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Voyager 2 observation of the multifractal spectrum in the heliosphere and the heliosheath

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Abstract. We look in detail at the multifractal scaling of the fluctuations in the interplanetary magnetic field strength as measured onboard Voyager 2 in the entire heliosphere and even in the heliosheath. More specifically, we analyse the spectra observed by Voyager 2 in a wide range of solar activity cycles during the years 1980–2009 at various heliospheric latitudes and distances from 6 to 90 astronomical units (AU). We focus on the singularity multifractal spectrum before and after crossing the termination heliospheric shock by Voyager 2 at 84 AU from the Sun. In addition, we investigate here the parameters of the model that describe the asymmetry of the spectrum, depending on the solar cycle. It is worth noting that the spectrum is prevalently right-skewed inside the whole heliosphere. Moreover, we have possibly observed a change in the asymmetry of the spectrum at the termination shock. We show that the degree of multifractality is modulated by the solar activity. Hence these basic results also bring significant support to some earlier claims suggesting that the solar wind termination shock is asymmetric.

1 Introduction

The general aim of this paper is to report on the developments in magnetic turbulence using multifractals based on a generalized two-scale weighted Cantor set as first proposed by Macek (2007). We have subsequently applied this model to intermittent multifractal turbulence in the solar wind magnetized plasma in the inner and the outer heliosphere at the ecliptic and at high heliospheric latitudes, and even in the heliosheath, beyond the termination shock. Admittedly, the solar wind is a very complex medium. Besides the current sheets (Borovsky, 2010; Li et al., 2011), this system has embedded magnetic coherent structures (Opher et al., 2011) and exhibits magnetic phase synchronisation (Chian and Miranda, 2009), which are both believed to contribute to intermittent space plasma turbulence (Sahraoui, 2008). It is also known that the solar wind is characterized by nonlinear Alfvén waves (e.g. Belcher and Davis, 1971), which are associated with directional discontinuities and magnetic decreases (Tsurutani et al., 2009, 2011a, b). Because of the complexity of the solar wind structure, a phenomenological approach can still provide important information about solar wind turbulence (Frisch, 1995). We therefore hope that our multifractal model will be a useful tool for analysis of intermittent turbulence in the heliospheric plasma. We also believe that multifractal analysis of various complex environments can shed light on the nature of turbulence (Bruno and Carbone, 2013).

We remind that a fractal is a rough or fragmented geometrical object that can be subdivided into parts, each of which is (at least approximately) a reduced-size copy of the whole. Strange attractors are often fractal sets that exhibit a hidden order within (irregular but deterministic) chaotic behaviour. We know that fractals are generally *self-similar* and independent of scale (with a particular fractal dimension). A generalization of this geometrical concept is a multifractal set. In fact, this object demonstrates various self-similarities, described by a multifractal spectrum of dimensions. One can say that self-similarity of multifractals is scale dependent, resulting in the singularity spectrum. A multifractal is therefore in a certain sense like a set of intertwined fractals.

Starting from seminal works of Kolmogorov (1941) and Kraichnan (1965), many authors have attempted to recover the observed scaling laws, and subsequently various multifractal models of turbulence have been developed (Meneveau and Sreenivasan, 1987; Carbone, 1993; Frisch, 1995). In particular, multifractal scaling of the energy flux at various scales in solar wind turbulence using Helios (plasma) data in the inner heliosphere has been analysed by Marsch et al. (1996). It is known that fluctuations in the solar magnetic fields may also exhibit multifractal scaling laws. The multifractal spectrum has been investigated using magnetic field data measured in situ by Advanced Composition Explorer (ACE) in the inner heliosphere (Macek and Wawrzaszek, 2011), by Voyager in the outer heliosphere up to large distances from the Sun (Burlaga, 1991, 1995, 2001, 2004; Macek and Wawrzaszek, 2009), and even in the heliosheath (Burlaga and Ness, 2010; Burlaga et al., 2005, 2006; Macek et al., 2011, 2012).

To quantify scaling of solar wind turbulence, we have developed a generalized two-scale weighted Cantor set model using the partition technique (Macek, 2007; Macek and Szczepaniak, 2008), which leads to complementary information about the multifractal nature of the fluctuations as the rank-ordered multifractal analysis (cf. Lamy et al., 2010). We have investigated the spectrum of generalized dimensions and the corresponding multifractal singularity spectrum depending on one probability measure parameter and two rescaling parameters. In this way we have looked at the inhomogeneous rate of the transfer of the energy flux indicating multifractal and intermittent behaviour of solar wind turbulence. In particular, we have studied in detail fluctuations in the velocity of the flow of the solar wind, as measured in the inner heliosphere by Helios (Macek and Szczepaniak, 2008), and by ACE in the inner heliosphere (Szczepaniak and Macek, 2008) and Voyager in the outer heliosphere (Macek and Wawrzaszek, 2009, 2011; Macek et al., 2011, 2012), including Ulysses observations at high heliospheric latitudes (Wawrzaszek and Macek, 2010).

It is well known that Voyager 1 crossed the termination heliospheric shock, which separates the Solar System plasma from the surrounding heliosheath, with the subsonic solar wind on 16 December 2004 at heliocentric distances of 94 AU (at present its distance to the Sun is about 126 AU after crossing the heliopause; Strumik et al., 2013; Gurnett et al., 2013). Please note that (using the pressure balance) the distance to the nose of the heliopause has been estimated to be ∼120 AU (Macek, 1998). Later, in 2007, Voyager 2 also crossed the termination shock at least five times at distances of 84 AU (now at 103 AU). The data have revealed a complex, rippled, quasi-perpendicular supercritical magnetohydrodynamic shock of moderate strength, with a reformation of the shock on a scale of a few hours, suggesting the importance of ionized interstellar atoms (so-called "pickup" protons) at the shock structure (Burlaga et al., 2008).

