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Nonlinear Processes in Geophysics

Nonlinear optimization set pair analysis model (NOSPAM) for assessing water resource renewability

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Abstract. There is much uncertain information which is very difficult to quantify in the water resource renewability assessment (WRRA). The index weights are the key parameters in the assessment model. To assess the water resource renewability rationally, a novel nonlinear optimization set pair analysis model (NOSPAM) is proposed, in which a nonlinear optimization model based on gray-encoded hybrid accelerating genetic algorithm is given to determine the weights by optimizing subjective and objective information, as well as an improved set pair analysis model based on the connection degree is established to deal with certain-uncertain information. In addition, a new calculating formula is established for determining certain-uncertain information quantity in NOSPAM. NOSPAM is used to assess the water resource renewability of the nine administrative divisions in the Yellow River Basin. Results show that NOSPAM can deal with the uncertain information, subjective and objective information. Compared with other nonlinear assessment methods (such as the gray associate analysis method and fuzzy assessment method), the advantage of NOSPAM is that it can not only rationally determine the index weights, but also measure the uncertain information quantity in the WRRA. This NOSPAM model is an extension to nonlinear assessment models.

1 Introduction

There are many uncertainties in human decision-making (Hsu et al., 2005; Oppenheim and Wehner, 2010). The water resource system is a highly complex nonlinear system influenced by the natural factors and human activities (Crutzen, 2002; Yang et al., 2004). There is also much uncertain information in the water resource renewability assessment (Milly et al., 2005; Zeng et al., 2007). Many hydrological scientists studied the water resource renewability with different methods, such as statistics methods (Shiklomanov, 1997), water-balance method (Oki et al., 2001; Vorosmarty et al., 2000; Alcamo et al., 2003), water scarcity index method (Oki and Kanae, 2006) and soft computing techniques (Muttil et al., 2006; Cheng et al., 2002; Wang et al., 2009; Chau et al., 2006). There are several main assessment approaches to nonlinear systems: the statistical approach, fuzzy assessment approach, analytic hierarchy process and the gray associate analysis approach (Zadeh, 1965; Saaty, 1972; Solow, 1994; Naschie, 2005; Kuo, 2007; Saaty, 2007; Yang et al., 2009; Su and Yang, 2009; Xu and Zhang, 2009). These approaches are difficult to assess uncertain systems, which causes new challenges to quantify the uncertain information.

But very complicated real systems can be solved with simple mathematical models (May, 1976). The emergence of set pair analysis (SPA) theory provides a new way in studying this kind of highly complex nonlinear system, and makes it possible to extract certain and uncertain information from the assessment systems with optimally interacting minds (Bahrami et al., 2010). SPA, which is an important part of nonlinear science, was first proposed by Keqin Zhao in 1989 (Hu et al., 2008; Su and Yang, 2009; Wang et al., 2009), and some scientists have done research on its theories and applications (Hu et al., 2008; Su and Yang, 2009; Wang et al., 2009). It can be effectively used to analyse such uncertain information as imprecise information, disagreement information, and find hidden rules (Hu et al., 2008; Su and Yang, 2009; Wang et al., 2009; Huang et al., 2009; Xu et al., 2010). But up to now, there is no feasible calculating formula of determining the uncertain information quantity in the SPA model. General nonlinear assessment methods, such as fuzzy set theory, gray system theory, analytic hierarchy process,

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and so on (Chu et al., 1979; Saaty, 2007), have some difficulties in assessing water resource renewability. The main reasons are given as follows: (1) It is very difficult to decide the index weights in the process of multiple objective decision-making because the indexes include much uncertain information, subjective and objective information from nature and human activities. (2) Although there were some discussions on uncertain information, it is difficult to take a quantitative analysis method with the above assessment models in real assessment systems.

In order to assess the water resource renewability rationally, a novel nonlinear optimization set pair analysis model (NOSPAM) is proposed in this study. The steps of NOSPAM are: First, a nonlinear optimization model based on gray-encoded hybrid accelerating genetic algorithm is introduced to determine the weights by synthesizing subjective and objective information. Second, an improved set pair analysis model is established to calculate the connection number, certain and uncertain information quantity. Finally, NOSPAM is used for evaluating the degree of water resource renewability of the nine administrative divisions in the Yellow River Basin.

2 The basic steps of the NOSPAM for water resource renewability assessment

In this paper, the NOSPAM model includes two parts: the nonlinear optimization model and improved set pair analysis model. The basic flow chart of this NOSPAM is shown in Fig. 1.

2.1 Model 1: Nonlinear optimization model

The basic steps of the nonlinear optimization model for determining weights are as follows.

