Nonlin. Processes Geophys., 21, 1043–1049, 2014
www.nonlin-processes-geophys.net/21/1043/2014/
doi:10.5194/npg-21-1043-2014


Nonlinear Processes in Geophysics
Open Access

# Spatial analysis of oil reservoirs using detrended fluctuation analysis of geophysical data

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Received: 17 March 2014 – Published in Nonlin. Processes Geophys. Discuss.: 30 April 2014
Revised: 13 August 2014 – Accepted: 9 September 2014 – Published: 20 October 2014

**Abstract.** We employ the detrended fluctuation analysis (DFA) technique to investigate spatial properties of an oil reservoir. This reservoir is situated at Bacia de Namorados, RJ, Brazil. The data correspond to well logs of the following geophysical quantities: sonic, gamma ray, density, porosity and electrical resistivity, measured in 56 wells. We tested the hypothesis of constructing spatial models using data from fluctuation analysis over well logs. To verify this hypothesis, we compare the matrix of distances of well logs with the differences in DFA exponents of geophysical quantities using a spatial correlation function and the Mantel test. Our data analysis suggests that the sonic profile is a good candidate for representing spatial structures. Then, we apply the clustering analysis technique to the sonic profile to identify these spatial patterns. In addition, we use the Mantel test to search for correlations between DFA exponents of geophysical quantities.

## 1 Introduction

To a great extent, information about petroleum reservoirs is obtained from well logs that measure geophysical quantities along drilled wells (see Asquith and Krygowski, 2004). As a rule, data are spatially sparse and present strong fluctuation; therefore, we have to rely on statistical methods for evaluating indices that describe the characteristics of the reservoirs (see, for instance, Hardy and Beir, 1994; Hewitt, 1998). The question of which methods are more appropriate for fulfilling this task is still open. In this work, we investigate the use of fluctuation analysis to tackle this problem.

The well log data comprise the most valuable information that can be obtained from geological volumes and from oil reservoirs. However, the cost of drilling imposes severe limitations on the number of wells. In this situation, we are faced with the problem of uncovering geophysical properties over long field extensions from data collected along a few drilled wells. To perform this task, we have to rely on data statistics that guarantee similarities between geological structures. One goal is to draw contour lines expressing the variation of proprieties in the subsurface by evaluating interpolation from the well log data. This will be justified if correlations show consistent spatial patterns. The question of this article is the following: can we use a detrended fluctuation analysis (DFA) exponent to discover spatial patterns? In other words, is a DFA exponent spatially correlated in such a way that we can employ it as a spatial parameter?

In the last decade, new techniques from the physics of complex systems were introduced in geophysics (see Lovejoy and Schertzer, 2007; Dashtian et al., 2011b). DFA is a powerful fluctuation analysis technique introduced by Peng et al. (1995) that was developed to deal with non-stationary time series. This tool is similar to the Hurst method (see,

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for instance, Mandelbrot, 1977) that is used to compare an aleatory time series with a similar Brownian series, as well as to evaluate correlation and anti-correlation in a series. The DFA technique has been used in many areas of the geophysical literature. In Padhy (2004), it is used to obtain information from seismic signals. In references Andrade et al. (2009), Chun-Feng and Liner (2005), Gholamy et al. (2008) and Tavares et al. (2005), DFA is employed to interpret and filter images of seismograms. In references Ribeiro et al. (2011), Lozada-Zumeta et al. (2012), Marinho et al. (2013) and Dashtian et al. (2011a), this technique is used, as in this manuscript, in the analysis of well logs.

When we treat complex systems that have a huge number of data, the DFA method is attractive because it allows us to summarize data in a suitable parameter. The DFA parameter summarizes fluctuation information of a time series. This parameter is related to the autocorrelation properties and the spectrum of frequency of the data. The DFA exponent in this sense is an overall measure of its complexity. This simple procedure allows a fast comparison between large samples. Furthermore, the first step in oil research is a geographical analysis of the surface. To have characteristics of the geological structure of the subsurface projected into a single measurement on the ground level is useful information. In addition, the spatial correlation between these quantities allows us to have a better understanding of the lithology, which is crucial in oil prospecting.