Admittedly, we have already analysed the interplanetary magnetic field strength as measured on the Voyager 2 spacecraft located below the solar equatorial plane (Macek et al., 2012), and compared our results with those for Voyager 1 located above the equatorial plane (Macek et al., 2011). The objective of this study is to test again the multifractal scaling for the wealth of data provided by Voyager 2, exploring in detail various regions of the heliosphere, especially before and after crossing the termination shock. Therefore, we analyse time series of the magnetic field fluctuations measured by Voyager 2 at a wide range of distances, heliolatitudes, and phases of solar cycles. This will allow us to investigate the parameters of the model that describe multifractality and asymmetry of the spectrum, depending on solar activity.

This paper is organized as follows. In Sect. 2 a generalized two-scale Cantor set model is summarised, and the Voyager 2 data are introduced in Sect. 3. The methods related to the concept of the multifractal singularity spectrum in the context of turbulence scaling are reviewed in Sect. 4. The results of our analysis are presented and discussed in Sect. 5. The importance of our more general asymmetric multifractal model is underlined in Sect. 6.

2 Two-scale weighted Cantor set model

Let us consider the generalized weighted Cantor set, as discussed by Macek (2007). Here this set with weight p and two scales is schematically shown in Fig. 1, taken from the paper by Macek (2007). This simple example of a multifractal is explained in several textbooks (e.g. Falconer, 1990; Ott, 1993) and provides a useful mathematical language for the complexity of turbulent dynamics, as discussed by Macek and Wawrzaszek (2009). At each stage of construction of this generalized Cantor set we thus have two rescaling parameters, l_1 and l_2, where $l_1 + l_2 \leq L = 1$ (normalized) and with two different probability measures, $p_1 = p$ and $p_2 = 1 - p$.

To obtain the generalized dimensions $D_q \equiv \tau(q)/(q-1)$ for this multifractal set, we use the following partition function (a generator) at the n-th level of construction (Hentschel and Procaccia, 1983; Halsey et al., 1986):

$$\Gamma_n^q(l_1, l_2, p) = \left(\frac{p^q}{l_1^{\tau(q)}} + \frac{(1-p)^q}{l_2^{\tau(q)}} \right)^n = 1. \tag{1}$$

We see that $\tau(q)$ does not depend on n, and after n iterations we have $\binom{n}{k}$ intervals of width $l = l_1^k l_2^{n-k}$, where $k = 1, \ldots, n$, visited with various probabilities. The resulting set of 2^n closed intervals (increasingly narrow segments of various widths and probabilities) for $n \to \infty$ becomes the weighted two-scale Cantor set. For any q one obtains $D_q = \tau(q)/(q-1)$ by solving numerically the following transcendental equation (e.g. Ott, 1993):

$$\frac{p^q}{l_1^{\tau(q)}} + \frac{(1-p)^q}{l_2^{\tau(q)}} = 1. \tag{2}$$

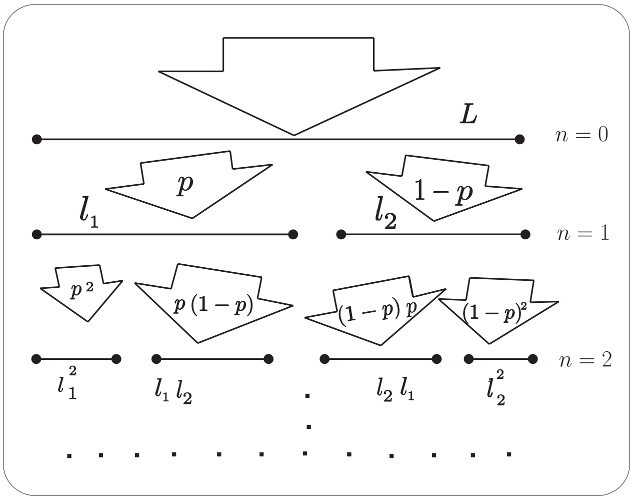

Fig. 1. Two-scale weighted Cantor set model for asymmetric solar wind turbulence, taken from Macek (2007).

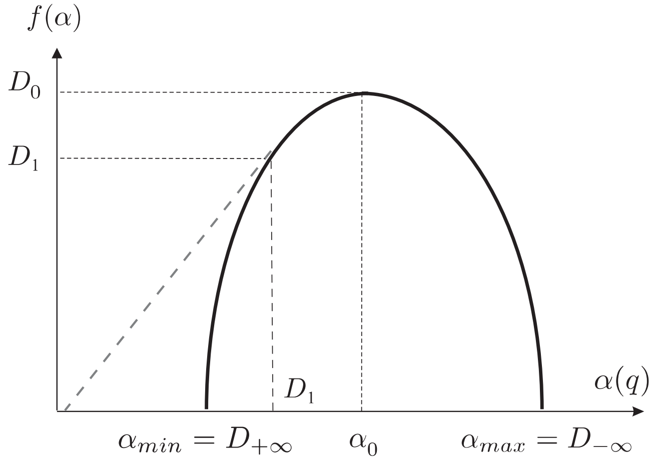

Fig. 2. The singularity multifractal spectrum $f(\alpha)$ versus the singularity strength α with some general properties: (1) the maximum value of $f(\alpha)$ is D_0; (2) $f(D_1) = D_1$; and (3) the line joining the origin to the point on the $f(\alpha)$ curve, where $\alpha = D_1$, is tangent to the curve, as taken from Ott (1993).