Step 1: Construction of objective vector function.

Supposing there are M indexes of water resource renewability assessment such as $f_1(x), f_2(x), \ldots, f_j(x), \ldots,$ and $f_M(x)$, objective vector function $F(x)$ can be expressed as

$$F(x) = [f_1(x), f_2(x), \ldots, f_j(x), \ldots, f_M(x)]^T \tag{1}$$

Step 2: Construction of objective vector of monitoring points.

Objective vector of monitoring points is given by

$$F_k = [f_{1,k}, f_{2,k}, \ldots, f_{j,k}, \ldots, f_{M,k}]^T \tag{2}$$

$k = 1, 2, \ldots, l$, where l is the number of monitoring points, $f_{j,k}$ is j-th index value of the k-th monitoring point.

Step 3: Construction of ideal interval vector.

Ideal interval vector is constructed with the range of the standard index value in each grade.

$$F_i^* = [f_{1,i}^*, f_{2,i}^*, \ldots, f_{j,i}^*, \ldots, f_{M,i}^*]^T, \tag{3}$$

$$f_{j,i}^* = [a_{j,i}, b_{j,i}], i = 1, 2, \ldots, n, \tag{4}$$

where n is the number of grades. $a_{j,i}$ and $b_{j,i}$ are the lower and upper endpoint of interval in which the j-th index value located, respectively.

To calculate scientifically, the value of $f_{j,k}$, $a_{j,i}$ and $b_{j,i}$ should be consistent with the value of grade, i.e., if the value of the grade is small, the value of $f_{j,k}$, $a_{j,i}$ and $b_{j,i}$ should also be small.

Step 4: Nonlinear optimization model based on gray-encoded hybrid accelerating genetic algorithm for determining index weights is constructed.

Supposing that the weight vector is $\lambda = (\lambda_1, \lambda_2, ..., \lambda_M)$, and i_0-th grade of the k_0-th monitoring point is recognized by considering experts' knowledge. $d(i, k_0, \lambda)$ is the distance between $F_{k_0}(x)$ and $F_i^*(x)$, where $F_{k_0}(x)$ is an objective

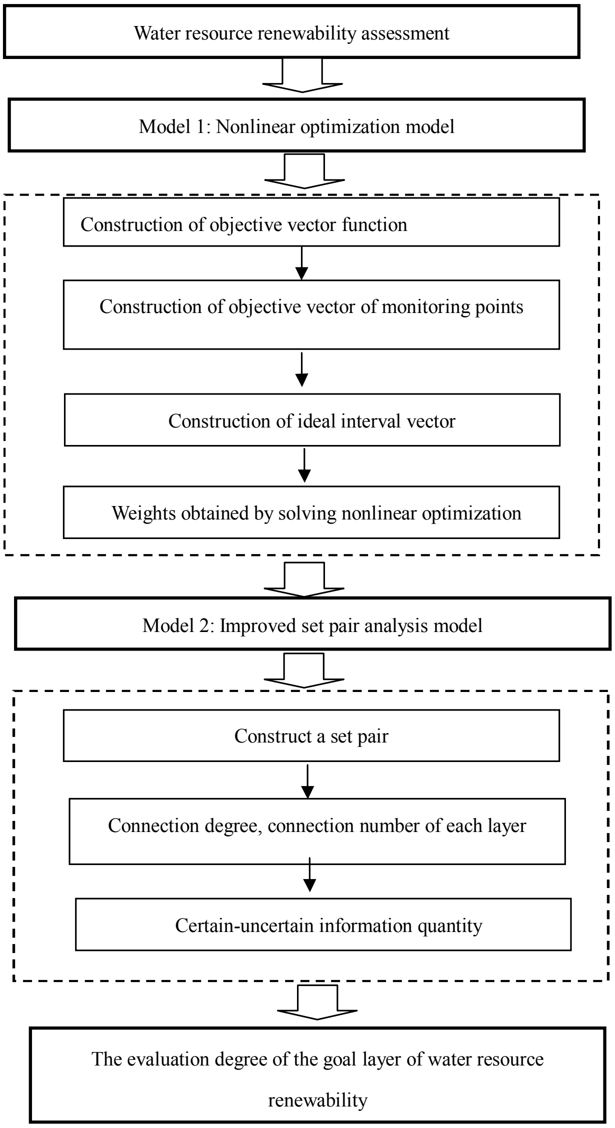

Fig. 1. The basic flow chart of NOSPAM for water resource renewability assessment.

