch\_params, 161  
ta0  
    params, 111  
tdelta  
    stoch\_params, 161  
tendencies  
    integrator, 78  
    rk2\_ss\_integrator, 122  
    rk2\_stoch\_integrator, 126  
tensor, 162  
    add\_check, 165  
    add\_elem, 165  
    add\_matc\_to\_tensor, 166  
    add\_matc\_to\_tensor4, 167  
    add\_to\_tensor, 168  
    add\_vec\_ijk\_to\_tensor4, 169  
    add\_vec\_ikl\_to\_tensor4, 169  
    add\_vec\_ikl\_to\_tensor4\_perm, 170  
    add\_vec\_jk\_to\_tensor, 171  
    coo\_to\_mat\_i, 172  
    coo\_to\_mat\_ij, 173  
    coo\_to\_mat\_ik, 173  
    coo\_to\_mat\_j, 173  
    coo\_to\_vec\_jk, 174  
    copy\_coo, 174  
    jsparse\_mul, 175  
    jsparse\_mul\_mat, 176  
    load\_tensor4\_from\_file, 176  
    load\_tensor\_from\_file, 178  
    mat\_to\_coo, 179  
    matc\_to\_coo, 179  
    print\_tensor, 180  
    print\_tensor4, 180  
    real\_eps, 191  
    scal\_mul\_coo, 181  
    simplify, 181  
    sparse\_mul2, 182  
    sparse\_mul2\_j, 183  
    sparse\_mul2\_k, 183  
    sparse\_mul3, 184  
    sparse\_mul3\_mat, 184  
    sparse\_mul3\_with\_mat, 185  
    sparse\_mul4, 186  
    sparse\_mul4\_mat, 186  
    sparse\_mul4\_with\_mat\_jl, 187  
    sparse\_mul4\_with\_mat\_kl, 188  
    tensor4\_empty, 188  
    tensor4\_to\_coo4, 189  
    tensor\_empty, 189  
    tensor\_to\_coo, 190  
    write\_tensor4\_to\_file, 190  
    write\_tensor\_to\_file, 191  
    tensor.f90, 260  
    tensor4\_empty  
        tensor, 188  
    tensor4\_to\_coo4  
        tensor, 189  
    tensor::coolist, 224  
        elems, 224  
        nelems, 224  
    tensor::coolist4, 224  
        elems, 225  
        nelems, 225  
    tensor::coolist\_elem, 225  
        j, 226  
        k, 226  
        v, 226  
    tensor::coolist\_elem4, 226  
        j, 227  
        k, 227  
        l, 227  
        v, 227  
    tensor\_empty  
        tensor, 189  
    tensor\_to\_coo  
        tensor, 190  
    test\_MAR.f90, 265  
        test\_mar, 265  
    test\_MTV\_int\_tensor.f90, 266  
        test\_mtv\_int\_tensor, 266  
    test\_MTV\_sigma\_tensor.f90, 266  
        test\_sigma, 267  
    test\_WL\_tensor.f90, 268  
        test\_wl\_tensor, 268  
    test\_aotensor  
        test\_aotensor.f90, 263  
    test\_aotensor.f90, 263  
        test\_aotensor, 263  
    test\_corr  
        test\_corr.f90, 264

