Supplement of Nonlin. Processes Geophys., 23, 159–173, 2016
http://www.nonlin-processes-geophys.net/23/159/2016/
doi:10.5194/npg-23-159-2016-supplement

Nonlinear Processes in Geophysics
Open Access

Supplement of

Detecting and tracking eddies in oceanic flow fields: a Lagrangian descriptor based on the modulus of vorticity

R. Vortmeyer-Kley et al.

Correspondence to: Rahel Vortmeyer-Kley (rahel.vortmeyer-kley@uni-oldenburg.de)

S1 The hydrodynamic model of a vortex street

The hydrodynamic model of the vortex street is an adapted version of the model by Jung et al. (1993). They model the flow in a channel behind a cylinder in its middle (cf. Jung et al. (1993) and Sandulescu et al. (2006) for details).
Here this model is modified in such a way that two counter rotating eddies that develop at times t and $t+T_c/2$ at position $(1,y_0)$ and $(1,-y_0)$ respectively, travel a distance L in positive x-direction within their lifetime T_c and fade out. The model is artificial because the impact of the cylinder on the eddy formation and its shading are neglected here making the eddy formation non-physical out of nowhere. However, since all quantities to be estimated by the eddy tracking tool are then given analytically, this artificial model makes up an ideal test bed for numerics.
Hence, the model is simplified as follows:

$$\Psi(x,y,t) = -wh_1(t)g_1(x,y,t) + wh_2(t)g_2(x,y,t) + u_0 y. \tag{S1}$$

The first two terms describe the life cycle of the eddies of opposite sense of rotation. The vortex strength is given by w. The dynamics of the eddy evolution is modelled as the modulation of the amplitudes by the function $h_1(t) = |\sin(\pi t/T_c)|$ resp. $h_2(t) = h_1(t - T_c/2)$. The function $g_i(x,y,t) = \exp(-\kappa_0((x - x_i(t))^2 + \alpha(y - y_i(t))^2))$ with $i = 1,2$ models the Gaussian shaped effect of the eddies on the stream function. The movement of the eddy centres is expressed by $x_1(t) = 1 + L(t/T_c \mod 1)$, $x_2(t) = x_1(t - T_c/2)$ and $y_1(t) = y_0 = -y_2(t)$. The factor $\kappa_0^{-1/2}$ is the radius and determined as a characteristic linear size of the eddies and α is the ratio between the elongations of the eddy in x and y direction. In our case α is set to 1 (circular eddies). The last term describes the background flow with the velocity u_0.
The parametrization of the flow is chosen as in Sandulescu et al. (2006). Lengths are measured in units of the eddy radius r and time in units of the lifetime T_c of an eddy. The dimensional and dimensionless parameters are given in Table S1.
The vortex strength is furthermore varied to study its impact on the eddy evolution.

Table S1. Dimensional and dimensionless parameters of the hydrodynamical model of a vortex street

Parameter	r	T_c	α	κ	L	u_0	y_0	w
dimensional	25 km	30 d	1	$1/r^2$	150 km	0.18 ms^{-1}	12.5 km	$55 \cdot 10^3$ m^{2}s^{-1}
dimensionless	1	1	1	1	6	18.66	0.5	200

S2 Algorithm of the eddy tracking

The key ideas of the algorithm (eddy detection, eddy tracking and eddy shape) inspired by the eddy tracking package by Nencioli et al. (2010) are schematically presented in this section.
Fig. S1 presents the concept of eddy detection. Similar algorithms can be constructed for the modulus of the vorticity and for the Okubo-Weiss criterion (taking into account the fact that eddy cores are minima in case of Okubo-Weiss).
Fig. S2 shows idea of the eddy tracking.
Fig. S3 presents the eddy shape detection based on the modulus of vorticity based Lagrangian descriptor M_V based on the modulus of vorticity. The eddy shape detection searches for the largest closed contourline with the largest gradient of M_V along the contour line. To maximise these two conditions at the same time, we maximise a quantity that combines this two ideas: (Area enclosed by the contour line)$\cdot$(($\sum$ gradient of M_V along contour line)/(length of contour line)). Because contour lines surround the eddy core like the layers of an onion, maximising the enclosed area includes maximising the length of the closed contourline. Maximising the gradient of M_V along the contourline is linked to the idea to search for a singular line (the eddy boundary).
In case of eddy shape detection for realistic oceanic velocity fields like the example of the western Baltic Sea, the coordinates of the eddy cores have to be understood as candidates for the eddy core.
From all candidates for eddy cores only those are kept which fulfil the following conditions:

- The convexity deficiency as defined in Haller et al. (2016) has to be smaller than a threshold.
- No land is enclosed in the contourline.
- The contourlines are longer than a threshold length. The reason for that criterion is mainly to speed up the computation. Very short contourlines can typically be found very close around the eddy cores. Therefore, they need not to be checked for the gradient of M_V, because they will not describe the eddy boundary.

This set of eddies can then be used as input for eddy tracking.

References

Haller, G., Hadjighasem, A., Farazmand, M., and Huhn, F.: Defining coherent vortices objectively from the vorticity, J Fluid Mech, 795, 136–173, 2016.

Jung, C., Tél, T., and Ziemniak, E.: Application of scattering chaos to particle transport in a hydrodynamical flow, Chaos, 3, 555–568, 1993.

Nencioli, F., Dong, C., Dickey, T., Washburn, L., and McWilliams, J. C.: A Vector Geometry-Based Eddy Detection Algorithm and Its Application to a High-Resolution Numerical Model Product and High-Frequency Radar Surface Velocities in the Southern California Bight, J Atmos Ocean Tech, 27, 564–579, 2010.

Sandulescu, M., Hernández-García, E., López, C., and Feudel, U.: Kinematic studies of transport across an island wake, with application to Canary islands, Tellus A, 58, 605–615, 2006.

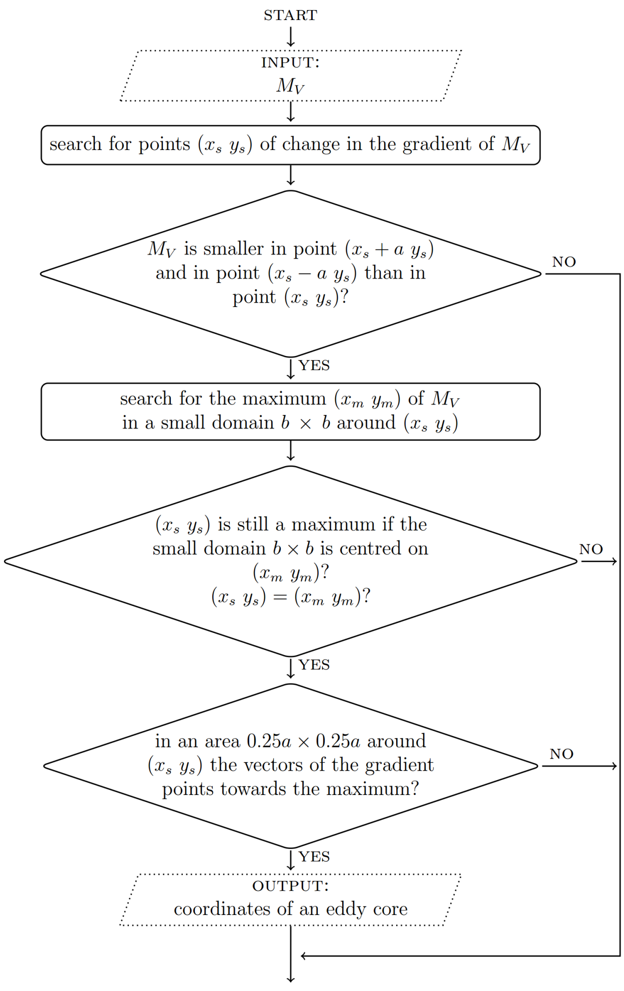

Figure S1. Schematic sketch of the eddy detection algorithm based on Lagrangian descriptor M_V.

