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Turbulence in the interstellar medium

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Abstract. Turbulence is ubiquitous in the insterstellar medium and plays a major role in several processes such as the formation of dense structures and stars, the stability of molecular clouds, the amplification of magnetic fields, and the re-acceleration and diffusion of cosmic rays. Despite its importance, interstellar turbulence, like turbulence in general, is far from being fully understood. In this review we present the basics of turbulence physics, focusing on the statistics of its structure and energy cascade. We explore the physics of compressible and incompressible turbulent flows, as well as magnetised cases. The most relevant observational techniques that provide quantitative insights into interstellar turbulence are also presented. We also discuss the main difficulties in developing a three-dimensional view of interstellar turbulence from these observations. Finally, we briefly present what the main sources of turbulence in the interstellar medium could be.

1 Introduction: the basics of turbulence

Turbulence is characterised by chaotic motions in a fluid (Rempel et al., 2004; He and Chian, 2005; Chian et al., 2006, 2007, 2010) that lead to diffusion of matter and dissipation of kinetic energy. It is to be stressed that not all chaotic motions in a fluid may be called "turbulent". Because of its chaotic nature turbulence can only be studied and modelled in terms of statistical quantities. Long-term deterministic local properties of a turbulent fluid are unpredictable.

For nearly incompressible and unmagnetised fluids, the temporal evolution of the fluid velocity field is given by the Navier–Stokes equation

$$\frac{\partial \boldsymbol{u}(\boldsymbol{x},t)}{\partial t} + \boldsymbol{u}(\boldsymbol{x},t)\cdot\nabla\boldsymbol{u}(\boldsymbol{x},t) = -\frac{\nabla p(\boldsymbol{x},t)}{\rho(\boldsymbol{x},t)} + \nu\nabla^2\boldsymbol{u}(\boldsymbol{x},t) + \boldsymbol{F}(\boldsymbol{x},t), \tag{1}$$

where $\boldsymbol{u}(\boldsymbol{x},t)$ represents the velocity field, p the pressure, ν the kinematic viscosity, and $\boldsymbol{F}$ an external force normalised by the local density. ρ is the gas mass density and is set constant in the incompressible case (with $\nabla\cdot\boldsymbol{u}=0$). Even in this simplified mathematical description the fluid dynamics is not a trivial solution. Equation (1) is non-linear, as seen from the advective term on the left-hand side, and non-local – in the sense that the local properties of the fluid are related to all the other regions – through the pressure term. The incompressibility condition results in an infinite sound speed, and in an instantaneous propagation of any perturbation in the fluid.

Burgers (1939) modelled the time evolution of the simplified version of the Navier–Stokes equation by considering $\nabla p = 0$. This equation has exact solutions, which may sound interesting, but it results in non-universal "turbulence", even though Burgers turbulence models have gained increasing interest due to their ability to describe the statistics of shock-induced structures and many other applications (see review by Bec and Khanin, 2007).

In the full Navier–Stokes equation, perturbations in $\boldsymbol{u}(\boldsymbol{x},t)$ are expected to have their distribution changed due to non-linear terms. These instabilities may drive local vorticity and

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result in the fragmentation of large-amplitude eddies into smaller ones, creating a turbulent pattern. As imagined by Richardson (1922), *big whirls have little whirls that feed on their velocity, and little whirls have lesser whirls, and so on to viscosity*. This statement represents one of the first conceptual descriptions of the energy cascade in turbulent flows. The shear drives unstable motions at large scales, which are broken and fragmented into smaller vortices, down to the smallest scales where they are damped, e.g. due to viscosity. In an incompressible viscous fluid this damping scale is that at which the timescale for viscous damping is of the order of the turnover dynamical time. At that scale, the eddy kinetic energy is transferred to internal energy due to viscosity. Turbulence is naturally developed over larger range of scales if viscosity is small, i.e. with a large Reynolds number ($Re = UL/\nu \gg 1$), the characteristic velocity U being injected at a length scale L.

Kolmogorov (1941, hereafter K41) realised that it would be possible to solve the Navier–Stokes equation for a turbulent flow if $\boldsymbol{u}(\boldsymbol{x},t)$ is considered a stochastic distribution. One of the key assumptions in the K41 theory is that the energy transfer rate ϵ should be constant at all scales. It is defined as $\epsilon \simeq \delta u_l^2/\tau_l$, where δu_l is the velocity fluctuation amplitude at length scale l, and $\tau_l = \tau_{\rm eddy} = l/\delta u_l$ its dynamical timescale[1]. Therefore, one obtains

$$\delta u_l \simeq (\epsilon l)^{\frac{1}{3}}. \tag{2}$$

Equation (2) means that turbulence can be modelled by scaling laws. This would be true within the so-called *inertial range of scales*, i.e. the scales where the energy transfer rate is constant, generally between the energy injection and the viscous damping scales. The velocity power spectrum $P_u(k)$ is defined[2] here by $\int_{k=1/l}^{\infty} P_u(k')\,dk' = \delta u_l^2$, from which we obtain the standard Kolmogorov power spectrum for the velocity field:

$$P_u(k) \propto \epsilon^{2/3} k^{-5/3}. \tag{3}$$

In other words, it is possible to reinterpret Kolmogorov's idea in Fourier space in terms of non-linear interaction between similar wavenumbers. This theory is a result of the so-called *locality*, i.e. similar wavenumbers, $k = 2\pi/\lambda$, of the non-linear wave–wave interaction that result in the energy cascade through smaller scales (Kraichnan, 1965a). From the spectral form of the Navier–Stokes equation, the three-wave interactions follow the selection rule $k_3 = k_1 + k_2$. The extrema are found at $k_3 \to 0$ and $k_1 = k_2$, which is the locality assumed in Kolmogorov's theory, resulting in $k_3 = 2k_1$.

The theory also predicts the scaling laws for the moments of velocity spatial increments, known as *velocity structure functions*, defined as

$$S_{\rm p}(l) = \left\langle \{[\boldsymbol{u}(\boldsymbol{r}+\boldsymbol{l}) - \boldsymbol{u}(\boldsymbol{r})]\cdot \boldsymbol{l}/l\}^p \right\rangle = C(p)\epsilon^{p/3} l^{p/3}, \tag{4}$$

where p is a positive integer representing the moment order and $\boldsymbol{l}$ is the spatial increment vector. In incompressible fluids, if the turbulence is considered *homogeneous*, *isotropic* and *self-similar*, i.e. scale invariant, then

$$S_{\rm p}(l) = C(p)\epsilon^{p/3} l^{p/3}, \tag{5}$$

where $C(p)$ was initially assumed by Kolmogorov to be constant with p.

One of the main successes of the Kolmogorov–Obukhov turbulence theory is the explanation of the empirical determination of the diffusion coefficient by Richardson (1926), done more than a decade before K41. The diffusion coefficient is related to the time evolution of the separation between Lagrangian points (e.g. particles dragged by the flow) in a turbulent medium. The probability distribution function Φ of pairs of points separated by a distance $\boldsymbol{r}$ may be described as

$$\frac{\partial \Phi(\boldsymbol{r},t)}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r} r^2 K(r) \frac{\partial \Phi(\boldsymbol{r},t)}{\partial r}, \tag{6}$$

where $K(r)$ represents the diffusion coefficient. It is easy to determine, from dimensional analysis, that if $\dot{r} = u(r) \propto r^{1/3}$ as in the Kolmogorov scaling, the diffusion coefficient for the inertial range will be $K(r) = k_0\,\epsilon^{1/3} r^{4/3}$, the scaling proposed by Richardson (1926). This diffusion coefficient for the inertial range substituted in Eq. (6) then results in

$$\Phi(\boldsymbol{r},t) = \frac{A}{(k_0 t)^3 \epsilon} \exp\left(-\frac{9r^{2/3}}{4k_0 \epsilon^{1/3} t}\right), \tag{7}$$

where A is a normalisation coefficient. The Richardson distribution is therefore non-Gaussian. Several experiments and numerical models have shown the validity of the turbulent diffusion scaling (Elliott Jr. and Majda, 1996; Fung and Vassilicos, 1998; Zouari and Babiano, 1994; Boffetta and Sokolov, 2002), as has also been recently used in the predictions of stochastic magnetic reconnection[3] (Lazarian et al., 2012).