vector of the k_0-th monitoring point, $F_i^*(x)$ is an ideal interval vector of the i-th grade. Nonlinear optimization model for determining weights is given by

$$\min f(\lambda) \tag{5}$$

where

$$f(\lambda)=\begin{cases} 1, & \min d(i,k_0,\lambda)=d(i_0,k_0,\lambda), i=1,2,...,n \\ 10^5, & \min d(i,k_0,\lambda)\neq d(i_0,k_0,\lambda), i=1,2,...,n \end{cases} \tag{6}$$

$$d(i,k_0,\lambda)=||F_{k_0}(x)-F_i^*(x)||=\sum_{j=1}^{M}\lambda_j\Delta(i,k,j) \tag{7}$$

$$s.t.\ \sum_{j=1}^{M}\lambda_j=1,\ \lambda_j\geq 0,\ j=1,2,...,M \tag{8}$$

here $\Delta(i,k_0,j)$ is given by

If i=1,

$$\Delta(i,k_0,j)=\begin{cases} (f_{j,k}0-a_{j,1})/(b_{j,1}-a_{j,1}) & f_{j,k_0}\in[a_{j,1},b_{j,1}] \\ 1+(f_{j,k_0}-a_{j,2})/(b_{j,2}-a_{j,2}) & f_{j,k_0}\in[a_{j,2},b_{j,2}] \\ 3 & f_{j,k_0}>b_{j,2} \end{cases} \tag{9}$$

If i=2,3,4,

$$\Delta(i,k_0,j)=\begin{cases} (f_{j,k_0}-a_{j,i})/(b_{j,i}-a_{j,i}) & f_{j,k_0}\in[a_{j,i},b_{j,i}] \\ 1+(f_{j,k_0}-b_{j,i-1})/(a_{j,i-1}-b_{j,i-1}) & f_{j,k_0}\in[a_{j,i-1},b_{j,i-1}) \\ 1+(f_{j,k_0}-a_{j,i+1})/(b_{j,i+1}-a_{j,i+1}) & f_{j,k_0}\in(a_{j,i+1},b_{j,i+1}] \\ 3 & f_{j,k_0}<a_{j,i-1}, f_{j,k_0}>b_{j,i+1} \end{cases} \tag{10}$$

If i=5,

$$\Delta(i,k_0,j)=\begin{cases} (f_{j,k_0}-a_{j,5})/(b_{j,5}-a_{j,5}) & f_{j,k_0}\in[a_{j,5},b_{j,5}] \\ 1+(f_{j,k_0}-b_{j,4})/(a_{j,4}-b_{j,4}) & f_{j,k_0}\in[a_{j,4},b_{j,4}] \\ 3 & f_{j,k_0}<a_{j,4} \end{cases} \tag{11}$$

The above weight nonlinear optimization model can be solved with gray-encoded hybrid accelerating genetic algorithm (Yang et al., 2005). In general, there are several solutions for the model. We can choose one of these solutions according to experts' ideas. The optimal weight vector can be marked by $\lambda^*=(\lambda_1^*,\lambda_2^*,...,\lambda_M^*)$.

The above-mentioned four steps form the nonlinear optimization model for determining weights, which is one part of the NOSPAM for water resource renewability assessment.

2.2 Model 2: Improved set pair analysis model

In this paper, set pair analysis model is improved by constructing the formulas of determining certain-uncertain information quantity. The basic steps of the improved set pair analysis model for water resource renewability assessment are as follows.

Step 1: Construct a set pair.

For water resource renewability assessment, the indexes of water resource renewability assessment are considered as set A, and the evaluation grades of water resource renewability assessment are considered as set B, then the two sets constitute a set pair H = (A, B).

Step 2: Determine the n-member connection degree μ_m of index layer I_m.

μ_m is given by

$$\mu_m=r_{m1}+r_{m2}i_1+r_{m3}i_2+\cdots+r_{m(n-1)}i_{n-2}+r_{mn}j, \tag{12}$$

where $r_{ml}\in[0,1]$ is the certain-uncertain component of I_m relative to $C_l\sim C_{l+1}$ levels which can deal with certain-uncertain information between assessment grades, $l=1,2,...n$. μ_m is determined according to the formula in Table 1. In Table 1, the measured value of I_m is t_m. And the evaluation grades are classified into $n+1$ levels $C_1,C_2,\cdots,C_{n+1}$ which are divided by the points $a_{m,1},a_{m,2},...,a_{m,n}$. The connection degree of index layer, criterion layer and goal layer can be obtained, and then when the value of "i" is determined, the evaluation degree of water resource renewability can be obtained.