3 Solar wind data

The heliospheric distances from the Sun and the heliographic latitudes during each year of the Voyager mission are given in Fig. 1 of the paper by Macek et al. (2012), cf. e.g. Richardson et al. (2004). Voyager 1 and 2 crossed the termination heliospheric shock in 2004 and 2007 at heliocentric distances of 94 and 84 AU respectively. However, Voyager 2 is of special interest in this paper. Therefore, we like to test again the multifractal scaling for the wealth of data provided by this spacecraft during various phases of the solar cycle, exploring thus in detail various regions of the entire heliosphere. Namely, we analyse time series of fluctuations in the daily averages of the magnetic field strength measured by Voyager 2 at a wide range of heliospheric distances (and heliolatitudes), especially before the termination shock crossing (during the years 1980–2006), namely between ∼6 and ∼80 AU from the Sun, and subsequently (2008–2009) at 85–90 AU, i.e. in the heliosheath.

4 Methods of data analysis

The generalized dimensions D_q as a function of index q (Grassberger, 1983; Grassberger and Procaccia, 1983; Hentschel and Procaccia, 1983; Halsey et al., 1986) quantify multifractality of a given system (Ott, 1993). Alternatively, we can describe intermittent turbulence by using the singularity spectrum $f(\alpha)$ as a function of a singularity strength α, which quantifies multifractality of a given system. This function sketched in Fig. 2 describes singularities occurring in considered probability measure, allowing a clearer theoretical interpretation by comparing the experimental results with those obtained from phenomenological models of turbulence (Halsey et al., 1986; Ott, 1993; Wawrzaszek and Macek, 2010). In this way in the case of turbulence these generalized measures are related to inhomogeneity, with which the energy (or magnetic) flux is distributed between different eddies (Meneveau and Sreenivasan, 1991). In the case of magnetic turbulence, high positive values of q (left part of the spectrum in Fig. 2) emphasize regions of intense magnetic fluctuations larger than the average, while negative values of q (right part) accentuate fluctuations lower than the average (Burlaga, 1995).

Following Burlaga (1995), let us take a stationary magnetic field $B(t)$ in the heliosphere. We can decompose this signal into time intervals of size Δt corresponding to the spatial scales $l = v_{\rm sw}\Delta t$. Then with each time interval one can associate a magnetic flux past the cross-section perpendicular to the plane during that time. In every considered year we use a discrete time series of daily averages, which is normalized so that we have $\langle B(t)\rangle = \frac{1}{N}\sum_{i=1}^{N} B(t_i) = 1$, where $i = 1, \ldots, N = 2^n$ (taking $n = 8$). Next, given this (normalized) time series $B(t_i)$, we associate some probability measure

$$p(x_j, l) \equiv \frac{1}{N}\sum_{i=1+(j-1)\Delta t}^{j\Delta t} B(t_i) = p_j(l) \tag{3}$$

with each interval of temporal scale Δt (using $\Delta t = 2^k$, with $k = 0, 1, \ldots, n$), where $j = 2^{n-k}$, i.e. calculated by using the successive average values $\langle B(t_i, \Delta t)\rangle$ of $B(t_i)$ between t_i and $t_i + \Delta t$ (Burlaga et al., 2006).

In the inertial range of the turbulence spectrum, the q-order total probability measure, the partition function (using probability defined in Eq. 3), should scale as

$$\sum p_j^q(l) \sim l^{\tau(q)}, \tag{4}$$

with $\tau(q)$ given in Eq. (6). In this case Burlaga (1995) has shown that the average value of the q-th moment of the magnetic field strength B at various scales $l = v_{\rm sw}\Delta t$ scales is

$$\langle B^q(l)\rangle \sim l^{\gamma(q)}, \tag{5}$$

with the similar exponent $\gamma(q) = (q-1)(D_q - 1)$. Using this measure we can construct both functions D_q and $f(\alpha)$, which are usually derived in the following way (see e.g. Macek and Wawrzaszek, 2009).

Namely, for a continuous index $-\infty < q < \infty$ using a q-order total probability measure, $I(q,l) \equiv \sum_{i=1}^N p_i^q(l)$ with p_i as given in Eq. (3) and a q-order generalized information entropy $H(q,l) \equiv -\log I(q,l) = -\log \sum_{i=1}^N p_i^q(l)$ defined by Grassberger and Procaccia (1983), one obtains the usual q-order generalized dimensions (Hentschel and Procaccia, 1983) $D_q \equiv \tau(q) \,/\, (q-1)$, where

$$\tau(q) = \lim_{l\to 0} \frac{[-\log \sum_{i=1}^N p_i^q(l)]}{\log(1/l)}. \tag{6}$$

Following Chhabra and Jensen (1989), we also define a one-parameter q family of generalized pseudoprobability measures (normalized):

$$\mu_i(q,l) \equiv \frac{p_i^q(l)}{\sum_{i=1}^N p_i^q(l)}. \tag{7}$$

For a given q with the associated index of the fractal dimension $f_i(q,l) \equiv \log \mu_i(q,l)/\log l$, the multifractal singularity spectrum of dimensions is defined directly as the averages denoted by $\langle \ldots \rangle$ taken with respect to the measure $\mu(q,l)$ in Eq. (7),

$$f(q) \;\equiv\; \lim_{l\to 0} \sum_{i=1}^N \mu_i(q,l)\; f_i(q,l) = \langle f(q)\rangle, \tag{8}$$

and the corresponding average value of the singularity strength is obtained by Chhabra and Jensen (1989):

$$\alpha(q) \;\equiv\; \lim_{l\to 0} \sum_{i=1}^N \mu_i(q,l)\; \alpha_i(l) = \langle \alpha(q)\rangle. \tag{9}$$