This theory of turbulence has been quite successful in reproducing most of the experimental data, and there is

[1] Note that we distinguish τ_l and $\tau_{\rm eddy}$ here, since τ_l represents the timescale for energy transfer at scale l, while $\tau_{\rm eddy}$ is the eddy turnover timescale. In the K41 theory both timescales are the same, but this is not true for other cases, e.g. as in some magnetised cases.

[2] The power spectrum is defined as the one-dimensional spectrum in Fourier space, while the energy spectrum, generally defined as $E_u(k) = k^2\,P_u(k)$, is the three-dimensional spectrum. For the sake of simplicity, we use the term *power spectrum* to represent the latter.

[3] This term accounts for the magnetic reconnection that is induced by turbulent motions near the current sheet – the separation layer between fields with components of opposed directions – which would then result in reconnection rates as a function of the stochastic motions of the fluid.

a flourishing literature with hundreds of works available, e.g. Armstrong et al., 1995; Leamon et al., 1998; Bale et al., 2005; Koga et al., 2007; Bourras et al., 2009; Chian and Miranda, 2009; Chepurnov and Lazarian, 2010; Sahraoui et al., 2010; Chian and Muñoz, 2011; Chang et al., 2012; Hurricane et al., 2012; Miranda et al., 2013, just to mention a few. Naturally, many authors criticised the fact that $C(p)$ is a constant in Kolmogorov's initial theory, given the breakdown of self-similarity at small scales and the possible non-universality of turbulence (given its "memory" related to the energy injection). These criticisms have been addressed later in the Kolmogorov–Obukhov turbulence theory (Kolmogorov, 1962; Obukhov, 1962), including the effects of *intermittency*. Intermittency results from rare and large local fluctuations in the velocity field which break the similarity condition (Frisch, 1995). One of the effects of intermittency is observed in the probability distribution function (PDF) of velocity longitudinal increments $\delta u_l = [\boldsymbol{u}(\boldsymbol{r}+\boldsymbol{l}) - \boldsymbol{u}(\boldsymbol{r})] \cdot \hat{l}$, which shows large deviations from the Gaussian distribution at small scales, with large-amplitude tails and peaked distributions at $\delta u_l \sim 0$ (see Fig. 1). Kraichnan (1991) pointed that sharp shocks could, for instance, result in more regions with smooth fluid flows and also more regions with sharp transitions in velocities, compared to the standard picture of the self-similar K41 turbulence. We would then expect non-Gaussian PDFs at both small and large scales.

Many authors have attempted to determine theoretically the scalings of turbulence with intermittency. One of the most successful approaches is the multifractal description of the energy dissipation field proposed by She and Levêque (1994). This theory results in $S_{\mathrm{p}}(l) \propto l^{\zeta(p)}$, with

$$\zeta(p)=\frac{p}{3}\left(1+\frac{2}{3}\right)+(3-D')\left[1-\left(1-\frac{2}{3(3-D')}\right)^{p/3}\right], \quad (8)$$

where D' represents the dimensionality of the dissipation structures. In Kolmogorov–Obukhov theory, structures of highest dissipation are filamentary, better described then by $D' \sim 1$, while recent numerical simulations reveal a dominance of two-dimensional intermittent structures at small scales (e.g. Moisy and Jimenez, 2004; Kowal et al., 2007; Kowal and Lazarian, 2007; Boldyrev and Perez, 2012), which is also supported by experimental data (e.g. Fredriksen et al., 2003; Thess and Zikanov, 2007). Multifractal analysis of Voyager 1 and 2 in situ data has also shown intermittent features in the magnetic turbulence at the solar wind and the termination shock (Macek and Szczepaniak, 2008; Macek et al., 2011, 2012). On the theoretical side, Birnir (2013) derived a statistical solution of the stochastic Navier–Stokes equation from the linear Kolmogorov–Hopf differential equation, accounting for the She–Levêque intermittency corrections. His results satisfactorily reproduce the PDFs built on observations and numerical simulations of turbulent flows. Compressibility and coupling between magnetic fields and the plasma flow – both present in the dynamics of the interstellar medium (ISM) – make the description of the interstellar turbulence even more complex.

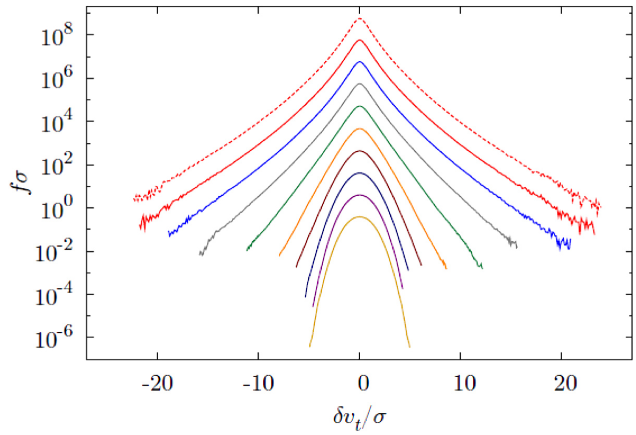

Fig. 1. PDF of velocity increments as a function of the lag length $|l|$, from small (top) to large scales (bottom) (extracted from Wilczek, 2010). The non-Gaussianity is clear for velocity increments at small scales.

1.1 Supersonic turbulence

Compressible plasmas are of great interest in astrophysics, and particularly in the case of interstellar turbulence. Compressibility in turbulent flows results in the formation of a hierarchy of density structures, viewed as dense cores nested in less dense regions, which are in turn embedded in low-density regions and so on. Such a hierarchical structure was described by von Weizsäcker (1951) as

$$\frac{\rho_\nu}{\rho_{\nu-1}}=\left(\frac{l_\nu}{l_{\nu-1}}\right)^{-3\alpha}, \quad (9)$$

where ρ_ν represents the average density of a structure at hierarchical level ν, at a length scale l, and α the compressibility degree, assumed to be the same at each level. The dimensionality of the system is obtained by $D' = 3 - 3\alpha$. Therefore, the average mass within each substructure must follow the relation $M_l \propto l^{3-3\alpha}$.

The density hierarchy as described above must then be coupled to the local turbulent motions. The energy density transfer rate must now be rewritten as $\epsilon_l = \rho_l\, \delta u_l^3 / l$ to account for the density changes at different scales (Lighthill, 1955). If, once again, one assumes the constancy of the energy transfer rate across scales within the inertial range (Fleck, 1996), one obtains the scaling of the amplitude of the velocity fluctuations

$$\delta u_l \propto l^{\frac{1}{3}+\alpha}, \quad (10)$$

and the velocity power spectrum is then given by

$$P_u(k) \propto k^{-5/3-2\alpha}. \quad (11)$$

Note that for stationary energy distribution solutions in compressible turbulence, $\alpha > 0$, which results in steeper velocity power spectra compared to the standard K41 scaling. The density power spectrum, on the other hand, instead of following the velocity field as a passive scalar would do, presents a distinct power spectrum given by

$$P_\rho(k) \propto k^{6\alpha-1}, \tag{12}$$

i.e. for $\alpha \sim 1/6$, the power spectrum of the density field becomes flat in the inertial range. One of the most striking results of the hierarchical model for the density field in compressible turbulence is its ability to recover the standard Kolmogorov scalings for the density-weighted velocity field $\boldsymbol{v} \equiv \rho^{1/3}\boldsymbol{u}$ (Fleck, 1996).

Numerical simulations of compressible turbulence have confirmed the scalings described above for $\alpha \simeq 0.15$ (Kritsuk et al., 2007; Kowal and Lazarian, 2007), close to $\alpha = 1/6$, for which the density power spectrum becomes flat. The velocity power spectrum on the other hand becomes $P_u(k) \propto k^{-2}$. Remarkably, this is the exact slope obtained for Burger's turbulence, despite the different framework of that theory.