The cost index refers to the smaller measured valued as the better evaluation grade, such as the index of water resource quantity of a unit area ($m^3\times m^{-2}\times a^{-1}$); but the benefit index is opposite, it refers to the smaller measured valued as the lower evaluation grade, such as the index of drought exponent (ratio).

Take the index of water resource quantity of unit area ($m^3\times m^{-2}\times a^{-1}$) for example, in Qinghai province, the connection degree can be written as μ, if $\mu=1$, it means that the degree of water resource quantity of unit area is strongest; if $\mu=-1$, it means that the degree of water resource quantity of the unit area is weakest; if $\mu=a+bi+cj\,(b\neq 0)$, it means that the degree of water resource quantity of the unit area is between strongest and weakest.

Step 3: Determine the n-member connection degree μ of the goal layer.

μ is given by

$$\mu=r_1+r_2i_1+r_3i_2+\cdots+r_{(n-1)}i_{n-2}+r_nj, \tag{13}$$

$$r_l=\sum_{m=1}^{M}w_mr_{ml}(1\leq l\leq n) \tag{14}$$

where r_l is the component of I_m related to $C_l\sim C_{l+1}$ degree. w_m is the weight of I_m which can be determined by model 1 (Nonlinear optimization model). μ represents n-member connection degree of goal layer, $r_1,r_2\cdots r_n$ represent correlative coefficient of each index layer, $i_1,i_2\cdots i_{n-2}$ represent identical-discrepancy-contrary; j represents contrary coefficient. The meaning of μ is the similarity with the index layer.

Step 4: Calculate the certain information quantity Q_i and uncertain information quantity Q_{ui}.

Q_i and Q_{ui} are expressed as

$$Q_i=1-\frac{\sum\limits_{k=2}^{n-1}r_k}{\sum\limits_{k=1}^{n}r_k} \tag{15}$$

Table 1. The formula of n-member connection degree I_m of index layer.

$n-$member connection degree μ_m	Cost index	Benefit index
$1+0i_1+\cdots+0i_{n-2}+0j$	$t_m \le a_{m,1}$	$t_m \ge a_{m1}$
$\frac{\lvert t_m-a_{m,2}\rvert}{\lvert a_{m,1}-a_{m,2}\rvert}+\frac{\lvert t_m-a_{m,1}\rvert}{\lvert a_{m,1}-a_{m,2}\rvert}i_1+0i_2\cdots+0j$	$a_{m,1} \le t_m \le a_{m,2}$	$a_{m,1} \ge t_m \ge a_{m,2}$
$0+\cdots+\frac{\lvert t_m-a_{m,(s+1)}\rvert}{\lvert a_{m,s}-a_{m,(s+1)}\rvert}i_{s-1}+\frac{\lvert t_m-a_{m,s}\rvert}{\lvert a_{m,s}-a_{m,(s+1)}\rvert}i_s+\cdots+0j$	$a_{m,s} \le t_{m,q} \le a_{m,q(s+1)}$	$a_{m,s} \ge t_m \ge a_{m,(s+1)}$
$0+\cdots+0i_{n-3}+\frac{\lvert t_m-a_{m,n}\rvert}{\lvert a_{m,(n-1)}-a_{m,n}\rvert}i_{n-2}+\frac{\lvert t_m-a_{m,(n-1)}\rvert}{\lvert a_{m,(n-1)}-a_{m,n}\rvert}j$	$a_{m,(n-1)} \le t_m \le a_{m,n}$	$a_{m,(n-1)} \ge t_m \ge a_{m,n}$
$0+0i_1+\cdots+0i_{n-2}+1j$	$t_m \ge a_{m,n}$	$t_m \le a_{m,n}$

Table 2. The meaning of each grade for water resource renewability assessment.

Grade	The meaning of each grade
Strongest	Water resource can be supplied and fully utilized repetitively by human beings through natural action or artificial management.
Stronger	Water resource can be utilized repetitively by human beings through natural action or artificial management.
Middle strong	Water resource can be mostly utilized repetitively by human beings through natural action or artificial management.
Weaker	Water resource can not be supplied and utilized repetitively by human beings through natural action or artificial management to some extent, it should arouse attention of humans
Weakest	Through natural action or artificial management, water resource cannot basically meet the requirement of supplement by human beings, some measures must be taken to improve the present situation.

$$Q_{ui}=\frac{\sum\limits_{k=2}^{n-1} r_k}{\sum\limits_{k=1}^{n} r_k} \tag{16}$$

Step 5: Calculate the n-member connection number of each layer.