Hence by using a q-order mixed Shannon information entropy $S(q,l) = -\sum_{i=1}^N \mu_i(q,l) \log p_i(l)$ we obtain the singularity strength as a function of q:

$$\alpha(q) = \lim_{l\to 0} \frac{[-\sum_{i=1}^N \mu_i(q,l)\log p_i(l)]}{\log(1/l)} = \lim_{l\to 0} \frac{\langle \log p_i(l)\rangle}{\log(l)}. \tag{10}$$

Similarly, by using the q-order generalized Shannon entropy $K(q,l) = -\sum_{i=1}^N \mu_i(q,l)\log \mu_i(q,l)$ we obtain directly the singularity spectrum as a function of q:

$$f(q) = \lim_{l\to 0} \frac{[-\sum_{i=1}^N \mu_i(q,l)\log \mu_i(q,l)]}{\log(1/l)} = \lim_{l\to 0} \frac{\langle \log \mu_i(q,l)\rangle}{\log(l)}. \tag{11}$$

The difference of the maximum and minimum dimensions, associated with the least dense and most dense regions in the considered probability measure, is given by

$$\Delta \equiv \alpha_{\rm max} - \alpha_{\rm min} = D_{-\infty} - D_{\infty} = \left| \frac{\log(1-p)}{\log l_2} - \frac{\log(p)}{\log l_1} \right|. \tag{12}$$

In the limit $p \to 0$ this difference rises to infinity. Hence, it can be regarded as a degree of multifractality (see e.g. Macek, 2006, 2007). The degree of multifractality Δ is naturally related to the deviation from a strict self-similarity. Thus Δ is also a measure of intermittency, which is in contrast to self-similarity (Frisch, 1995, chapter 8). In the case of the symmetric spectrum using Eq. (12) this degree of multifractality becomes

$$\Delta = D_{-\infty} - D_{+\infty} = \ln(1/p - 1)/\ln(1/\lambda). \tag{13}$$

In particular, the usual middle one-third Cantor set without any multifractality, i.e. with $\Delta = 0$, is recovered with $p = 1/2$ and $\lambda = 1/3$.

Moreover, using the value of the strength of singularity α_0 at which the singularity spectrum has its maximum $f(\alpha_0) = 1$, we define a measure of asymmetry by

$$A \equiv \frac{\alpha_0 - \alpha_{\rm min}}{\alpha_{\rm max} - \alpha_0}. \tag{14}$$

Please note that the value $A = 1$ ($l_1 = l_2 = 0.5$) corresponds to the one-scale symmetric case, e.g. for the so-called p model.

5 Results and discussion

As usual we have analysed the slopes $\gamma(q)$ of $\log_{10}\langle B^q\rangle$ versus $\log_{10} l$ and identified the range of scales from 2 to 16 days, over which the multifractal spectra are applicable. This has allowed us to obtain the values of D_q as a function of q according to Eq. (6). Equivalently, as discussed in Sect. 4, the multifractal spectrum $f(\alpha)$ as given by Eq. (11) as a function of scaling indices α, Eq. (10), exhibits universal properties of multifractal scaling behaviour.

In this paper the results for the singularity multifractal spectrum $f(\alpha)$ obtained using the Voyager 2 data of the solar wind magnetic fields are presented in a wide range of distances in the whole heliosphere. The calculated spectra in the relatively near heliosphere at 6–40 AU (1980–1992), i.e. within the planetary orbits, and in the distant heliosphere beyond the planets at 40–60 AU (1994–1999), are shown in Figs. 3 and 4 respectively.

Voyager 2 crossed the heliospheric termination shock at 84 AU in 2007. Therefore, in Fig. 5 we show the results in the very deep heliosphere before shock crossing, at the following heliocentric distances: (a) 66–68 AU, (b) 69–71 AU, (c) 75–77 AU, (d) 78–81 AU, and after that shock crossing, namely at (e) 85–88 AU and (f) 88–90 AU respectively. Here