1.2 Magnetized turbulence

Magnetic fields introduce further complexity in the plasma dynamics that can be described by the magnetohydrodynamic (MHD) equations in the fluid approximation and assuming perfect coupling between the field and the plasma:

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{u} = -\frac{\nabla p}{\rho} + \nu\nabla^2\boldsymbol{u} + \frac{(\nabla\times\boldsymbol{B})\times\boldsymbol{B}}{4\pi\rho} + \boldsymbol{F}, \tag{13}$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla\times(\boldsymbol{u}\times\boldsymbol{B}) + \eta\nabla^2\boldsymbol{B}, \tag{14}$$

where $\boldsymbol{B}$ is the magnetic field and η the plasma resistivity ($\eta = 0$ for ideal plasmas).

Let us first consider an external uniform magnetic field B_0. Any perturbation in the fluid velocity field will be coupled to the magnetic field. The magnetic tension/pressure results in a decrease in the non-linear growth of perturbations, but only in those perpendicular to the magnetic field lines. This complex coupling between the flow and the magnetic field makes the modelling of turbulence in magnetised plasmas an interesting task[4].

1.2.1 The Iroshnikov–Kraichnan model

A useful simplification of the equations above is made by considering $\boldsymbol{B} = \boldsymbol{B}_0 + \boldsymbol{\delta B}$ and using the Elsässer variable $\boldsymbol{z}^{\pm} = \boldsymbol{u} \pm \boldsymbol{\delta}\breve{\boldsymbol{B}}$, where $\breve{B} = B/(4\pi\rho)^{1/2}$. This was derived independently by Iroshnikov (1964) and Kraichnan (1965a, b) (IK hereafter). From this change in variables, Eqs. (13) and (14) result in (see Schekochihin and Cowley, 2007)

$$\frac{\partial z^{\pm}}{\partial t} \mp v_A \nabla_{||} z^{\pm} + z^{\mp}\cdot\nabla z^{\pm} = -\nabla p + \frac{\nu+\eta}{2}\delta z^{\pm} + \frac{\nu-\eta}{2}\delta z^{\mp} + \boldsymbol{F}, \tag{15}$$

where $v_A = B_0/\sqrt{4\pi\rho}$ is the Alfvén velocity and $\nabla_{||}$ is the spatial derivative parallel to the direction of the mean magnetic field.

In their model, Iroshnikov and Kraichnan proposed that incompressible magnetised turbulence results from the non-linear interactions of counter-propagating wave packets. The timescale for the two wave packets to cross each other is of the order of the Alfvén time $\tau_A \sim l_{||}/v_A$, where $l_{||}$ is the length scale of the wave packet parallel to the mean magnetic field. In their phenomenological description of the MHD turbulence, the interactions between the wave packets are *weak*, i.e. $|z^{\pm}| \ll \breve{B}_0$, or the field perturbations are much smaller than B_0. Notice that, superimposed on the magnetic fluctuations, the fluid is also perturbed and the dynamical timescale of a fluid "eddy" is $\tau_{\rm eddy} \equiv l/\delta u_l$. The different wave modes (mechanical and magnetic perturbations) thus interact with each other. For the interaction between modes to be weak the Alfvén time must be much smaller than the dynamical timescale, i.e. $\tau_A \ll \tau_{\rm eddy}$. The non-linear decay of the wave packets in such weak interactions, and subsequently the turbulent cascade, can only occur after several interactions. Since interactions are random, the wave packet amplitude changes in a random walk fashion, i.e. $N = (\tau_l/\tau_A)^{1/2}$ interactions are needed for the wave packet to change significantly. At the same time, N is also defined by the number of crossings in a decay timescale $N = \tau_l/\tau_{\rm eddy}$, which results in

$$\tau_l \sim \frac{\tau_{\rm eddy}^2}{\tau_A} \sim \frac{l^2 v_A}{l_{||}\delta u_l^2}. \tag{16}$$

Therefore, the turnover time at scale l is longer by a large factor and, as expected, the non-linear cascade proceeds much more slowly.

The second major assumption in the IK theory of weak turbulence is its isotropy, i.e. $l_{||} \sim l$. Substituting this scaling into the relation $\epsilon = \delta u_l^2/\tau_l$, one obtains

$$\delta u_l \sim (\epsilon v_A)^{1/4} l^{1/4} \quad \text{and} \quad P_u(k) \sim (\epsilon v_A)^{1/2} k^{-3/2}. \tag{17}$$

There is evidence of an IK cascade in the solar wind and interplanetary medium (e.g. Bamert et al., 2008; Ng et al., 2010). However, many observations of the solar wind turbulence also suggest a more Kolmogorov-like turbulence, i.e. $\propto k^{-5/3}$ (e.g. the early studies of Coleman, 1968, 1968; Matthaeus and Goldstein, 1982; and the more recent papers by Alexandrova et al., 2008; Chian and Miranda, 2009; Sahraoui et al., 2010; Li et al., 2011; Chian and Muñoz, 2011; Kozak et al., 2012; Hellinger et al., 2013). It is possible though that a mix of both cascades may occur, as pointed out by e.g. Salem et al. (2009) and Alexakis (2013), which

[4]More details on MHD turbulence may be found in Biskamp (2003).

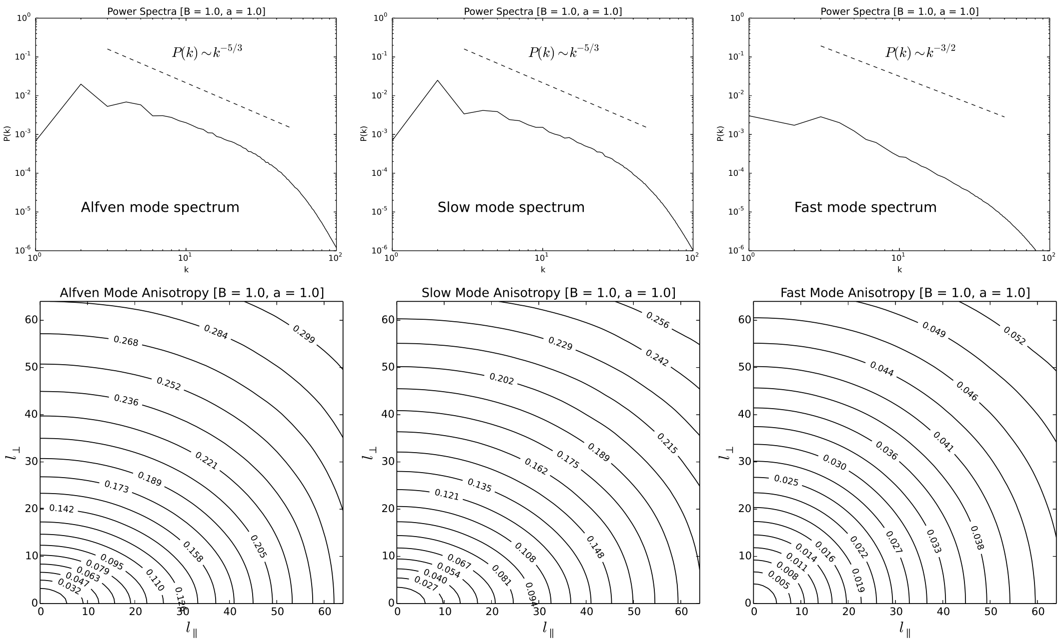

Fig. 2. Spectra and second-order structure function anisotropy of dispersion (δv) of the different wave modes in MHD turbulence. The Alfvén and slow modes present the K41 power cascade and strong anisotropy of dispersion of velocity at small scales, while the fast waves present the IK cascade and are basically isotropic at all scales. Data from a 1024^3 isothermal, sub-Alfvénic and subsonic turbulence model.

showed a mix of K41 and IK cascades for the magnetic and velocity field fluctuations, respectively. Moreover, most of these data also reveal the solar wind turbulence to be highly anisotropic (i.e. $\delta u_l^{||} \neq \delta u_l^{\perp}$) with respect to the local magnetic field (Horbury et al., 2008, 2012).