The n-member connection number of each layer in water resource renewability is given as follows. Supposing $\mu=r_1+r_2i_1+r_3i_2+\ldots+r_{(n-1)}i_{n-2}+r_nj$ as n-member connection degree, $\mu\in[-1,\,1]$, equally divide $[-1,1]$ interval as the value of $i_{n-2},i_{n-1},\cdots,i_2,i_1$, the value of n-member connection number can be calculated for each layer.

Step 6: Determine the degree of water resource renewability.

Equally dividing interval $[-1,1]$, every interval corresponds to the degree of $C_1,C_2,\cdots,C_n,C_{n+1}$. By comparing the value of evaluation degree and connection number, we can obtain the degree of water resource renewability.

2.3 General model of NOSPAM

Now we can see that nonlinear optimization model can scientifically calculate the weight of each index in the water resource renewability system, and then the improved set pair analysis model can solve the problem of all kinds of uncertainties, complexity and hierarchy. Based on model 1-Nonlinear optimization model and model 2-Improved Set pair analysis model, we can obtain the general model of NOSPAM for water resource renewability assessment as Fig. 1.

3 Assessment of water resource renewability in the Yellow River Basin

3.1 Assessment indexes and standard

Aiming at Yellow River Basin, the degree of water resource renewability can be divided into 5 grades (Yang et al., 2004): strongest (1), stronger (2), middle strong (3), weaker (4), and weakest (5). The meaning of each grade of water resource renewability is shown as Table 2. Table 3 shows the assessment indexes of each layer for water resource renewability, No. 1 index indicates water resource quantity of unit area ($\mathrm{m}^3\times\mathrm{m}^{-2}\times\mathrm{a}^{-1}$); No. 2 index indicates surface water resource quantity of the unit area ($\mathrm{m}^3\times\mathrm{m}^{-2}\times\mathrm{a}^{-1}$); No. 3 index indicates ground water resource quantity of the unit area ($\mathrm{m}^3\times\mathrm{m}^{-2}\times\mathrm{a}^{-1}$); No. 4 index indicates water resource quantity of the unit area in the highest flow years ($\mathrm{m}^3\times\mathrm{m}^{-2}\times\mathrm{a}^{-1}$); No. 5 index indicates water resource quantity of the unit area in the lowest flow years ($\mathrm{m}^3\times\mathrm{m}^{-2}\times\mathrm{a}^{-1}$); No. 6 index indicates drought exponent

Table 3. The index meaning of each layer.

Goal layer	Criterion layer	Index layer
Water resource renewability assessment	The effects of natural evolution	No. 1 Water resource quantity of unit area
		No. 2 Surface water resource quantity of unit area
		No. 3 Ground water resource quantity of unit area
		No. 4 Water resource quantity of unit area in the highest flow years
		No. 5 Water resource quantity of unit area in the lowest flow years
		No. 6 Drought exponent
		No. 7 Precipitation deep
	The effects of human activities	No. 8 Annual growth rate of GDP
		No. 9 Annual growth rate of overall agriculture production value
		No. 10 Agriculture consumed water quantity per ten thousand Yuan production value
		No. 11 Water use ration for single livestock

Table 4. Assessment standard based on the data of the whole China.

Index No.	Assessment grade				
	Strongest (1)	Stronger(2)	Middle strong(3)	Weaker (4)	Weakest(5)
No. 1	> 0.85	0.45–0.85	0.17–0.45	0.05–0.17	< 0.05
No. 2	> 0.85	0.45–0.85	0.15–0.45	0.05–0.15	< 0.05
No. 3	> 0.20	0.13–0.20	0.08–0.13	0.04–0.08	< 0.04
No. 4	> 1.5	1.0–1.5	0.4–1.0	0.15–0.4	< 0.15
No. 5	> 0.5	0.3–0.5	0.1–0.3	0.03–0.1	< 0.03
No. 6	< 0.5	0.5–3.0	3.0–15.0	15.0–20.0	> 20.0
No. 7	> 1500	1000–1500	500–1000	100–500	< 100
No. 8	> 8.25	7.75–8.25	7.25–7.75	6.75–7.25	< 6.75
No. 9	> 10	8–10	6–8	4–6	< 4
No. 10	< 500	500–1000	1000–1500	1500–2000	> 2000
No. 11	< 3.5	3.5–5.5	5.5–7.5	7.5–9.5	> 9.5

(ratio); No. 7 index indicates precipitation deep (mm); No. 8 index indicates annual growth rate of GDP (%); No. 9 index indicates annual growth rate of the overall agriculture production value (%); No. 10 index indicates agriculture consumed water quantity per ten thousand Yuan production value ($10^{-4} \times \mathrm{m}^3 \times \mathrm{Yuan}^{-1}$); No. 11 index indicates water use ration for single livestock ($\mathrm{m}^3 \times \mathrm{capita}^{-1}$).