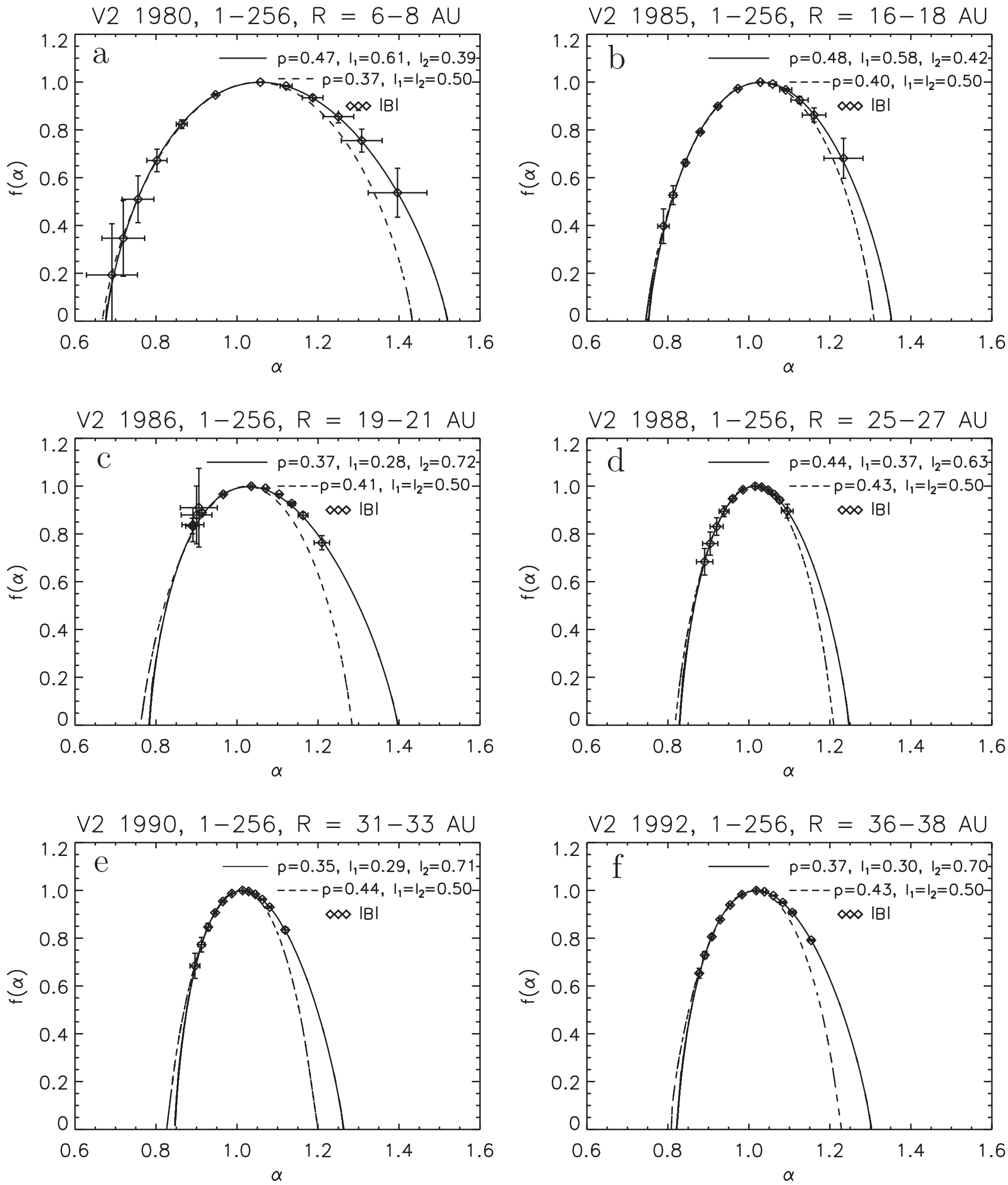

Fig. 3. The singularity spectrum $f(\alpha)$ as a function of a singularity strength α. The values are calculated for the weighted two-scale (continuous lines) model and the usual one-scale (dashed lines) p model with the parameters fitted using the magnetic fields (diamonds) measured by Voyager 2 in the relatively near heliosphere (within planetary orbits) at **(a)** 6–8 AU, **(b)** 16–18 AU, **(c)** 19–21 AU, **(d)** 25–27 AU, **(e)** 31–33 AU, and **(f)** 36–38 AU, respectively.

it is rather difficult to argue that there is an asymmetry in the calculated spectra: we see that there is some difference in the symmetric and asymmetric spectrum for $q < 0$ (right part of the spectrum) for the years 2003 and 2009, Fig. 5b and f, where only one point determines asymmetry. Admittedly, during the period of 2003–2009 shown in Fig. 5, spurious large amplitude fluctuations in the magnetic field are present and the average magnetic field strength is very weak close to the measurement uncertainties.

Next, we look for the parameters of our model: weight p and scale l_1 sketched in Fig. 1 depending on solar activity ($p \leq 0.5$, $l_1 + l_2 \simeq 1$). The results obtained during four time intervals, namely solar minimum (MIN), solar maximum (MAX), and declining (DEC) and rising (RIS) phases of solar cycles are shown in Fig. 6a for p and Fig. 6b for l_1 respectively. The crossing of the termination shock (TS) by Voyager 2 in 2007 (at 84 AU) is marked by a vertical dashed line. Certainly, these parameters p and l_1 are related to the