The case study employed in this work is an oil reservoir, and we apply the DFA technique to data logs of drilled wells. The oil reservoir is situated at Bacia de Namorados, an offshore field in Rio de Janeiro State, Brazil. The five geophysical measurements available in the well logs are sonic (DT, sonic transient time), gamma ray (GR, gamma emission), density (RHOB, bulk density), porosity (NPHI, neutron porosity) and electrical resistivity (ILD, deep induction resistivity). The manuscript can be summarized as follows. In Sect. 2 we perform three tasks: show the geological data in some detail, briefly introduce the mathematics of the DFA, and present the statistical methods we use in this work: spatial correlation, the Mantel test and the $k$ means clustering analysis technique. In Sect. 3 we show the results of the spatial correlation function and the Mantel test; we estimate that the sonic profile is the best candidate for modeling spatial patterns. In addition, we apply clustering analysis to this geophysical quantity to create a spatial model. Finally, in Sect. 4, we conclude the work and give our final remarks.

## 2 Model background

### 2.1 The geological data

The geological data used in this work are from well logs located in the oilfield of Bacia de Namorados, Rio de Janeiro State, Brazil. The wells are situated in an area of approximately 100 km$^2$ and at a distance of 150 km from the coast. The spatial arrangement of the well logs is illustrated in Fig. 4, and the matrix of distance among pairs of wells $i$ and $j$ is done by $d_{i,j}$. The number of records for each well is not constant. The sonic register was recorded in $N = 17$ well logs, gamma ray $N = 53$), density ($N = 51$), porosity ($N = 48$), and, finally, resistivity ($N = 54$). The time series of the geophysical quantities of each well log has around $N_S \approx 1000$; the exact value depends on the measurement. This data series length guarantees a good statistic for the use of the DFA method (Kantelhardt et al., 2001). An example of a segment of the time series corresponding to each of the five geophysical variables is visualized in Fig. 1.

### 2.2 The detrended fluctuation analysis – DFA

DFA is a fluctuation analysis technique (see for instance Peng et al., 1994 and Kantelhardt et al., 2001). We present a concise description of the DFA algorithm; a comprehensive introduction to the method is in Peng et al. (1995) and Ihlen (2012). Consider a time series $x_t = (x_1, x_2, \ldots, x_{N_S})$ with $N_S$ elements. To calculate the DFA algorithm, we initially integrate the series $x(t)$, producing a new variable $y(t)$:

$$y(t) = \sum_{i=1}^{t} |x_i|. \tag{1}$$

In the second step of the algorithm, we perform an equal partitioning of the time series into boxes of length $n$. A data fitting is performed inside each box by using the least squares method. The generated auxiliary curve is called the local trend $y_n(t)$ of the data. In the third step, we detrend the integrated series, $y(t)$. To execute this procedure, we subtract $y(t)$ from the local trend $y_n(t)$. The root mean square fluctuation is found with the help of the relation

$$F(n) = \sqrt{\frac{1}{N_S} \sum_{i=1}^{N_S} (y(t) - y_n(t))^2}. \tag{2}$$

The fourth step consists in estimating Eq. (2) over all boxes of size $n$. Usually $F(n)$ increases with $n$; a linear increase in $F(n)$ with $n$ on a log–log scale is a typical signature of fractal behavior. The exponent $\alpha$ of the relation

$$F(n) = n^{\alpha} \tag{3}$$

is known as the DFA exponent. The most important equation of this theoretical development is Eq. (3), which provides a relationship between the average root mean square fluctuation, $F(n)$, as a function of the box size $n$. In this work, we have computed $\alpha$ with the help of the algorithm available in Matlab. A similar algorithm is also available in the C language (Peng et al., 1995). In Fig. 2, we show, as an illustration, the curve of $F(n)$ versus $n$ for two distinct wells for gamma-ray and sonic data.