Here eleven indexes are called index layer, the factors of human activities and the factors of nature evolution are called criterion layer and water resource renewability assessment is called goal layer, i.e., water resource renewability assessment system is divided into water resource social renewability assessment system and water resource natural renewability assessment system shown in Table 3. Table 4 shows the assessment standard based on data of the whole China (Yang et al., 2004).

3.2 Assessment result

From Table 4, we can see that the assessment standard was divided into several parts, take No. 1 index as an example: if the measured value is above 0.85, the connection degree can be written as $\mu = a$; if the measured value is between 0.45 and 0.85, the connection degree can be written as $\mu = a + b_1 i_1$; if the measured value is between 0.17 and 0.45, the connection degree can be written as $\mu = b_1 i_1 + b_2 i_2$; if the measured value is between 0.05 and 0.17, the connection degree can be written as $\mu = b_2 i_2 + cj$; if the measured value is under 0.05, the connection degree can be written as $\mu = cj$. So we can draw a conclusion that the four-member connection degree can be obtained in this study. The measured values of indexes of the Yellow River Basin were given in the relative reference (Yang et al., 2004).

Considering the experts' ideas, we take Neimenggu with the fifth grade water resource renewability as a basic point to solve the weight problem of subjective

Table 5. The four-member connection degree of I_m of Qinghai province.

Indexes	The weight of each index	The four-member connection degree of I_m
Water resource quantity of unit area	0.124	$\mu_1 = 0.7250 i_2 + 0.2750 j$
Surface water resource quantity of unit area	0.124	$\mu_2 = 0.8700 i_2 + 0.1300 j$
Ground water resource quantity of unit area	0.124	$\mu_3 = 0.5250 i_2 + 0.4750 j$
Water resource quantity of unit area in the highest flow years	0.124	$\mu_4 = 0.0320 i_2 + 0.9680 j$
Water resource quantity of unit area in the lowest flow years	0.124	$\mu_5 = 0.0850 i_1 + 0.9150 i_2$
Drought exponent	0.079	$\mu_6 = 0.2800 + 0.7200 i_1$
Precipitation deep	0.142	$\mu_7 = 0.8583 i_2 + 0.1417 j$
Annual growth rate of GDP	0.027	$\mu_8 = 1.0000$
Annual growth rate of overall agriculture production value	0.039	$\mu_9 = -1.0000$
Agriculture consumed water quantity per ten thousand Yuan production value	0.044	$\mu_{10} = 0.1880 i_2 + 0.8120 j$
Water use ration for single livestock	0.047	$\mu_{11} = 0.3000 i_1 + 0.7000 i_2$

Table 6. The connection degree, connection number, certain-information quantity and uncertain-information quantity of water resource renewability in the Yellow River Basin.

Divisions	Connection degree	Certain-information quantity	Uncertain-information quantity	Total connection number
Qinghai	$\mu_1 = 0.0632 + 0.1003 i_1 + 0.5105 i_2 + 0.3240 j$	0.3872	0.6108	−0.3961
Sichuan	$\mu_2 = 0.0942 + 0.4023 i_1 + 0.3843 i_2 + 0.1172 j$	0.2114	0.7866	−0.0171
Gansu	$\mu_3 = 0.0833 + 0.1283 i_1 + 0.3310 i_2 + 0.4553 j$	0.5386	0.4593	−0.4389
Ningxia	$\mu_4 = 0.0203 + 0.1341 i_1 + 0.1132 i_2 + 0.7303 j$	0.7506	0.2473	−0.7031
Neimenggu	$\mu_5 = 0.0531 + 0.0856 i_1 + 0.0927 i_2 + 0.7667 j$	0.8198	0.1783	−0.7159
Shanxi	$\mu_6 = 0.0398 + 0.1330 i_1 + 0.3349 i_2 + 0.4903 j$	0.5301	0.4679	−0.5172
Shaanxi	$\mu_7 = 0.0348 + 0.1311 i_1 + 0.4315 i_2 + 0.4006 j$	0.4354	0.5626	−0.4650
Henan	$\mu_8 = 0.0316 + 0.1804 i_1 + 0.6431 i_2 + 0.1429 j$	0.1745	0.8235	−0.2640
Shandong	$\mu_9 = 0.1209 + 0.2143 i_1 + 0.5012 i_2 + 0.1616 j$	0.2825	0.7155	−0.1353
Yellow River Basin	$\mu_{10} = 0.0284 + 0.1123 i_1 + 0.4312 i_2 + 0.4261 j$	0.4545	0.5435	−0.5029

Table 7. The evaluation results of water resource renewability of the nine administrative divisions in the Yellow River Basin with different methods.

