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Nonlinear Processes in Geophysics

Comment on "Chaos and magnetospheric dynamics" by G. P. Pavlos, D. Diamandidis, A. Adamopoulos, A. G. Rigas, I. A. Daglis and E. T. Sarris

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In a recent paper, Pavlos et al. (1994) have analyzed the AE index using a combination of singular value decomposition (Broomhead and King, 1987) and correlation dimension (Grassberger and Procaccia, 1983). Based on these results, they claim that the magnetosphere exhibits deterministic chaos. In this comment, it is demonstrated that there is no evidence to support this view; using the same data as was used by Pavlos et al., no evidence whatsoever is found that the magnetosphere can be described by a strange attractor, in agreement with the results of a number of other papers (Takalo et al., 1993; Prichard and Price, 1992, 1993).

The nonlinear time series methods used in the paper by Pavlos et al. have previously been applied to the AE index by Sharma et al. (1993), who also found low estimated correlation dimensions. However, Prichard and Price (1993) have shown that the dimensions estimated by Sharma et al. cannot be distinguished from those for stochastic signals with have the same power spectrum and amplitude distribution as the original data. In this comment, it is shown that the same thing appears to be true of the data used by Pavlos et al.

The method used by Pavlos et al. is as follows: first, the data is embedded using the standard time delay embedding procedure (Packard et al., 1980) $\vec{x}(t) = (x(t), x(t+\tau), \ldots, x(t+(m-1)\tau))$. They then perform a singular value decomposition on the vectors $\vec{x}(t)$, project onto the singular vectors corresponding to the d largest singular values, and then estimate the correlation dimension in this d-dimensional space. Unfortunately, Pavlos et al. never state what value of d was used. Since it is not known what parameters were used by the authors, $m = 30$, $\tau = 20$, $d = 6, 7, 8, 9$, and an autocorrelation parameter (Theiler, 1986) $W = 500$ will be used in this comment. Using the first 131072 points of the AE index from 1978, the correlation integral is calculated using the above parameters. The correlation integral is also computed for 39 surrogate data sets (Theiler et al., 1992; Smith, 1992) which have the same power spectrum, and amplitude distribution as the original data, but are otherwise stochastic. The results of these calculations are shown in figure 1a. The solid line is the dimension estimated by the method of Takens (1985), which can be expressed in terms of the correlation integral (Grassberger and Procaccia, 1983):

$$D_{\text{Takens}} = \frac{C(r_o)}{\int_0^{r_o} (C(r)/r)dr} \tag{1}$$

where the upper cutoff r_o is taken to be half the standard deviation of the time series. The dotted lines are the result of the same calculation for the surrogate data sets. It is clear that the original data cannot be distinguished from the surrogate data sets using the Takens estimator, which calls into question the results of low dimensionality claimed by Pavlos et al. It should be pointed out that Pavlos et al. did compare the dimensions they calculated to those for surrogate data sets made by the scrambling the phases of the Fourier transform. However, such surrogate data sets will have a roughly gaussian distribution. For data such as the AE index, it is important to make surrogates which also preserve the non-gaussian distribution. That is, one should test the null hypothesis that the data is the result of a static transform of a linear stochastic process, since it is obvious by inspection that the data is not the direct result of a gaussian linear stochastic process.

Even though no evidence for low dimensionality was found, there does appear to be some evidence for nonlinearity in the data. In figure 1b the correlation integral for $\tau = 20$ and $m = 7$ (no SVD embedding was used) is shown, for both the original data (solid) and for 39 surrogate data sets (dashed). It is clear that the original data can be distinguished from the surrogates for $3 < r < 100$, so there is some evidence for nonlinearity in the data. However, the AE index is largely controlled by the solar wind, and it has be found (Prichard and Price, 1994) that the solar wind exhibits more evidence for nonlinearity that the AE index does. By examining

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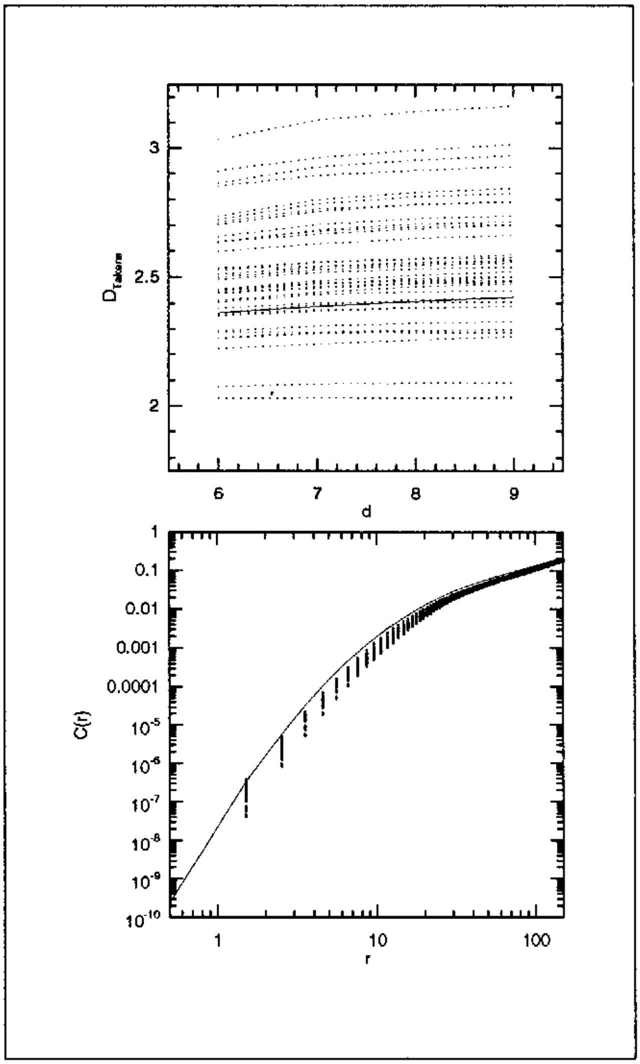

Fig. 1. (a) Takens dimension estimator as a function of reduced embedding dimension d for the AE data (solid) and for 39 surrogate data sets. (b) Correlation intergal and a function of radius for the AE data (soild) and for 39 surrogate data sets.

the AE index alone, it is not possible to determine if the nonlinearity is the result of the intrinsic dynamics of the magnetosphere, or the result of the nonlinearity in the solar wind. In order to answer this question one needs to use nonlinear input-output methods (Casdagli, 1992; Price et al., 1994).

In conclusion, these results suggest that there is no evidence that the AE index can be described by a low dimensional strange attractor as has been suggested by Pavlos et al. The fact that the magnetosphere is largely controlled by the solar wind makes it a randomly driven non-autonomous system. This alone should argue against the existence of a strange attractor in the AE index. In order to address the role of nonlinear dynamics in the Earth's magnetosphere one needs to use input-output methods where the solar wind driver is explicitly taken into account.

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