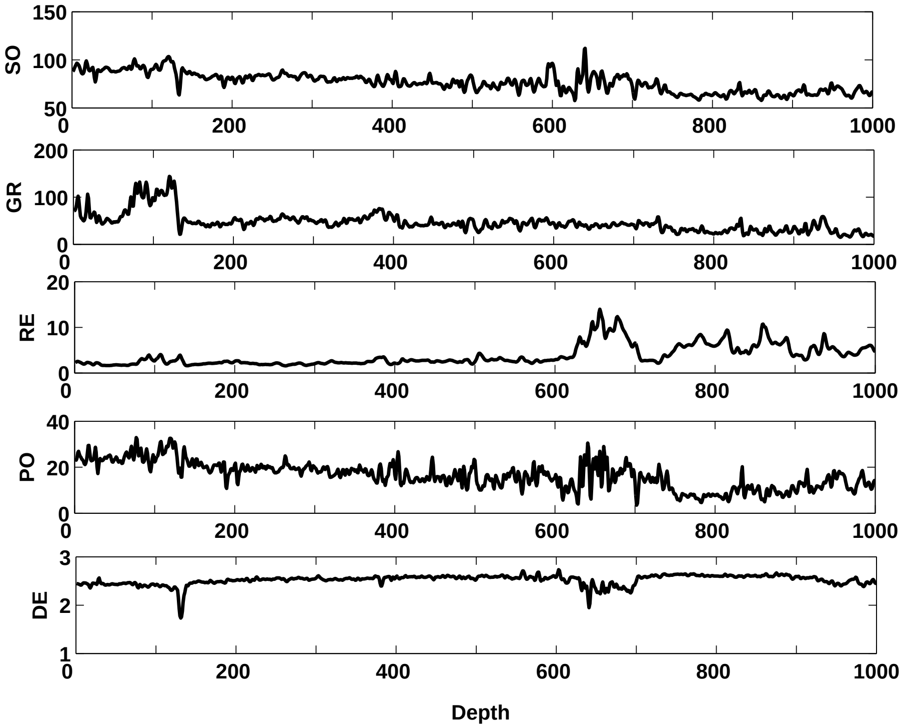


**Figure 1.** A segment of a typical measurement, for an arbitrary well, of the geophysical properties versus depth (in meters): sonic (SO), gamma ray (GR), density (DE), porosity (PO), and resistivity (RE).

We performed a similar analysis for the available well logs of all geophysical quantities. For 98 % of cases, the correlation coefficient of the adjusted line in the log–log plot fulfils the relation $R^2 \leq 0.95$, for $R$ the linear correlation coefficient (Sokal and Rohlf, 1995). The cases that do not follow this condition were discarded from the statistics.

### 2.3 Statistical analysis

In the paragraphs that follow, we show the statistical methods explored in the paper. All statistical analyses were performed using the R language (see reference R Development Core Team, 2008).

#### 2.3.1 Spatial correlation

To test the spatial correlation between variables, the most simple statistics comprise the correlation function, $\mathrm{Corr}(\tau)$, for $\tau$ the correlation length. To test the spatial correlation between DFA exponent and distance, we start ranking all $d_{i,j}$ of the distance matrix. We compute the difference of the matrix of the DFA exponent: $\Delta^t \alpha_{i,j} = |\alpha_i^t - \alpha_j^t|$ for all geophysical variables $g^t$. The quantity $\Delta\alpha^t$ is ordered according to distances $\tau$. $\mathrm{Corr}^t(\tau)$ is estimated as follows:

$$\mathrm{Corr}^t(\tau) = \frac{\sum_{l=1}^{\mathrm{Num}} \Delta\alpha^t(d)\,\Delta\alpha^t(d+\tau)}{\mathrm{Num} > \mathrm{sd}\,(\Delta\alpha^t)} \qquad (4)$$

where the sum in the equation is performed over all possible pairs Num. To compute $\mathrm{Corr}(\tau)$, the quantity $\Delta\alpha$ is transformed to $\Delta\alpha \to \Delta\alpha - \mu$ for $\mu$ the average of $\Delta\alpha$. The correlation function is evaluated over a zero mean series. The standard deviation, $\mathrm{sd}(\Delta\alpha)$, in the denominator normalizes adequately the function such that $\mathrm{Corr}(0) = 1$.