Divisions	Evaluation degree of natural aspect with NOSPAM	Evaluation degree of human activities with NOSPAM	General assessment grade with different methods		
			Gray associate analysis method	Fuzzy assessment method	NOSPAM
Qinghai	4	3	4	4	4
Sichuan	3	3	3	3	3
Gansu	4	3	4	4	4
Ningxia	5	3	5	4	5
Neimenggu	5	3	5	4	5
Shanxi	4	3	4	4	4
Shaanxi	4	3	3	4	4
Henan	4	3	3	3	4
Shandong	4	3	3	3	3
The Yellow River Basin	4	3	4	4	4

* 1, 2, 3, 4, 5 stand for the evaluation degree of I, II, III, IV, V.

and objective information. From formula (5), one optimal weight vector λ^* is obtained by gray-encoded hybrid-accelerated genetic algorithm (Yang et al., 2005), λ^*=(0.124,0.124,0.124,0.124,0.124,0.079,0.142,0.027, 0.039,0.044,0.049) The detail steps of applying NOSPAM are as follows.

Firstly, the four-member connection degree of each index can be obtained according to Table 1. Take Qinghai province, as an example, the weight of each index and the four-member connection degree of I_m are given as Table 5. Then, based on above weight vector λ^*, we calculate the four-member connection number of each province when summarizing the connection degree of these eleven indexes. For example, the four-member connection degree of Qinghai can be written as $\mu = 0.0632 + 0.1003 i_1 + 0.5105 i_2 + 0.3240 j$.

Let $i_1 = 1/3, i_2 = -1/3, j = -1$, we can get the connection number of first grade indexes as Table 6.

Take Qinghai province, as an example, according to Table 1 and formula (12), the four-member connection degree can be written as $\mu = 0.0632 + 0.1003 i_1 + 0.5105 i_2 + 0.3240 j$.

And then, inducing $i_1 = 1/3, i_2 = -1/3, j = -1$ into calculating formula, the four-member connection number can be obtained, and the value is -0.3961. Certain and uncertain information quantity of water resource renewability in the Yellow River Basin is given in Table 6. And the connection degree, connection number, certain-information quantity and uncertain-information quantity of water resource renewability in the Yellow River Basin are given in Table 6.

$[-1, 1]$ interval is divided into 5 parts: (0.6, 1], (0.2, 0.6], $(-0.2, 0.2]$, $(-0.6, -0.2]$, $[-1, -0.6]$. These intervals corresponds to the strongest, stronger, middle strong, weaker, weakest evaluation grade of water resource renewability, respectively. Also taking Qinghai, as an example, the four-member connection number is -0.3961, which corresponds to the interval $(-0.6, -0.2]$, so the evaluation grade of water resource renewability of Qinghai is weaker.

From the diagram above, it can be concluded that: as to water resource renewability, Sichuan and Shandong are middle strong; Qinghai, Gansu, Shanxi, Shaanxi Henan and the Yellow River Basin are weaker, Ningxia and Neimenggu are the weakest. The assessment result is shown in Fig. 2.

3.3 Discussion

In this paper, the influencing factors of water resource renewability can be divided into two parts: natural factors and human factors. This model can calculate the degree of water resource renewability from these two parts, which makes the research have realistic significance. For example, if we only compare the natural aspects of water resource renewability, we can add the connection degree of index (1) $\sim$ (7) which represents the nature aspects together, and other steps are the same. Simultaneously, we can get the evaluation degree of the human aspects of water resource renewability. The evaluation degree of water resource renewability in subsystems is given in Table 7.

Table 7 shows the results of the human aspect of water resource renewability are nearly the same, and they are middle strong. But the natural aspects of water resource renewability are discrepant: Ningxia and Neimemggu are weakest; Sichuan is middle strong; other regions are weaker. So it can be seen that society environment of nine administrative divisions in the Yellow River Basin are nearly the same, which indicates human beings are intervening in the hydrological process in all administrative divisions of the Yellow River Basin. And the natural environment of these regions is not so optimistic. So if we want to improve water resource renewability in the Yellow River Basin, we can focus on not only the natural aspect of water resource renewability but also society environment, for example, we can draft fitting water-saving planning, groundwater-protecting planning or other programming to improve water resource renewability in Yellow River Basin. So NOSPAM can not only be applied to the water resource renewability comprehensive assessment, but it can also be applied to the subsystem of water resource renewability comprehensive assessment. And certain and uncertain information quantity of water resource renewability in the Yellow River Basin can be measured clearly as Table 6.