As pointed out by Goldstein (2001), one of the main issues raised by the solar wind is *why is the power spectrum of this anisotropic, compressible magnetofluid often Kolmogorov-like?*

1.2.2 The Goldreich–Sridhar model

Marsch (1990) remarked that if, instead of an Alfvén time, the timescale for the waves to interact non-linearly with each other was the regular eddy turnover time, i.e. $\tau_l \simeq \tau_{\rm eddy} \sim l_{||}/\delta u_l$, one would get a K41 cascade for the magnetised turbulence. This would be true also for the case of strong turbulence, $|z^{\pm}| > \breve{B}_0$. The isotropy condition was retained, which raised a problem, most of the observational data mentioned above revealing strongly anisotropic turbulence.

Hereafter Goldreich and Sridhar (1995) proposed a turbulent model based on anisotropic fluctuations, with strong coupling between the wave modes. Strictly speaking the GS95 model assumes a critical balance between mechanical and Alfvénic modes in such a way that $l_{\perp}/\delta u_l \simeq l_{||}/v_A$. Therefore

$$l_{||} \sim v_A \epsilon^{-1/3} l_{\perp}^{2/3} \quad \text{and} \quad P_u(k) \propto k^{-5/3}. \tag{18}$$

From Eq. (18), not only is the magnetized turbulence anisotropic, but it is *local* in the sense that the anisotropy is measured in the reference frame of the local magnetic field. Such an anisotropy is expected to occur in both the dispersion of velocity (δv) and wave vectors $\boldsymbol{k}$, though it is easier to observe velocity dispersion anisotropies from the interstellar medium, as discussed below. Therefore, statistically, a large number of eddies with local fields randomly distributed in space result in an average zero anisotropy (even at small scales). In the strong magnetised cases though, the anisotropy would be more clearly detected in experiments and observations.

Several direct numerical simulations of magnetised turbulence in a quasi-incompressible regime have been performed in the past decade. Many numerical experiments reveal that MHD turbulence indeed has a large part of its energy cascade close to a K41 distribution. However, as shown by Cho and Lazarian (2002, 2003), Cho et al. (2002) and Kowal and Lazarian (2010), the decomposition of the different modes in MHD turbulence actually reveals that, although Alfvén and slow modes behave as a K41 type of turbulence and are anisotropic, the fast modes are isotropic and follow IK statistics (see Fig. 2).

The effects of imbalanced (or cross-helicity) turbulence in the cascade and statistics of the local fields have also been addressed in the past few years (Lithwick et al., 2007; Beresnyak and Lazarian, 2008, 2010; Wicks et al., 2011; Markovskii and Vasquez, 2013, and references therein). Imbalanced turbulence occurs when waves travelling in opposite directions along the mean magnetic field are of unequal amplitudes, i.e. carry different energy fluxes to small length scales, so that $z_l^+/z_l^- \neq 1$ and $\epsilon_l^+/\epsilon_l^- \neq 1$. The imbalance may arise in MHD turbulence since the interaction timescales between waves z_l^+ and z_l^- are different, and the cascade generally occurs faster for z_l^-. This is understood as the number of interactions (N) being much larger for counter-propagating wave packets, resulting in $\epsilon_l^+/\epsilon_l^- > 1$. In such a scenario, numerical simulations show that the anisotropy is not equal for the different wave modes.

Locality of scales for wave–wave interactions has also been the subject of recent studies in turbulence (Carati et al., 2006; Alexakis, 2007; Mininni et al., 2008; Aluie and Eyink, 2009; Beresnyak and Lazarian, 2010). Magnetic fields are responsible for long-range interactions, from the Lorentz force acting over the whole fluid frozen to it. Therefore, different wavelengths may interact with each other non-linearly. Bispectra of fluctuations of density are discussed in Burkhart et al. (2009), and the non-local interactions appear to be important in MHD and supersonic turbulence models. A similar approach is used for studying the non-local interactions of Elsässer modes (Cho, 2010), resulting in a substantial fraction of non-local interactions in MHD turbulence. The role of the non-local interactions in the turbulent cascade is still not clear though. Turbulence in magnetised collisionless plasmas has been also studied in the past few years (e.g. Hellinger et al., 2006; Schekochihin et al., 2008; Bale et al., 2009, and others) in order to determine the role of collisionless plasma instabilities in the dynamics of plasma turbulence. Simulations by Kowal et al. (2011) and Santos-Lima et al. (2013) reveal that the statistics are still dominantly Kolmogorov-like, though strong asymmetries that may also arise due to instabilities (firehose, mirror and cyclotron instabilities) are small scale.

2 Signatures of a turbulent ISM

In the previous section some theoretical aspects of turbulence have been presented. Its direct comparison to the dynamics of the interstellar gas is not trivial, as we discuss in the following. However, we will present here some observational evidence of a turbulent ISM, and discuss the possible turbulent regimes that may be inferred from these.

The recognition of a turbulent interstellar medium dates back to the 1950s with the work of von Weizsäcker (1951) on the spatial distribution of dense structures in the plane of the sky. He recognised the hierarchy of structures and suggested its turbulent origin. The identification of turbulent motions was provided shortly after it was measured from velocity dispersions (von Hoerner, 1951). Later on, the observational and theoretical supports for a turbulence-dominated ISM have grown considerably (see reviews by Elmegreen and Scalo, 2004; Mac Low and Klessen, 2004; Hennebelle and Falgarone, 2012, and references therein), causing a major shift in the understanding of the ISM nature, from a thermal pressure-dominated system, as thought before, to a very dynamic multi-phase system.

2.1 Density distributions

As mentioned above, one of the main signatures of the turbulent character of the ISM is related to the density distribution of its contents. Up to now, tracers of the gas density distributions of the ISM at large scales have been predominantly indirect[5]. They rely on spectral lines and continuum emissions from the different phases of the ISM: the hot and fully ionised (HIM), the warm and fully/partially ionised medium (WIM/WNM), and the cold weakly ionised (CNM). These emissions being integrated along lines of sight and projected in the plane of the sky, sophisticated inversion methods have to be implemented. It is the statistical approach of the temporal and spatial variability of these emission fluxes that are the readily accessible observational techniques for studying interstellar turbulence.

With hydrogen being the most abundant element in the universe, the λ 21 cm line of neutral hydrogen is a key diagnostic. Its line-integrated emission is proportional to the bulk of the hydrogen column density, since its opacity remains low over most of the ISM. Statistics of the HI intensity spatial distributions have therefore been used to probe interstellar turbulence, but the results are far from homogeneous. Green (1993) studied the power law of the spatial power spectrum of the HI emission from different fields in our galaxy. He obtained power spectra with slopes between -2.2 and -2.8 at a scale range between 35 and 200 pc. From the HI 21 cm absorption towards Cas A. Roy et al. (2009) derived a power law with index -2.7, consistent with Kolmogorov turbulence in the diffuse interstellar medium. However, Miville-Deschênes et al. (2003) find an impressive power law in the nearby ISM at high galactic latitude with the same slope of -3.6 over two orders of magnitude in scales (between 0.1 and 25 pc). Similar studies have been performed since then, including other density tracers such as the CO and ^{13}CO line emission of molecular clouds, and power laws have also been inferred (e.g. Bensch et al., 2001; Hill et al., 2008). A review of the scatter of the power-law slopes measured is given in Hennebelle and Falgarone (2012). The scatter of the slope values is certainly affected by projection effects: one would expect a 2-D power spectrum $k^{-8/3}$ for an intrinsic Kolmogorov scaling. However, the integration along lines of sight crossing often large amounts of turbulent ISM with different properties tends to blur such a simple law. Moreover, the different tracers originate in truly different phases of the ISM, with varying amounts of small-scale structure that may affect the power spectrum of the density distributions (i.e. in many cases, like supersonic turbulence, density fluctuations

[5]In situ data have been obtained at the nearby interstellar plasma by Voyager 1 (Gurnett et al., 2013), though no direct study of the local turbulence has been discussed yet.

are not simply advected by turbulence as passive scalars, see e.g. Audit and Hennebelle, 2005). Indeed, as seen in Fig. 10 of Hennebelle and Falgarone (2012), many studies (including the power spectrum of the dust thermal emission) give power-law indices close to −2.7. It is not possible though to presume that a Kolmogorov-like cascade operates in the ISM, with scalings given by Eqs. (2) and (3). Even though compressibility seems to have little effect on the statistics of the ISM, except for small scales (∼ pc scales) and cold and dense regions, magnetisation effects may be important, as we discuss further below.