test\_corr.f90, 263  
     test\_corr, 264  
 test\_corr\_tensor  
     test\_corr\_tensor.f90, 264  
 test\_corr\_tensor.f90, 264  
     test\_corr\_tensor, 264  
 test\_dec\_tensor  
     test\_dec\_tensor.f90, 264  
 test\_dec\_tensor.f90, 264  
     test\_dec\_tensor, 264  
 test\_inprod\_analytic.f90, 265  
     inprod\_analytic\_test, 265  
 test\_m3  
     memory, 86  
 test\_mar  
     test\_MAR.f90, 265  
 test\_memory  
     test\_memory.f90, 266  
 test\_memory.f90, 266  
     test\_memory, 266  
 test\_mtv\_int\_tensor  
     test\_MTV\_int\_tensor.f90, 266  
 test\_sigma  
     test\_MTV\_sigma\_tensor.f90, 267  
 test\_sqrtm  
     test\_sqrtm.f90, 267  
 test\_sqrtm.f90, 267  
     test\_sqrtm, 267  
 test\_tl\_ad  
     test\_tl\_ad.f90, 267  
 test\_tl\_ad.f90, 267  
     test\_tl\_ad, 267  
 test\_wl\_tensor  
     test\_WL\_tensor.f90, 268  
 theta  
     aotensor\_def, 28  
 tl  
     tl\_ad\_tensor, 201  
 tl\_ad\_integrator, 192  
     ad\_step, 192  
     buf\_f0, 194  
     buf\_f1, 194  
     buf\_ka, 194  
     buf\_kb, 194  
     buf\_y1, 194  
     init\_tl\_ad\_integrator, 193  
     tl\_step, 193  
 tl\_ad\_tensor, 195  
     ad, 196  
     ad\_add\_count, 196  
     ad\_add\_count\_ref, 197  
     ad\_coeff, 197  
     ad\_coeff\_ref, 198  
     adtensor, 203  
     compute\_adtensor, 198  
     compute\_adtensor\_ref, 198  
     compute\_tltensor, 199  
     count\_elems, 203  
     init\_adtensor, 199  
     init\_adtensor\_ref, 199  
     init\_tltensor, 200  
     jacobian, 200  
     jacobian\_mat, 201  
     real\_eps, 203  
     tl, 201  
     tl\_add\_count, 201  
     tl\_coeff, 202  
     tltensor, 203  
     tl\_ad\_tensor.f90, 268  
     tl\_add\_count  
         tl\_ad\_tensor, 201  
     tl\_coeff  
         tl\_ad\_tensor, 202  
     tl\_step  
         tl\_ad\_integrator, 193  
     tl\_tendencies  
         rk2\_ss\_integrator, 122  
     tltensor  
         tl\_ad\_tensor, 203  
     to0  
         params, 112  
     trapzd  
         int\_comp, 71  
     triu  
         util, 209  
     tw  
         params, 112  
     typ  
         inprod\_analytic::atm\_wavenum, 223  
     use  
         LICENSE.txt, 242  
     util, 203  
         cdiag, 204  
         choldc, 204  
         cprintmat, 205  
         diag, 205  
         floordiv, 205  
         init\_one, 205  
         init\_random\_seed, 206  
         invmat, 206  
         ireduce, 206  
         isin, 206  
         mat\_contract, 207  
         mat\_trace, 207  
         piksrt, 207  
         printmat, 208  
         reduce, 208  
         rstr, 208  
         str, 209  
         triu, 209  
         vector\_outer, 209  
     util.f90, 269  
         lcg, 270  
     utot  
         mtv\_int\_tensor, 96

v  
  stat, 152  
  tensor::coolist\_elem, 226  
  tensor::coolist\_elem4, 227

var  
  stat, 151

vector\_outer  
  util, 209

vtot  
  mtv\_int\_tensor, 96

w  
  inprod\_analytic::ocean\_tensors, 229  
  mar, 83

WL\_tensor.f90, 270

wl\_tensor, 209  
  b1, 214  
  b14, 214  
  b14def, 214  
  b2, 214  
  b23, 215  
  b23def, 215  
  b3, 215  
  b4, 215  
  dumb\_mat1, 215  
  dumb\_mat2, 215  
  dumb\_mat3, 216  
  dumb\_mat4, 216  
  dumb\_vec, 216  
  init\_wl\_tensor, 211  
  I1, 216  
  I2, 216  
  I4, 217  
  I5, 217  
  ldef, 217  
  ltot, 217  
  m11, 217  
  m12, 217  
  m12def, 218  
  m13, 218  
  m1tot, 218  
  m21, 218  
  m21def, 218  
  m22, 218  
  m22def, 219  
  mdef, 219  
  mtot, 219

work  
  sqrt\_mod, 149

wred  
  mar, 83

write\_tensor4\_to\_file  
  tensor, 190

write\_tensor\_to\_file  
  tensor, 191

writeout  
  params, 112

x  
  memory, 87

  x1  
    rk2\_wl\_integrator, 134

  x2  
    rk2\_wl\_integrator, 134

  x\_int\_mode  
    stoch\_params, 162

  xa  
    corrmod, 43

  xs  
    memory, 88

  y2  
    corrmod, 43

  ya  
    corrmod, 43

  ydy  
    corr\_tensor, 32

  ydyy  
    corr\_tensor, 32

  yy  
    corr\_tensor, 33

  zs  
    memory, 88