#### 2.3.2 Mantel test

The Mantel test is a statistical tool to test correlation between two symmetrical matrices of the same rank. The rationale of this test is to employ matrix elements in the same way as vectors of objects. In this way, the Mantel test is quite similar to the Pearson test that searches for a correlation between two vectors. In the Mantel test, matrices are transformed into vectors to evaluate the linear correlation (see Sokal and Rohlf, 1995).

We compute two distinct sets of tests: in the first, we check for correlation between the matrix of distances of well logs $d_{i,j}$ and the differences matrix of the DFA exponent $\Delta^t \alpha_{i,j}$ of any geophysical variable $g^t$. In a second moment, we compare the DFA of geophysical quantities by applying the Mantel test between matrices of $\Delta^t \alpha_{i,j}$ and $\Delta^s \alpha_{i,j}$ of geophysical quantities $g^t$ and $g^s$. Of course, we evaluate this test only over pairs $i$ and $j$ of well logs that have available data for both $g^t$ and $g^s$.

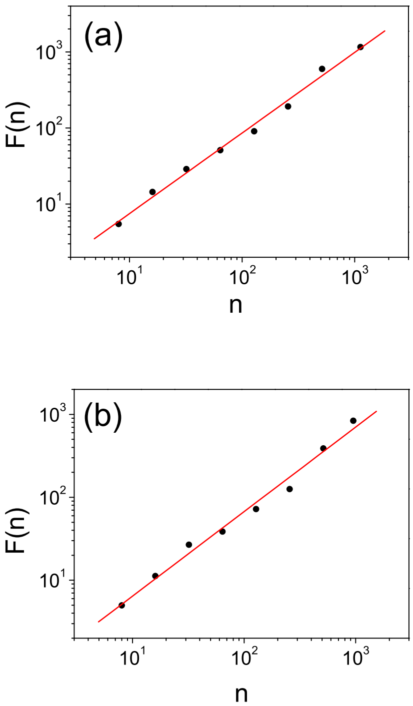


**Figure 2.** A typical plot illustrating a DFA scaling property: $F(n)$ versus $n$, the curve of Eq. (3). The good fitting of most curves on a log–log scale reveals the fractal characteristic of geophysical data. In **(a)**, well 2 of gamma ray data, and in **(b)**, well 17 of sonic data.

### 2.3.3 Clustering analysis

For the geophysical quantities that show spatial correlation, we search for spatial patterns. In this article, we use $k$ means, a standard tool of clustering analysis, to perform this task. The $k$ means methodology works by creating groups using a metric criterion. The user of the method chooses a fixed number $k$ of subsets, or clusters, and an optimization algorithm selects elements according to the distance to $k$ centroids.

In our study, we find that only one geophysical quantity presents a significant spatial correlation: the sonic variable. To use the $k$ means methodology, it is necessary to have at least three input variables. To obtain the two additional parameters, we employ the following strategy: we use the upper and lower values of the error interval of the fitting of the curve defined by Eq. (3).

We use a Monte Carlo test, or a randomization test, to check if the $k$ cluster method creates groups that are closer, in a metric sense, than groups generated in an aleatory way. We define an index $\Omega$ of neighborhood in the following way. Consider the map of the field with all wells. Over each well, we attach a geometric ball (or a disk) of radius $b$. The wells that are spatially closer, share overlapping balls, in opposition to distant wells. This schema of overlapping balls is used to measure if two wells that are in the same $k$ group are close or not. For all pairs of well logs, we perform this computation: if the balls of two well logs overlap and belong to the same group, we count $\Omega \to \Omega + 1$; otherwise, we do nothing. The index $\Omega$ is normalized by the number of groups and the maximal number of elements in each $k$ group. After that, we shuffle the well logs over the $k$ groups and compute $\Omega_{\text{shuffled}}$ over the shuffled data. The idea of this method is to see if the $k$ groups are more distant from each other than groups chosen at random. We estimate a $p$ value as the probability of $\Omega$ being larger than the $\Omega_{\text{shuffled}}$ distribution.