Armstrong et al. (1995) used another tracer of density fluctuations, the scintillation of the background radiation (i.e. changes in the refraction index due to the turbulent motions in the ionised components of the ISM) in order to obtain the density spectrum along the line of sight. As a complementary method, fluctuations of the Faraday rotation measurements (RM) in the plane of sky are also used to estimate density fluctuations (once the magnetic field is known) in the line of sight (e.g. Minter and Spangler, 1996). The combined data provide the density fluctuations along the line of sight, but for different length scales, as seen in Fig. 3. The turbulence probed by both methods (scintillations and RM) presents a most impressive spectrum, with a unique Kolmogorov-like slope across more than ten orders of magnitude in wavenumber.

Similar works have been done for external galaxies. Turbulence has been characterised based on similar techniques for the Small Magellanic Cloud (see Stanimirovic et al., 1999; Stanimirovic and Lazarian, 2001; Chepurnov et al., 2008; Burkhart et al., 2010) and has revealed spatial variations in HI morphology. Dutta et al. (2013) calculated an HI intensity fluctuation power spectrum for a sample of 18 spiral galaxies and found slopes in the range of −1.9 to −1.5. Shallower spectra, compared to K41, could be evidence of two-dimensional eddy-dominated turbulence at scales larger than the disk thicknesses.

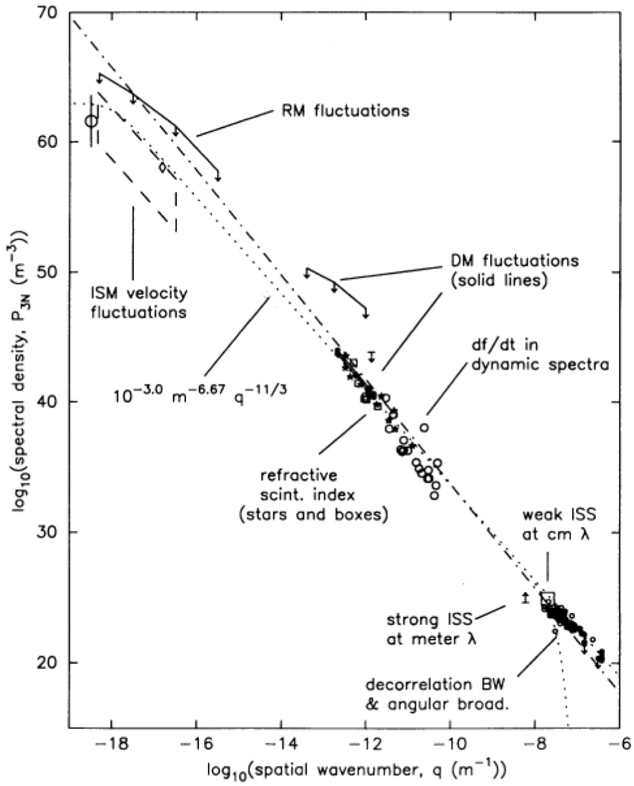

Fig. 3. Power spectrum of density along the line of sight from different data sets, and the dashed line as a reference for a Kolmogorov-like spectrum for one dimension ($k^{-11/3}$) (extracted from Armstrong et al., 1995).

2.2 Velocity fields

2.2.1 Direct statistical analysis

Spectral lines of several species observed with high spectral resolution may be used to infer the turbulence velocity distributions in the different phases of the ISM, such as hydrogen lines (mostly) and some ions for the diffuse ISM (e.g. Bowen et al., 2008), and molecular spectral lines (^{12}CO and ^{13}CO in most surveys) for the molecular clouds. The early surveys of Larson (1981) and Solomon et al. (1987) revealed the universal line-width and mass-distribution scalings among molecular clouds. Notably, both works pointed to a velocity dispersion relation $\sigma_v \propto l_v^{\alpha}$, with $\alpha \sim 0.5$ (see Fig. 4, left panel). Many similar studies were carried out to study the velocity distribution in molecular clouds, such as the work by Goldsmith et al. (2008), Yoshida et al. (2010), Qian et al. (2012) and Heyer and Brunt (2012) in the Taurus Molecular Cloud; Gustafsson et al. (2006) and Liu et al. (2012) for the Orion complex, and many others.

More recent studies confirmed the same scaling relation although with slopes varying significantly (Heyer and Brunt, 2004; Qian et al., 2012). Qian et al. (2012) for instance used the variance of the velocity difference of cores in molecular clouds, instead of the line width, and obtained $\alpha_v \sim 0.7$. On the other hand, massive cores are known to exhibit shallower slopes compared to what is frequently assumed (i.e. $\alpha_v < 0.5$).

Recently, Ballesteros-Paredes et al. (2011) compiled different observational surveys and concluded that, while in general terms the typical CO clouds observed by Heyer et al. (2009) lie close to Larson's relation, this is clearly not the case for the dense and massive cores, which exhibit large velocity dispersions for their relatively small sizes (Fig. 4). Those authors propose that the large dispersions observed at small scales are related to increased velocities as the clouds become gravitationally bound. However, the increased dispersion at small scales has already been reported in Falceta-Gonçalves et al. (2010a), based on numerical simulations without self-gravitating objects. For these authors the large

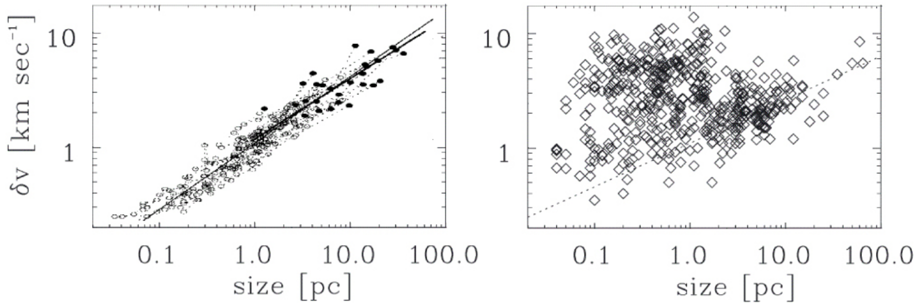

Fig. 4. Velocity dispersion relations from different surveys by Heyer and Brunt (2004) (left panel) and compilation from several surveys done by Ballesteros-Paredes et al. (2011) (right panel). As the later authors point out, while large CO clouds from the survey by Heyer et al. (2009) exhibit the typical Larson relationship, denser structures show a larger dispersion of velocity. This fact has been interpreted by those authors as being due to gravity in collapsing cores, while Falceta-Gonçalves et al. (2010b) argued for projection effects and compressibility.

dispersion observed at small scales is an intrinsic feature of the turbulent gas. The broad dispersion of the scaling relation indicates a turbulent regime dominated by compressible motion at small scales, as discussed in Sect. 1.1, though regular incompressible turbulence dominates at larger scales. Compressibility, as described in Fleck's model (see Eq. 10), naturally gives larger slopes for the dispersion relation, with a value of $\alpha \sim 0.16$ favoured by observations. It is not clear though what the actual role of gravity is in the statistics of the molecular cloud emissions.

At the large-scale end of the cascade, the apparent uniqueness of the scaling of the velocity dispersion with size scale suggests a universal source (or mixture of sources) of energy for the molecular gas turbulence in our galaxy. Chepurnov and Lazarian (2010) presented a statistical analysis of high-latitude HI turbulence in the Milky Way based on the velocity coordinate spectrum (VCS) technique. They found a velocity power spectrum $P_u(k) \propto k^{-3.8}$ and an injection scale of $\sim 140 \pm 80$ pc. The slightly steeper slope, compared to K41, can be the result of shock-dominated (compressible) turbulence, with averaged sonic Mach numbers ~ 7–8 (see Sect. 1.1 above).