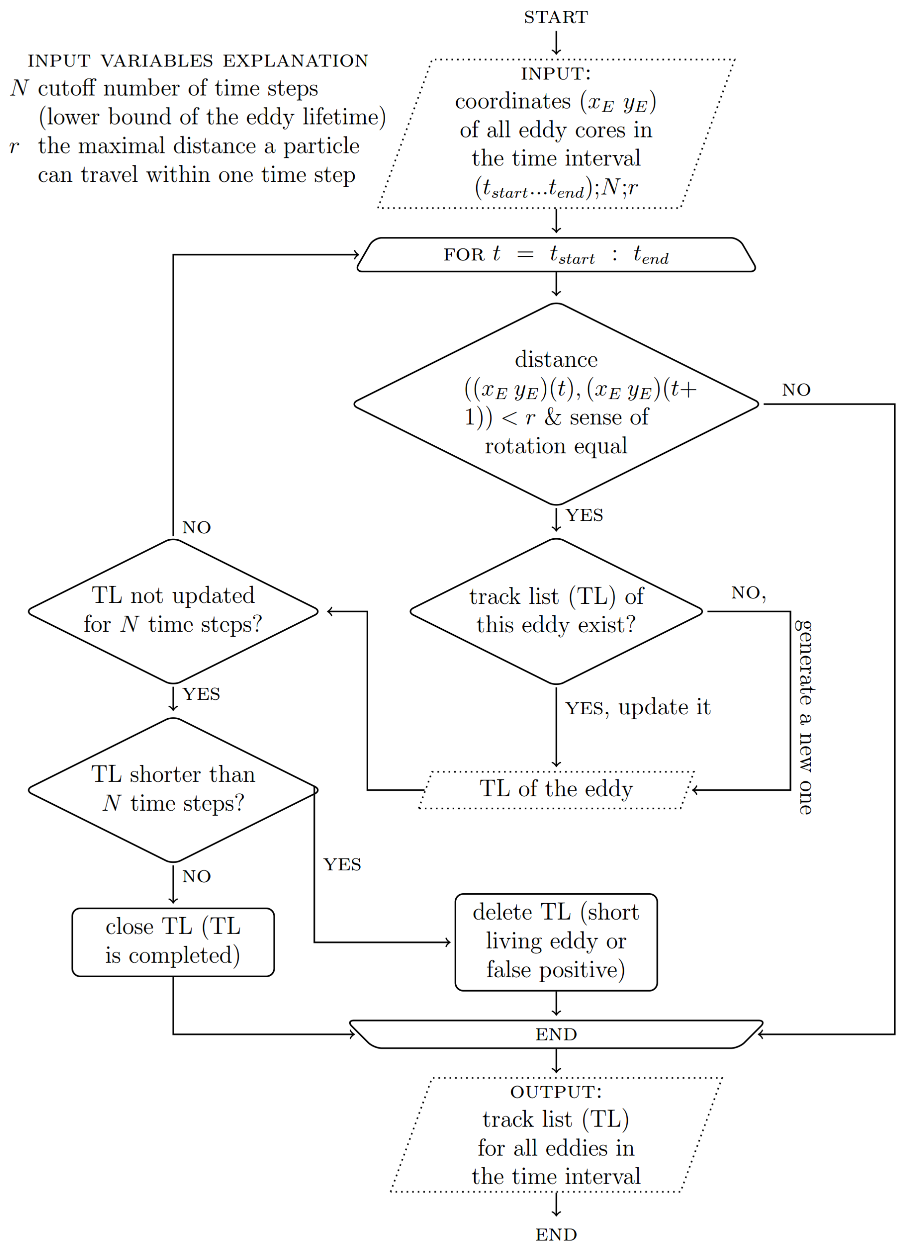

Figure S2. Schematic sketch of the eddy tracking algorithm based on Lagrangian descriptor M_V.