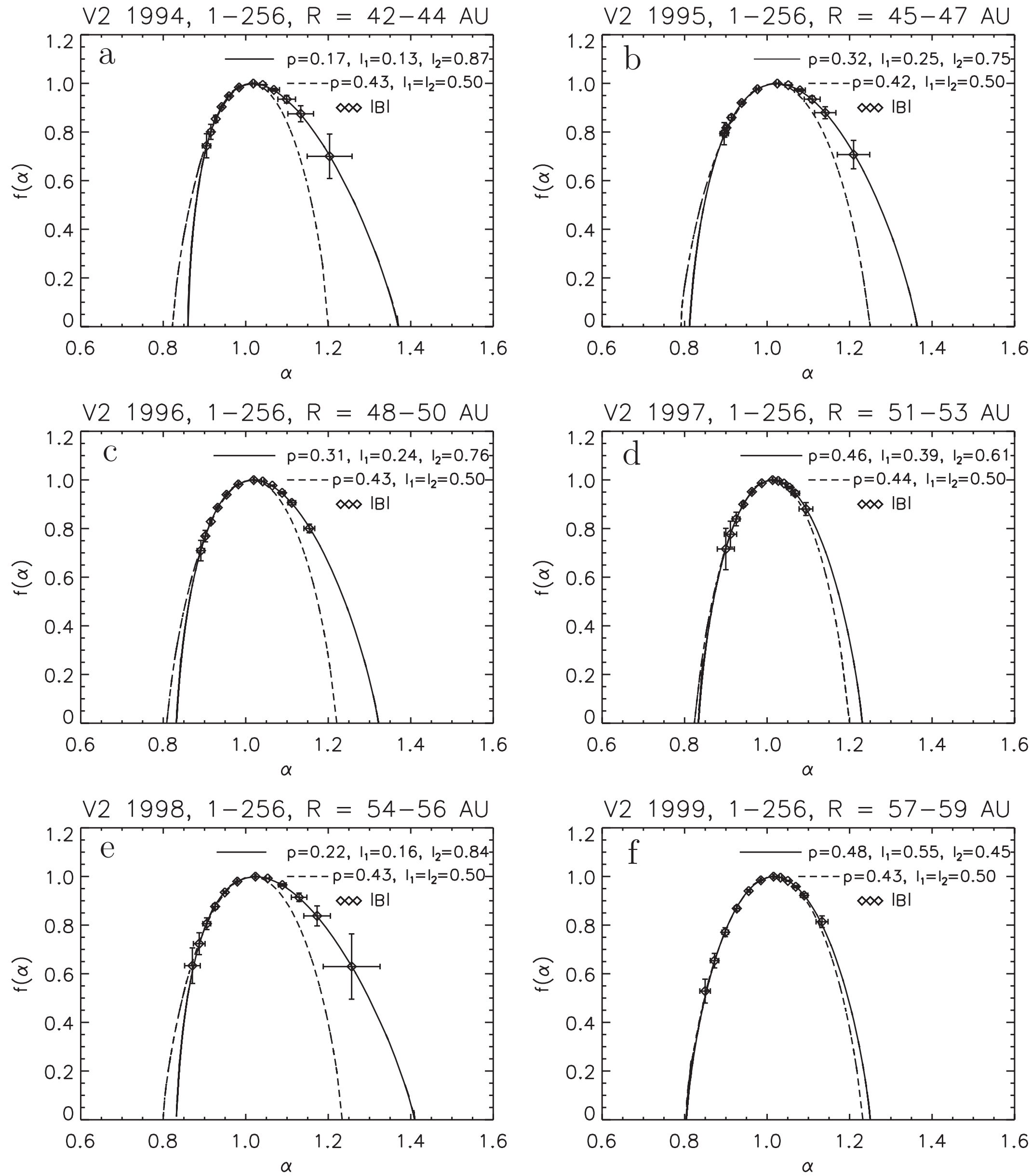

Fig. 4. The singularity spectrum $f(\alpha)$ as a function of a singularity strength α. The values are calculated for the weighted two-scale (continuous lines) model and the usual one-scale (dashed lines) p model with the parameters fitted using the magnetic fields (diamonds) measured by Voyager 2 in the distant heliosphere (beyond the planetary orbits) at **(a)** 42–44 AU, **(b)** 45–47 AU, **(c)** 48–50 AU, **(d)** 51–53 AU, **(e)** 54–56 AU, and **(f)** 57–59 AU, respectively.

degree of multifractality Δ and asymmetry A in the heliosphere according to Eqs. (12) and (14). Please note that the value $l_1 = 0.5$ (dotted) corresponds to the one-scale symmetric model. In this case for $p = 0.5$ there is no multifractality; $\Delta = 0$ in Eq. (13).

In fact, as noted by Macek et al. (2012), the value of the degree of asymmetry A decreases (the spectrum becomes more asymmetric) with increasing distance between 5 AU and 50 AU (during the period 1980–1996). This is very possible, since merged interaction regions, which could be ultimately responsible for the asymmetry, form and develop in this region (Burlaga et al., 2003). However, as also seen in Fig. 6b, the degree of asymmetry is roughly constant ($l_1 \sim 0.5$ ($A \sim 1$)), suggesting a symmetric spectrum between the year 1996 (at 50 AU) and the termination shock crossing in 2007 (at 84 AU), cf. Macek et al. (2012), Fig. 8. This is plausible, because the merged interaction regions damp out here, as discussed by Burlaga et al. (2007).

We see that similarly as for Voyager 1 in the heliosphere, there could only be a few points above $l_1 = 0.5$ at large