Two-point statistics are also used, but, since in situ measurements are not yet available, one easily accessible observable turns out to be the variations in the plane of the sky of the line-of-sight centroid velocity of spectral lines. Lis et al. (1996) showed that they trace the plane-of-the-sky projection of the vorticity. Using a sample of about one million independent CO spectra in a diffuse field, Hily-Blant and Falgarone (2009) identified, on statistical grounds, the ensemble of positions at which vorticity departs from a Gaussian distribution. These form coherent elongated structures at the parsec scale that are found to harbour sub-structures of most intense velocity shears down to the milliparsec scale (Falgarone et al., 2009). These coherent structures are proposed to be the manifestations of the intermittency of turbulent dissipation in diffuse molecular clouds (see the review of Hennebelle and Falgarone, 2012), which may be compared to Eq. (8) above.

Li and Houde (2008) studied the scaling relations of the velocity dispersions from different neutral and ionised molecular species, namely HCN and HCO^+, in the region of M17. As occurs in many other star-forming regions, the ionised molecules systematically present a smaller dispersion of velocity compared to the neutral molecules. Such a difference arises as turbulent energies dissipate differently for the species due to ambipolar diffusion. Falceta-Gonçalves et al. (2010b) showed that the dispersion for ions is typically smaller than that for the neutral species basically due to the damping of the ion turbulence at the ambipolar diffusion scales ($\simeq 0.01$ pc).

The direct comparison between statistics of observational data and the theory must be done with caution. Column density projections, or in another sense emission maps, are influenced by projection effects. Different structures projected onto the same line of sight, but decorrelated at a given length scale, may be observed as a single structure on the projected emission map. Some deconvolution is possible though once the velocity profile is known, with a high spectral resolution.

2.2.2 Indirect access to the velocity field via maser emission

The low surface brightness of the above tracers and projection effects make the direct analysis of turbulent flows in the ISM difficult. Maser spots that are bright point sources and are transported by turbulence as passive scalars (because they are tiny and low-mass structures) turn out to be powerful tracers of the turbulent velocity field. Maser radiation in molecular lines appear in dense regions where population level inversion can be generated by radiative pumping, for instance in the dense molecular gas of star-forming regions (SFRs) associated with ultra-compact HII regions, embedded IR sources, hot molecular cores, Herbig–Haro objects, and outflows (Litvak, 1974; Reid and Moran, 1981; Elitzur, 1992; Lo, 2005). Maser emissions are often characterised by high brightness temperatures and high degrees of polarisation. Intense maser emission is detected in the molecular lines of hydroxyl (OH), water (H_2O), silicon monoxide (SiO), ammonia (NH_3), methanol (CH_3OH), among others.

Walker (1984) used the *very long baseline interferometry* (VLBI) maps of the H_2O maser source in W49N to demonstrate that both two-point velocity increments and two-point spatial correlation functions exhibit power-law dependencies on the maser spot separation, which is indicative of a turbulent flow. Gwinn (1994) performed statistical analysis of VLBI data for W49N to confirm the power-law dependence of the velocity dispersion and spatial density of masing spots on a spatial scale, and interpreted this observation as evidence of turbulence. Imai et al. (2002) reported sub-milliarcsec structures of H_2O masers in W3 IRS 5. A cluster of maser spots (emission spots in individual velocity channels) displays velocity gradients and a complicated spatial structure. Two-point spatial correlation functions for the spots can be fitted by the same power laws in two very different spatial ranges. The Doppler-velocity difference of the spots as a function of spot separation increases as expected in Kolmogorov-type turbulence. Strelnitski et al. (2002) used VLBI data to investigate the geometry and statistical properties of the velocity field traced by H_2O masers in five star-forming regions. In all sources the angular distribution of the H_2O maser spots shows approximate self-similarity over almost four orders of magnitude in scale. The lower order structure functions for the line-of-sight component of the velocity field can be fitted by power laws, with the exponents close to the Kolmogorov value. Similar results were also obtained for other regions (e.g. Richards et al., 2005; Strelnitski, 2007; Uscanga et al., 2010).

2.3 Turbulent magnetic fields

The magnetic fields in our galaxy are modelled as a superposition of different components: (i) a large-scale field, following a spiral structure in the plane of the galactic disk, and extending high above the plane into the galactic halo, and (ii) a complex component of locally disturbed magnetic fields, which are related to molecular clouds and star formation regions. The spiral pattern in the disk aligns with the spiral arms (e.g. Han, 2006). This is expected, since the shear of gaseous motion around the centre of the galaxy stretches the field lines in this direction (see the review of mean field dynamo by Beck et al., 1996).

There are four main methods of studying the fluctuations in the ISM magnetic field, namely the polarisation of dust thermal emission (both in emission in the far-infrared (FIR) and absorption in the visible and near-IR), the Zeeman effect of spectral lines, Faraday rotation and polarisation of the synchrotron emission. Polarised synchrotron emission can also be mapped in order to provide the geometry of the field lines in the plane of the sky. Faraday rotation and synchrotron polarisation measurements excel in probing the magnetic field of the diffuse ionised medium of the ISM, i.e. they are excellent tools for studying the large-scale fields of galaxies in general. More extensive reviews both of magnetic fields in star-formation regions and galactic-scale magnetic fields are given in Crutcher (2012) and Han (2006), respectively.

As mentioned earlier, synchrotron emission polarisation can be used for mapping the large-scale structure of the magnetic fields in galaxies (see review by Beck, 2009). The fields traced by the polarised synchrotron emission present intensities of the order of $\sim$ 10–15 µG. However, the synchrotron emission probes the ionised medium only, which is less useful in determining the turbulence properties of the star formation regions, dominated by the dense and neutral components of the ISM. Therefore, a magnetic field with intensity $\sim$ 10–15 µG is supposed to thread most of galactic disk, except the dense regions of the arms where the local properties of the plasma and stellar feedback may dramatically change the field properties.

Oppermann et al. (2012) compiled an extensive catalogue of Faraday rotation measure (RM) data of compact extragalactic polarised radio sources in order to study the angular distribution of the all-sky RMs. The authors found an angular power spectrum $P(k) \propto k^{-2.17}$ for the Faraday depth, which is given by the product of the line-of-sight magnetic field component $B_{\rm LOS}$ and the electron number density $n_{\rm e}$. The combination of the RM and polarisation vectors of the synchrotron emission allows one to reconstruct the three-dimensional structure of galactic magnetic fields. Such angular fluctuations of the Faraday depth are thought to be related to the turbulent ISM. However, the relationship between the fluctuations of the RM and the local fluctuations of electron density and magnetic fields is not clear yet. This, for instance, is an interesting subject for further comparisons with simulations (as in Gaensler et al., 2011).

Possibly the most direct method for estimating the magnetic field intensity in the dense and cold ISM relies on the detection of the Zeeman effect (see Robishaw et al., 2008, for details). For instance, Sarma et al. (2002) detected and studied the Zeeman effect in H_2O masers in several SFRs and determined line-of-sight magnetic field strengths ranging from 13 to 49 mG. They found a close equilibrium between the magnetic field energy and turbulent kinetic energy in masing regions.

Alves et al. (2012) showed that shock-induced H_2O masers are important magnetic-field tracers of very high-density gas in low-mass protostellar core IRAS 16293-2422. They investigated whether the collapsing regime of this source is controlled by magnetic fields or other factors such as turbulence, and concluded that the magnetic field pressure derived from data is comparable to the ram pressure of the outflow dynamics. This indicates that the magnetic field is energetically important for the dynamical evolution of the protostellar core.

Due to its brightness, maser emission is better for probing magnetic fields, but they are rare and limited in extent. The Zeeman effect in non-masing regions has been detected for HI, OH, and CN lines for which the turbulent broadening is typically larger than the Zeeman splitting in frequency. The compilation by Crutcher (1999) and recent CN Zeeman observations in SFRs by Falgarone et al. (2008) show that the turbulent motions within the SFRs and molecular clouds are supersonic but sub-Alfvénic. The *upper limit* magnetic field intensity scales with density, estimated from a Bayesian analysis, as $B \propto n^{\kappa}$, with $\kappa \sim 0.47$ (Crutcher et al., 2010). Collapsed structures along the mean field would produce $\kappa \to 0$, while shock compressions perpendicular to the field lines result in $\kappa \to 1$. The observed relation with $\kappa \sim 0.47$ is expected, for instance, in Alfvénic perturbations and is in agreement with MHD simulations (e.g. Burkhart et al., 2009). It was also claimed in that work that, despite its relative importance in the overall dynamics of clouds, the uniform magnetic fields in these clouds are in general not strong enough to prevent gravitational collapse based on the mass-to-flux (M/Φ) ratios observed. Other major compilations of Zeeman measurements in molecular clouds are given, for example, in Bourke et al. (2001) and Troland and Crutcher (2008) with similar results.