**Table 1.** The results of spatial correlation: the decaying of the spatial correlation and the Mantel test. The linear fitting of the correlation function is indicated in the table, as well as the output of the Mantel test. The result indicates that only sonic data are appropriate for constructing spatial analysis. The geophysical quantities are indicated in the table: sonic (SO), density (DE), gamma ray (GR), electrical resistivity (RE), and porosity (PO).

| | Spatial correlation | | | Mantel test | |
|---|---|---|---|---|---|
| | $F$ | $\rho$ | $p$ | $r$ | $p$ |
| PO | 0.002 | 0.00003 | 0.96 | −0.021 | 0.64 |
| RE | 0.11 | 0.002 | 0.74 | 0.016 | 0.51 |
| GR | 1.05 | 0.015 | 0.31 | −0.028 | 0.73 |
| SO | 9.03 | 0.12 | 0.004 | 0.181 | 0.06 |
| DE | 0.64 | 0.01 | 0.43 | 0.023 | 0.34 |

## 3 Results

To check for spatial correlation, we use three independent statistical tests: the spatial correlation, the Mantel test and the clustering analysis. To improve the visualization of our analysis, we introduce a couple of spatial pictures of the DFA exponent computed over the well logs (Fig. 3). We depict five figures, one for each geophysical variable: porosity (PO), resistivity (RE), gamma ray (GR), density (DE), and sonic (SO), as indicated in the picture. The spatial image uses arbitrary distance unities $x$ and $y$. To help the perception of the system, we depict contour plots with colors. Regions sharing the same color assume close DFA values.

### 3.1 Spatial correlation

We initially compute the function $\text{Corr}(\tau)$ for $0 \le \tau \le 80$ for all geophysical variables; we checked that 80 is a number large enough for $\text{Corr}(\tau)$ decay, and start oscillating around zero. We expect that in case $\alpha$ variables of any geophysical quantity $g^t$ show spatial correlation, the function $\text{Corr}(\tau)$ should decrease with $\tau$. To analyze the decay of $\text{Corr}(\tau)$ of the geophysical variables, we fit a linear curve and test how significant its decay is. The result of the fitting of the geophysical quantities is shown in Table 1. This result indicates that the only quantity that reveals a significant decay is the

**Table 2.** This symmetric table shows the $p$ value of the Mantel test of hypothesis for correlations between the DFA exponents of geophysical quantities. The test is performed between each pair of five geophysical variables: porosity (PO), resistivity (RE), gamma ray (GR), density (DE), and sonic (SO).

| | RE | GR | DE | SO |
|---|---|---|---|---|
| PO | $p = 0.088$ | $p = 0.74$ | $p = 0.95$ | $p = 0.21$ |
| RE | – | $p = 0.73$ | $p = 0.62$ | $p = 0.44$ |
| GR | – | – | $p = 0.62$ | $p = 0.61$ |
| DE | – | – | – | $p = 0.13$ |

sonic data; all the other quantities show $p > 0.05$ for the linear fitting test.

### 3.2 Mantel test

Table 1 also shows the results of the analysis of the Mantel test for all geophysical variables. Here it computes the correlation between two matrices: $d_{i,j}$, the matrix of distance between two wells, and $\Delta\alpha_{i,j} = \alpha_i - \alpha_j$, the matrix of difference between DFA exponent $\alpha$ for the same wells. The correlation parameter of the test is indicated by $r$, while $p$ is the $p$ value of the significance test. In agreement with the output of the correlation function analysis, the smallest $p$ value is attributed to the sonic variable. This result justifies the use of sonic data for constructing spatial patterns, the subject of the next section.

We use the Mantel test not only to analyze the correlation between distances and the DFA exponent, but also to perform a comparison between distinct geophysical quantities. That means we compare matrices $\boldsymbol{\Delta\alpha}^t$ and $\boldsymbol{\Delta\alpha}^s$ of geophysical quantities $g^t$ and $g^s$. The result of this analysis is shown in Table 2. We plot only the $p$ value of the test in the table. The major agreement observed was between variables: resistivity and porosity, which is followed by density and sonic.