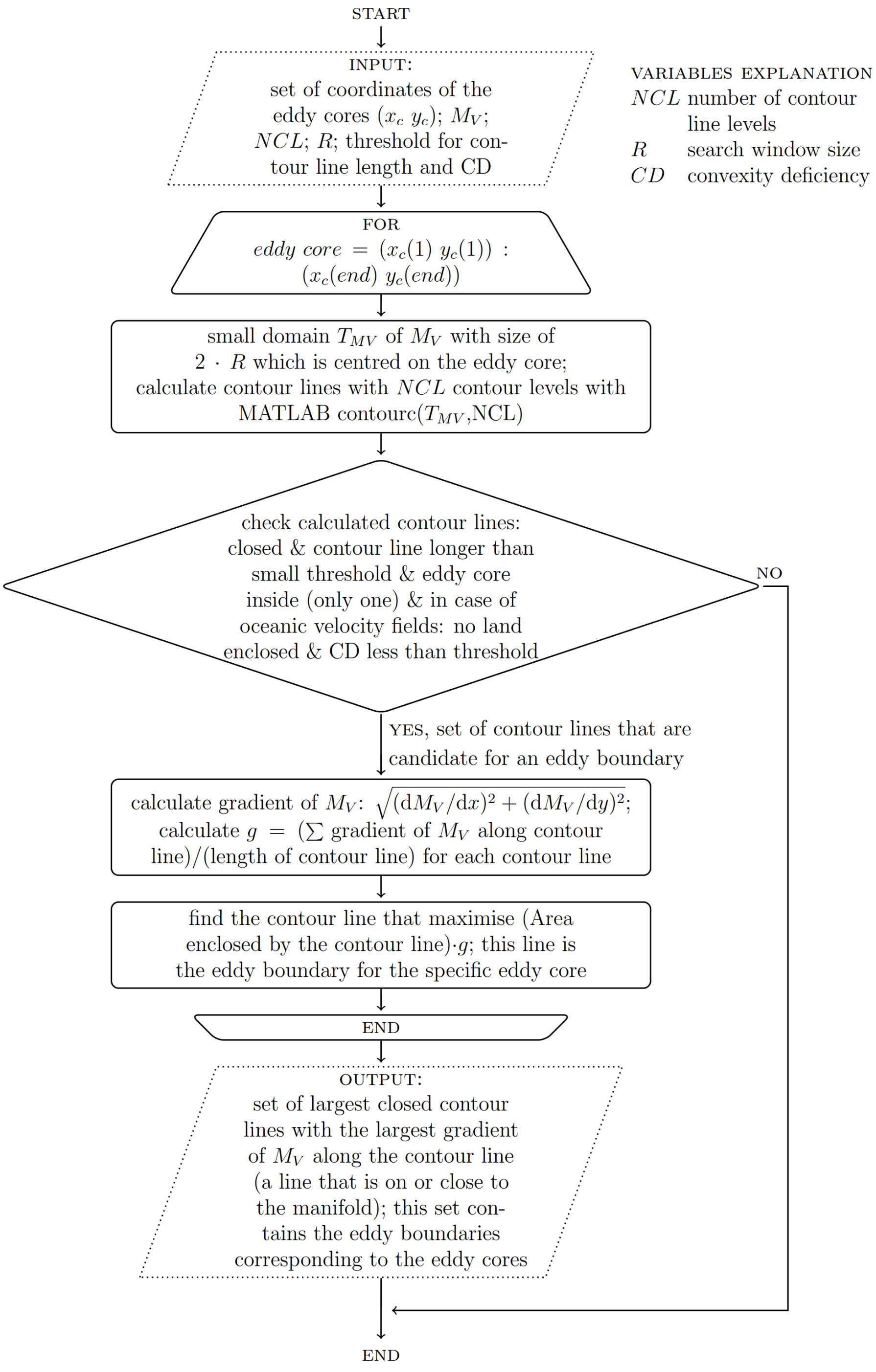

Figure S3. Schematic sketch of the eddy shape algorithm based on the Lagrangian descriptor M_V.