Polarisation maps of molecular clouds

Radiation may be polarised due to a preferred direction for emission/absorption from aspherical dust grains, as well as by some molecules and atoms. The ISM in known to be populated by a complex distribution of grain sizes and shapes. Depending on its composition an aspherical rotating dust particle may align with the magnetic field line. The orientation of the polarisation of radiation is then linked to the orientation of the magnetic field itself (see review by Lazarian, 2007). Many observational data have been made available in the past decade both on absorption and emission dust polarisation (e.g. Heiles, 2000; Chapman et al., 2011).

The strengths of magnetic fields can be estimated from polarisation maps by the Chandrasekhar and Fermi (1953) (CF) technique. The CF method is based on the assumption that the magnetic and turbulent kinetic pressures are the dominant ones within the cloud, and that the fluid motions are coupled to the magnetic field lines. In this sense, any perturbation from the fluid turbulence will result in a change in the orientation of the field lines. Major improvements in the CF technique are given e.g. by Falceta-Gonçalves et al. (2008), Hildebrand et al. (2009) and Houde et al. (2009). If the velocity dispersion $\delta v_{\rm los}$ is known, e.g. from spectral lines, the mean magnetic field in the plane of sky can be estimated as (Falceta-Gonçalves et al., 2008)

$$B_{\rm sky}^{\rm uniform} \simeq \delta v_{\rm los}\,(\pi < \rho >)^{1/2}\left[1+\tan^{-1}(\delta\phi)\right], \qquad (19)$$

where $\delta\phi$ represents the dispersion in the polarisation angle.

From the equation above, the ratio $\delta B/B_{\rm sky}$ – assumed to be $\sim \tan^{-1}\delta\phi$ – is directly related to the Alfvénic Mach number of the turbulence. Notice that the dependence of the projected $\delta B/B_{\rm sky}$ on the actual 3-D MHD turbulence may be

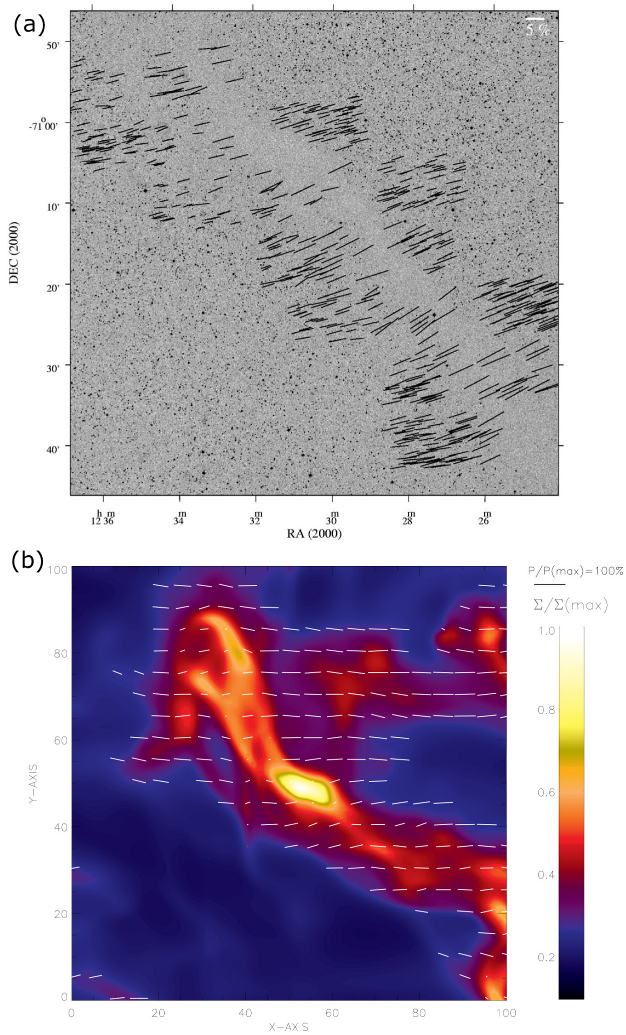

Fig. 5. Left panel: optical polarisation map of the Muska dark cloud (extracted from Pereyra and Magalhães, 2004). Right panel: simulated polarisation map from a three-dimensional simulation of MHD sub-Alfvénic turbulence (extracted from Falceta-Gonçalves et al., 2008).

removed from higher order statistical analysis, as proposed in Falceta-Gonçalves et al. (2008).

The left image of Fig. 5 presents the polarisation map of the Muska dark cloud (Pereyra and Magalhães, 2004) in the optical wavelengths as a result of dust absorption. Vectors represent the magnetic field orientation. The filamentary morphology of the dark cloud is perpendicular to the external field, which is very uniform, indicating a sub-Alfvénic turbulent regime. On the right-hand side of Fig. 5, the polarisation map is overplotted on the column density projection of a 3-D MHD numerical simulation of sub-Alfvénic turbulence (Falceta-Gonçalves et al., 2008). Such comparisons between MHD numerical simulations and measurements of magnetic fields in the ISM are important in unveiling the physics of

MHD turbulence, and its role in other phenomena such as star formation.

Spatial dispersion of magnetic fields in molecular clouds from polarisation maps may be used to characterise the power spectrum of magnetised turbulence in the inertial and dissipation ranges. Houde et al. (2011) found a power law inertial range for the magnetic field spatial distribution that is $\propto k^{-2.9\pm0.9}$, and a cutoff at scales ~ 0.009 pc, which is claimed by the authors to be related to the ambipolar diffusion scales.

Again, as another issue in a proper modelling of the statistics of velocity, gravity is claimed to interfere with the statistics of the observed polarisation maps (Koch et al., 2012a, b). Gradients in emission towards the cores of molecular clouds have been shown to be associated with gradients in the polarisation angles. A transition from a magnetically subcritical to a supercritical state[6] could then explain the trend, and this technique could provide an independent way of estimating the local magnetic force compared to gravity.

Heyer and Brunt (2012) showed that the turbulence in the densest regions of the Taurus molecular cloud is super-Alfvénic, while the reverse is true in the surrounding lower density medium, threaded by a strong magnetic field. This observational result is in agreement with the transition expected between scales as dense structures are formed, e.g. by shocks, in a supersonic but sub-Alfvénic large-scale turbulence (see the discussion in Falceta-Gonçalves et al., 2008; Heyer et al., 2008; Burkhart et al., 2009).

Similar to the synchrotron radiation case, by combining dust polarisation maps with Zeeman measurements in molecular clouds one can determine the three-dimensional structure of the magnetic field. Poidevin et al. (2013) recently succeeded in testing this approach for a number of objects of the SCUBA Polarimeter Legacy (SCUPOL) data catalogue. The authors were able to determine the orientation of the mean field with respect to the line of sight, as well as to estimate the turbulence regime within several molecular clouds. The authors also claimed that all observed clouds seem to present a universal large-scale turbulence that is supersonic ($M_\mathrm{s} \sim 6$–8) and sub-Alfvénic ($M_\mathrm{a} \sim 0.5$–0.9), at scales as large as 50pc.

In terms of comparing these data with basic theories of magnetised turbulence, most observations point towards a magnetically dominated turbulence at scales larger than a few tenths of parsecs. Heyer et al. (2008) also showed some of the first evidence of anisotropic turbulence in molecular clouds, with respect to the large-scale magnetic field orientation. The observations of the Taurus Molecular Cloud revealed a significant anisotropy in the dispersion of velocity (δv), being larger for lags perpendicular to the mean large-scale field lines. Even though a Goldreich–Sridhar similarity relation is not obtained, the anisotropy observed is a strong indication of strong coupling between MHD wave modes in the insterstellar turbulence, as predicted by the GS95 model. We could extrapolate a bit and say that a GS95 model combined with fractal density distributions, as given in Fleck (1996), is favoured for the ISM turbulence based on current observations.