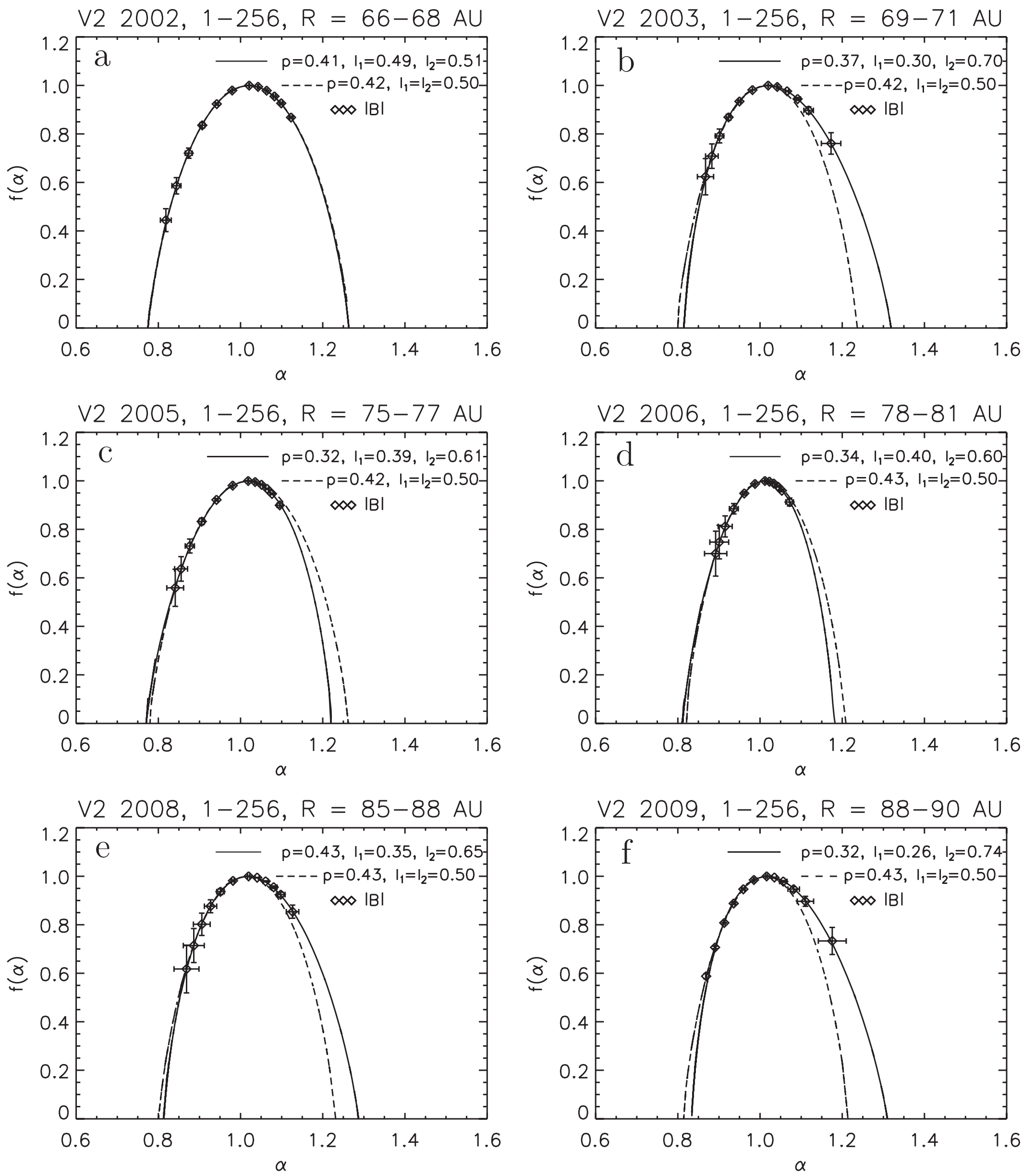

Fig. 5. The singularity spectrum $f(\alpha)$ as a function of a singularity strength α. The values are calculated for the weighted two-scale (continuous lines) model and the usual one-scale (dashed lines) p model with the parameters fitted using the magnetic fields (diamonds) measured by Voyager 2 in the very deep heliosphere (before) at **(a)** 66–68 AU, **(b)** 69–71 AU, **(c)** 75–77 AU, **(d)** 78–81 AU, and after crossing the termination shock at **(e)** 85–88 AU and **(f)** 88–90 AU respectively.

heliospheric distances (cf. Macek et al., 2011, Fig. 3). Really, inside the distant heliosphere the spectrum is prevalently right-skewed ($l_1 < 0.5$, $A < 1$) and only twice (during the declining phase) was the left-skewed spectrum ($l_1 > 0.5$, $A > 1$) possibly observed, even though it is still consistent with a symmetric spectrum. Whatever the case, we see that the right-skewed spectrum ($A < 1$) before the crossing of the termination shock is preferred for both Voyager 1 and 2 data (Macek et al., 2012).

Table 1 summarizes the values of Δ and A calculated for Voyager 2 data in the relatively near heliosphere (within the planetary system up to 40 AU, see Fig. 3), the distant heliosphere (i.e. beyond the planets, 40–60 AU), Fig. 4, and the very deep heliosphere (65–80 AU), Fig. 5a–d, together with those in the heliosheath (85–90 AU), Fig. 5e and f.

Again the multifractal scaling is still asymmetric before shock crossing, with the calculated degree of asymmetry at distances 78–81 AU equal to $A = 1.14 \pm 0.23$. We see that this value changes to $A = 0.83 \pm 0.17$ when crossing the