We also calculated water resource renewability of the nine administrative divisions in the Yellow River Basin by the gray associate analysis method, fuzzy method with our weights, which is shown in Table 7. Although NOSPAM gives the similar results as other methods, the advantage of NOSPAM is that it can not only rationally determine the weight, but also calculate the certain and uncertain information quantity of water resource renewability in the Yellow River Basin. NOSPAM is a new way to assess water resource renewability.

4 Conclusions and prospect

To assess the water resource renewability rationally, NOSPAM model is established, which takes genetic algorithm and set pair analysis as a theory basis. The NOSPAM is used to assess the water resource renewability for nine administrative divisions in the Yellow River Basin, the main conclusions are as follows.

1. Set pair analysis model is improved by constructing the formulas (12)$\sim$(16) for determining certain-uncertain information quantity. Certain and uncertain information quantity of water resource renewability in the Yellow River Basin is calculated by these formulas. And the expression of n-member connection degrees in NOSPAM is given. The NOSPAM can fully take advantage of certain and uncertain information.

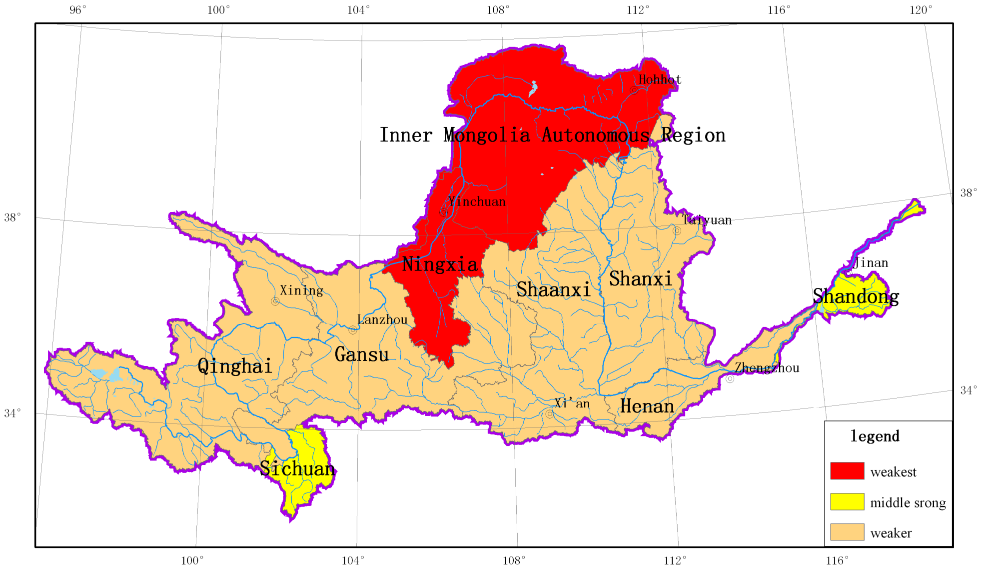

Fig. 2. The result of water resource renewability assessment in the Yellow River Basin.

2. Nonlinear optimization model is given to determine the weight by use of global convergent gray-encoded hybrid accelerating genetic algorithm in NOSPAM. NOSPAM can fully take advantage of subjective and objective information, which makes the weights more reasonable.

3. As to water resource renewability of nine administrative divisions in the Yellow River Basin from the total system, we can see that Sichuan and Shandong are middle strong, Qinghai, Gansu, Shanxi, Shaanxi, Henan and the Yellow River Basin are weaker, Ningxia and Neimenggu are the weakest. From the subsystem, we can see that the results of the human aspect of water resource renewability are almost the middle strong, which indicates human beings have intervened in the hydrological process in all administrative divisions of the Yellow River Basin. But the natural aspects of water resource renewability are discrepant, Ningxia, Neimemggu are weakest, Sichuan is middle strong, and other regions are weaker. And the natural environment of these regions is not so optimistic. The results show that the NOSPAM can play an important role in the application and analysis.

4. Compared with the gray associate analysis method and fuzzy assessment method, NOSPAM can not only rationally determine the index weight, but also measure the certain-uncertain information quantity in the WRRA. NOSPAM can be widely used in the certain-uncertain water resource assessment systems. This research will have significant theoretical and practical impacts on the studies of the nonlinear assessment methods. The new nonlinear optimization set pair analysis model (NOSPAM) can be used in assessing other nonlinear systems in the future and its theory will be further studied.

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