### 3.3 Clustering analysis

The sonic variable has proven a good candidate for generating spatial patterns. In Fig. 4, we plot the oil reservoir area with well logs. Axes $x$ and $y$ represent the spatial coordinates; we use arbitrary metric units. The points in the figure represent the coordinates of the well logs. In Fig. 4a, we use the fixed number of clusters $k = 3$, while in Fig. 4b, we use $k = 4$. Elements in the same cluster are indicated by a common symbol. These two pictures suggest that the sonic variable is indeed a good geophysical quantity for modeling spatial formations.

To test how good the spatial formation of the clustering analysis is, we employ a Monte Carlo test. We estimated the proper $\Omega$ value and found $p = 0.005$ for $k = 3$ and $p = 0.16$ for $k = 4$ using an optimal ball size $b$. We checked the $k$ means clustering technique for the other quantities: sonic,

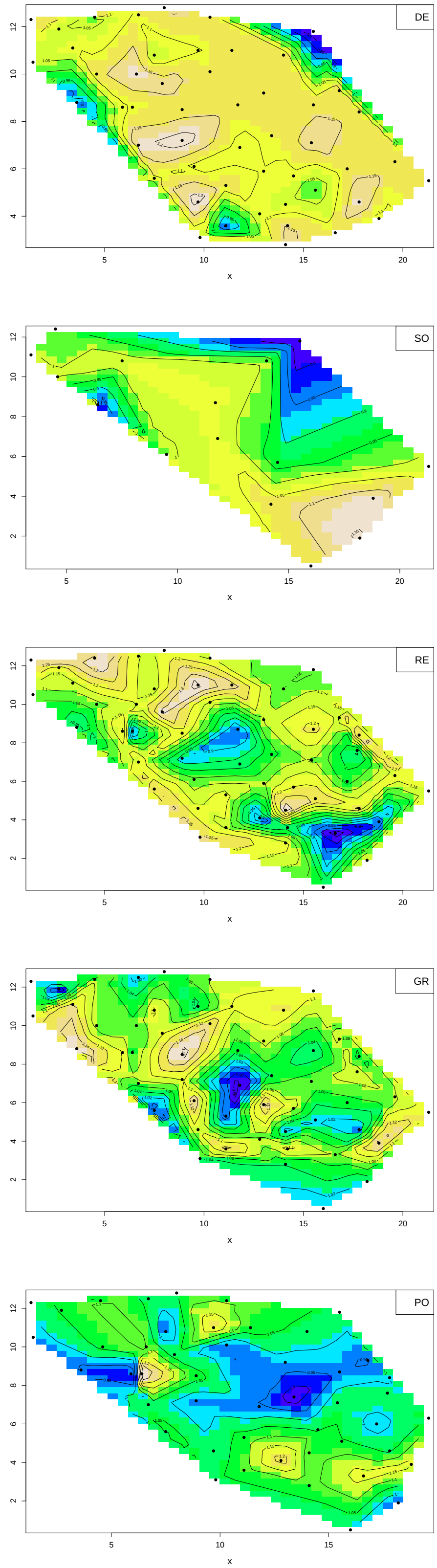


**Figure 3.** Contour plots of DFA values over spatial data of the oil reservoir of Campo dos Namorados, RJ, Brazil. We depict five figures, one for each geophysical variable: porosity (PO), resistivity (RE), gamma ray (GR), density (DE), and sonic (SO). The dots correspond to well logs. We use arbitrary length unities $x$ and $y$.