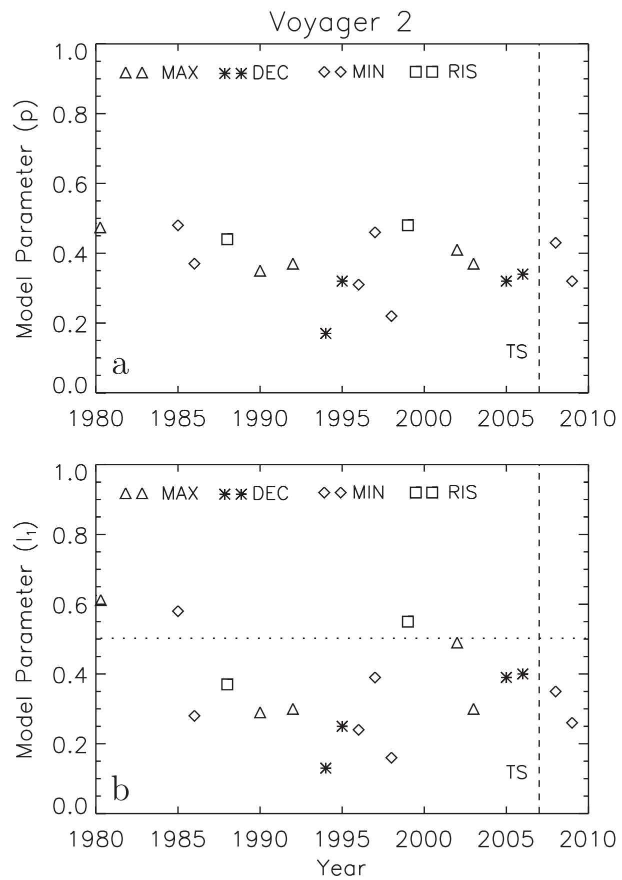

Fig. 6. Evolution of both parameters **(a)** weight p and **(b)** scale l_1 of the generalized Cantor set model ($p \leq 0.5$, $l_1 + l_2 \simeq 1$) obtained during solar minimum (MIN) and solar maximum (MAX), and declining (DEC) and rising (RIS) phases of solar cycles (at various heliospheric distances and latitudes). The value $l_1 = 0.5$ corresponds to the one-scale symmetric model (dotted). The crossing of the termination shock by Voyager 2, TS, is marked by a vertical dashed line.

Table 1. Degree of multifractality Δ and asymmetry A observed by Voyager 2 in the heliosphere and the heliosheath.

Heliocentric Distance	Year	Multifractality Δ	Asymmetry A
6–8 AU	1980	0.85 ± 0.12	0.90 ± 0.25
16–18 AU	1985	0.60 ± 0.05	0.82 ± 0.12
19–21 AU	1986	0.62 ± 0.06	0.70 ± 0.16
25–27 AU	1988	0.43 ± 0.03	0.77 ± 0.09
31–33 AU	1990	0.41 ± 0.02	0.61 ± 0.06
36–38 AU	1992	0.48 ± 0.01	0.75 ± 0.03
42–44 AU	1994	0.50 ± 0.06	0.44 ± 0.09
45–47 AU	1995	0.55 ± 0.04	0.67 ± 0.09
48–50 AU	1996	0.49 ± 0.02	0.69 ± 0.04
51–53 AU	1997	0.40 ± 0.04	0.91 ± 0.18
54–56 AU	1998	0.58 ± 0.08	0.53 ± 0.12
57–59 AU	1999	0.45 ± 0.02	0.96 ± 0.08
66–68 AU	2002	0.49 ± 0.02	1.02 ± 0.06
69–71 AU	2003	0.51 ± 0.03	0.74 ± 0.08
75–77 AU	2005	0.46 ± 0.04	1.09 ± 0.18
78–81 AU	2006	0.38 ± 0.04	1.14 ± 0.23
85–88 AU	2008	0.48 ± 0.05	0.83 ± 0.17
88–90 AU	2009	0.48 ± 0.05	0.66 ± 0.11

termination shock at 85–88 AU, but because of large error bars and a rather limited sample, a symmetric spectrum is still possible beyond the termination shock (cf. Burlaga and Ness, 2010). Nevertheless, it would be worth noting a possible change in the symmetry of the spectrum at the shock relative to its maximum at a critical singularity strength $\alpha = 1$, where $A = 1$. Since the density of the measure $\varepsilon \propto l^{\alpha-1}$, this is in fact related to changing properties of the magnetic field density ε at the termination heliospheric shock.

Naturally, the multifractal scaling of the fluctuations in the interplanetary magnetic field strength should be somehow related to the physical properties of our environment in space. Even though it is far beyond the scope of this paper, we can speculate that the multifractal turbulence in the heliosphere and heliosheath could be associated with magnetic coherent structures or magnetic bubbles (Opher et al., 2011), reflecting the structure of solar corona (Telloni et al., 2009) as suggested by Sahraoui (2008), or possibly with magnetic phase coherence (Chian and Miranda, 2009). In fact, the increase in phase synchronisation follows the increase in the measure of intermittency (multifractality) (Chian and Miranda, 2009). Alternatively, the multifractal spectrum can be related to nonlinear Alfvén waves, or magnetic decreases associated with mirror mode structures (generated by plasma instabilities) identified in the Voyager 1 heliosheath data (Tsurutani et al., 2011a, b).

6 Conclusions

We have studied the fluctuations in the interplanetary magnetic field strength indicating multifractal and intermittent behaviour of solar wind magnetic turbulence in the entire heliosphere and even the heliosheath. We have demonstrated that, for the general weighted two-scale Cantor set model with two different scaling parameters, a better agreement of the multifractal spectrum with the real data is obtained, especially for $q < 0$.

We show that for Voyager data the degree of multifractality for magnetic field fluctuations in the solar wind is modulated by the solar activity. That would certainly require a specific mechanism responsible for such a correlation, resulting in the normal or lognormal distribution of the observed magnetic fields (Burlaga et al., 2005; Chen et al., 2008; Burlaga et al., 2009). This in turn can hopefully allow us to infer some new important information about the nature of the heliospheric shock and also about the evolution of the whole heliosphere.

In particular, we have again observed a possible change in the asymmetry of the spectrum when crossing the termination shock by Voyager 2 in 2007. Consequently, a concentration of magnetic fields stretches, resulting in fatter flux tubes or weaker current concentration in the heliosheath. Admittedly we can still have an approximately symmetric spectrum in the heliosheath, where the plasma is expected to be roughly in equilibrium in the transition to the interstellar medium. We also confirm that before the shock crossing turbulence is more multifractal than that in the heliosheath (Macek et al., 2011, 2012).

We believe that the solar wind could be considered a turbulence laboratory (Bruno and Carbone, 2013). We therefore propose our somewhat more general asymmetric multifractal model as a useful tool for analysis of intermittent turbulence in space environments. We hope that this model could also shed light on the nature of turbulence.

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