[6]i.e. the system becomes supercritical once gravity overcomes the magnetic pressure.

3 Origins of interstellar turbulence

Surveys of different atomic and molecular line emissions have shown us that the diffuse ISM is turbulent at scales > 150 pc, with $\delta v \geq 50\,\mathrm{km\,s^{-1}}$. This results in a specific energy transfer rate[7] of $\epsilon \simeq m_H n_H \delta v_L^3 / L \sim 10^{-25} - 10^{-24}\,\mathrm{erg\,cm^{-3}\,s^{-1}}$. Brunt et al. (2009) estimated the driving scales of turbulence for molecular clouds by comparing observed and synthetic CO velocity dispersions from numerical simulations. They found that only models with large-scale sources of turbulence, such as supernovae-driven outflows (SNe) and galactic dynamics, fit well to the observed data.

Supernovae have been claimed as main turbulence drivers by many authors (e.g. Norman and Ferrara, 1996; Mac Low and Klessen, 2004; Avillez and Breitschwerdt, 2004, 2005; Joung and Mac Low, 2006; Hill et al., 2012). They certainly correspond to an important driving mechanism for turbulence in starburst regions and small galaxies (e.g. Falceta-Gonçalves et al., 2010a; Ruiz et al., 2013). However, their impact on galactic turbulence, in a more generalized sense, is still a matter of debate.

One issue is that numerical simulations of SNe-driven turbulence create superbubbles that are far too hot and diffuse (see Avillez and Breitschwerdt, 2005; Joung and Mac Low, 2006; Hill et al., 2012). Other critical arguments disfavour SNe as a main driver mechanism as well, at least for our galaxy. Zhang et al. (2001) analysed the CO emission lines from the Carina complex and obtained a turbulent energy flux per mass density unit cascading over scales $\sim 10^{-7}\,(\mathrm{km\,s^{-1}})^2\,\mathrm{yr^{-1}}$ that could not be explained from stellar feedback, but which is in rough agreement with the injection rate of energy from the gravitational interaction of the ISM gas and the galactic spiral arms. Sánchez-Salcedo et al. (2007) also showed that HI mapping of our galaxy is consistent with a turbulence injection rate that is not directly related to the star formation rate, but is about constant with respect to the galactocentric radius. Also, the correlation lengths related to SNe turbulence are strongly dependent on local properties (such as local density and temperature) (see Leão et al., 2009). Such local dependence also occurs with respect to the height related to the galactic plane, since SNe energy is easily released outwards (e.g. Melioli et al., 2009; Marasco et al., 2013). The universality of the observed properties of

[7]This estimate is at least one order of magnitude larger than that of Mac Low and Klessen (2004), since these authors considered a lower injection velocity at the largest scales ($\delta v_L = 10\,\mathrm{km\,s^{-1}}$).

turbulence in our galaxy, together with the extremely large injection scales (>100 pc), suggest a galactic-scale driving source, which is later amplified, as second-order effects, by local stellar feedback. Qian et al. (2012), for instance, obtained similar core and ambient turbulent statistics, which suggested that molecular cores condense from more diffuse gas, and that there is little (if not no) additional energy from star formation into the more diffused gas.

Turbulence driven by galactic dynamics models, such as driven by velocity shears in the galactic disk, were posed long ago (e.g. Fleck, 1981). Instabilities such as the magneto-rotational instability have also been proposed (e.g. Sellwood and Balbus, 1999; Kim and Ostriker, 2002). Interactions between the arms of the galaxy and the disk gas also generate perturbations, as large as $20\,\mathrm{km\,s^{-1}}$ (Gómez and Cox, 2002), that could explain most of the injection of energy into turbulent motions. It is not clear yet which of these mechanisms (SNe or galactic dynamics) is more important for the observed turbulence in the ISM. Certainly, it is a promising subject for studies in the upcoming years, both from theoretical and observational sides.

4 Conclusions

In this work, we briefly reviewed part of the current understanding of incompressible, compressible and/or magnetised turbulence, which can be applied to characterise the interstellar medium. There is a vast literature available for each of these and a complete review of turbulence is out of the scope of this work. We discussed the recent theoretical improvements made in the modelling and characterisation of the different turbulent regimes. Multifractal description, statistics of probability functions, and spectral analysis are just a few that have been currently employed to characterise spatial and temporal variations of plasma properties associated with turbulent motions.

Phenomenological descriptions of turbulence in Fourier space, such as that of Kolmogorov–Obukhov, are particularly simple and still very useful in the diagnostics of interstellar turbulence. Since scaling relations for compressible, incompressible and magnetised turbulence of these theories may differ among each other, observations can be used to determine the turbulent regime of the ISM.

Spectroscopy has been long used to probe the velocity distributions along the line of sight. The observed amplitudes of the turbulent motions indicate that the ISM transits from a supersonic turbulent regime at scales of tens to hundreds of parsecs, at which the turbulence is driven, to subsonic at smaller scales. The scales where turbulence is subsonic depend on the "phase" of the ISM plasma. Dense molecular clouds present lower temperatures, which result in subsonic turbulence only at very small scales ($\ll 1$ pc). The warmer and more diffuse media, such as the warm neutral medium and the warm diffuse medium, present subsonic flows at scales of a few parsecs due to the larger local sound speeds. It is interesting to mention that this transition is deeply related to the origin of the dense molecular clouds. These objects originate either due to the large-scale compressible motions of the gas (e.g. Williams et al., 2000), or at small scales due to other mechanisms, such as thermal instabilities. Current observations favour the first, given their length scales and internal dynamics (see Poidevin et al., 2013). Spatial gas distributions over the plane of the sky are also provided observationally. The filamentary structure observed reveals a compressible dominated turbulent regime, at least at most of the observed scales. Observations also reveal a magnetised ISM. All these ingredients combined result in compressible and magnetised ISM turbulence, challenging theorists to provide a phenomenological description of the combined effects of supersonic flows and strong magnetic fields. Despite the good agreement between observations and the Goldreich–Sridhar model for magnetised turbulence, and Fleck's model for compressible ones, such as spectral slopes, scaling relations and multifractal analysis applied to emission maps, a complete unified theory has yet to be developed.

One of the major problems in comparing statistics of observed quantities with theories of turbulence relies on projection effects. Observations are spatially limited in the sense that all statistics are done either along the line of sight (e.g. scintillation, velocity dispersion from spectral lines, Faraday rotation) or in the plane of the sky. In addition, even the plane-of-the-sky maps are related to integrated quantities (e.g. emission lines, column density, Stokes parameters for the polarisation maps). One must therefore be careful when comparing these with theories of three-dimensional turbulence.

Other effects may also make a direct comparison between theory and observations challenging. Self-gravity of dense gas and stellar feedback, for instance, have been neglected in this paper. These processes are responsible for extra sources of energy and momentum, but are not easily linked to the turbulent cascade. Despite their obvious importance for the process of e.g. star formation, their role in the statistics of the turbulence is not completely clear. Naturally, fragmentation and clumping would be enhanced if self-gravity is considered (Vázquez-Semadeni et al., 1996; Ballesteros-Paredes et al., 2011; Cho and Kim, 2011); however, its role in the cascade itself and in intermittency is unknown.

Future studies from the theoretical side are possibly to be focused on the understanding of combined processes, such as magnetic fields, gravity, compressibility and radiation, and on the energy transfer among scales. The formation of coherent structures and how their statistics relate to the bulk of the fluid are vital for theories of star formation. New data are also expected for the upcoming years. Although the Herschel mission ended in early 2013 its data have not yet been fully explored. Other major observational facilities, such as the Planck[8] satellite and the Atacama Large Millimeter Array (ALMA), will provide complementary data at radio and microwave frequencies with very large sensitivity, thereby going "deeper" than the sensitivity reached by other instruments. Also, Gurnett et al. (2013) recently presented the first in situ measurement of the interstellar plasma as Voyager 1 crossed the heliopause and started to probe the nearby interstellar plasma. This opens new possibilities in studying interstellar turbulence locally. It is clear that the future of interstellar turbulence science is going to be very exciting.

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