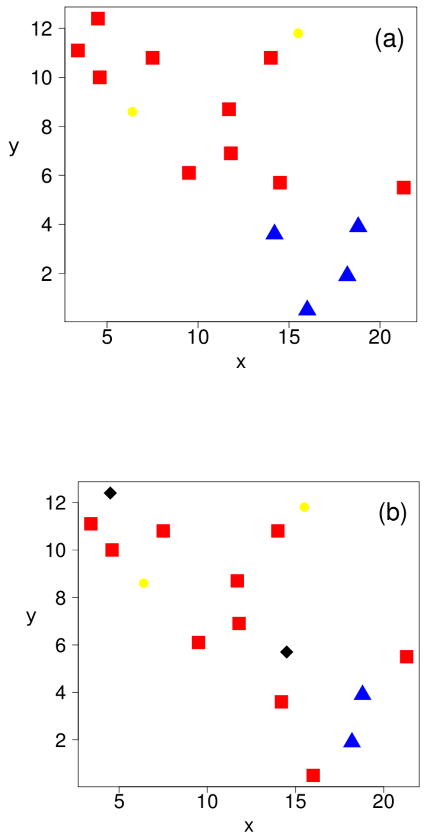


**Figure 4.** Clustering analysis patterns for sonic data: **(a)** $k = 3$ and **(b)** $k = 4$. Both figures show a satisfactory cluster formation in these data, as confirmed by a Monte Carlo test. We use arbitrary length unities $x$ and $y$.

resistivity, porosity and gamma ray. We use $3 \leq k \leq 6$ for all these geophysical data sets, and we found no $p$ greater than 0.05, which means no evidence of significant spatial cluster formation. This result is indirect evidence that only a sonic variable is a good choice for the formation of spatial patterns.

## 4 Final remarks

The issue of this manuscript is to test the hypothesis that we can use DFA exponents $\alpha$ from log wells as integrated indices projected over the Earth's surface to reveal spatial structures. Each $\alpha$ is an index that summarizes the structure of fluctuation of a geophysical quantity over geological layers of thousands of meters deep. The challenge is to use the information about the fluctuation from a set of distinct well logs distributed over several kilometers to construct spatial patterns.

The results of the Mantel test and the spatial correlation function indicate that the only geophysical parameter we can rely on in this global approach to modeling spatial patterns is sonic. We use partitioning by $k$ means, a standard technique of cluster analysis appropriate for representing spatial models. A visual inspection of the spatial patterns, as well as a Monte Carlo test, verify that sonic data form good spatial models for $k = 3$ and $k = 4$. By contrast, other geophysical quantities do not show significant results in the Monte Carlo test.

In addition to spatial analysis, we also used the Mantel test to search for correlations between geophysical quantities. In a previous work (Ribeiro et al., 2011), using the same data set but applying a different methodology, it was found that the only pair of geophysical variables that shows significant correlation was density and sonic ($p = 0.01$). In this work, the pairs of quantities that show greater significance were porosity and resistivity ($p = 0.088$), closely followed by density and sonic ($p = 0.13$). The paper Ferreira et al. (2009) also found a major correlation between sonic and density by using a standard correlation matrix. For both the methodologies, the density and sonic pair seems to be correlated. This property is probably related to the trivial fact that sound speed increases with density (see, for instance, Feynman and Leighton, 1964), a result that is close to our result. As the methodologies of these works are not identical, we do not expect the same result. Indeed, small discrepancies are acceptable in statistical treatments. This last result is in agreement with Dashtian et al. (2011a), who used cross-correlation analysis between well logs and found that sonic, porosity and density are more correlated between them than gamma ray.

To conclude the work, we go back to the initial question of the manuscript: is it possible to create spatial models using fluctuation analysis? The sonic variable has shown enough spatial correlation to perform this task, but the density, which is the quantity most correlated with the sonic variable, does not share the same property. However, a visual inspection of the couple of Fig. 3 suggests that the porosity has a consistent spatial distribution. In a future work, we intend to test the combination of distinct geophysical quantities in the formation of spatial patterns.

*Acknowledgements.* The authors thank the ANP (National Oil Agency – Brazil) for the concession of the geological data and K. C. O. Costa and F. W. F. Silva for the use of data from their Masters dissertations. The authors also gratefully acknowledge the financial support of CNPq (INCT-Nano(Bio)Simes, Procad-Casadinho), CAPES (Rede NanoBioTec and PNPD) and CTPERTRO/FINEP.

Edited by: A. G. Hunt
Reviewed by: two anonymous referees

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