Supplementary Note

Results of the linear stability analysis

Main paper: Patch behaviour and predictability properties of modelled finite-amplitude sand ridges on the inner shelf

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1 Introduction

In the main paper results are shown of the linear stability analysis for the default setting of the parameters only. In this note the sensitivity of the initial growth rate, alongshore spacing and migration speed to changes in the transverse bottom slope β, the offshore wave height $H_{rms,s}$ and the wave period are presented. The sensitivity of these variables to variations in the offshore angle of incidence can be found in Vis-Star et al. (2008). Finally, an explanation, based on earlier literature, will be given of the physical mechanism that causes the initial growth of sand ridges.

2 Dependence linear results on inner shelf slope and offshore wave height

In Fig. 1 cross-shore profiles of U_w are shown for different offshore root-mean-square wave heights. The larger the offshore root-mean-square wave height the larger the offshore wave orbital velocity. The onshore increase in the wave orbital velocity is stronger for higher waves due to a more intense shoaling.

In Fig. 2 contour plots of longshore spacing, growth rate and migration speed of the most preferred mode in the $H_{rms,s} - \beta$ plane are shown. Note that for offshore wave heights smaller than 1.2 m the model assumption that the wave orbital velocity is larger than the current amplitude is no longer satisfied.

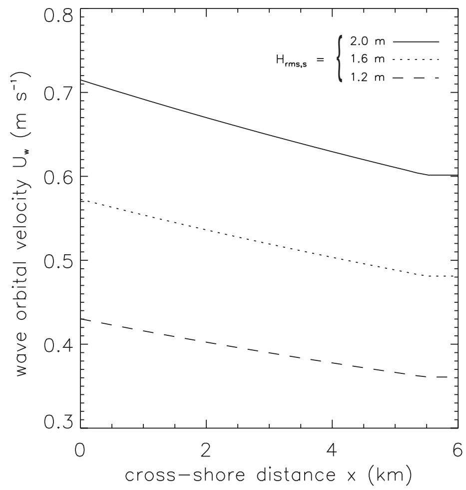

Figure 1: Cross-shore profiles of the wave orbital velocity in the basic state for different offshore root-mean-square wave heights.

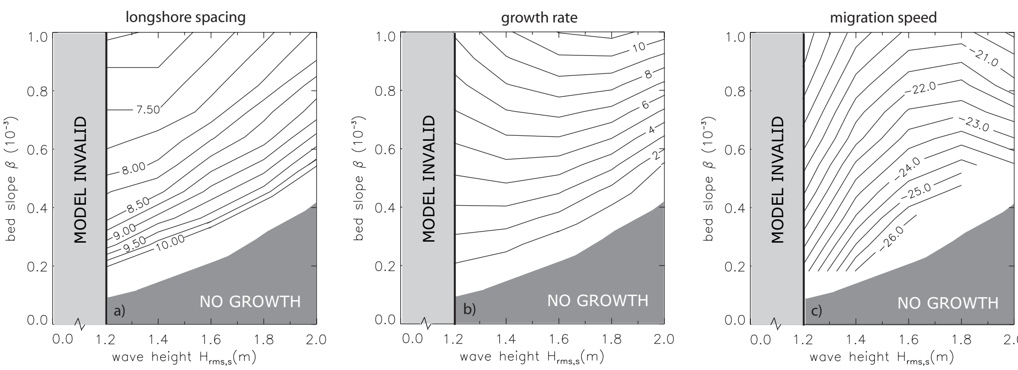

Figure 2: Contour plots of equal (a) longshore spacing (km), (b) growth rate (10^{-3} yr^{-1}) and (c) migration speed (m yr^{-1}) of the initially most preferred mode in the $H_{rms,s} - \beta$ plane.

The longshore spacing of the most preferred mode is smaller for steeper transverse bottom slopes. The longshore spacing increases with an increase in the offshore root-mean-square wave height. The critical transverse bottom slope increases from 1.0×10^{-4} to 4.3×10^{-4} when the offshore root-mean-square wave height is increased from 1.2 m to 2.0 m, respectively. As long as the critical inner shelf slope is exceeded, the growth rate of the bedforms increases and the migration speed decreases with increasing transverse bottom slope of the inner shelf. The dependence of these variables on the offshore wave height is more complicated. In general, the growth rate decreases with an increase in offshore wave height. However, for offshore wave heights between 1.2 m and 1.6 m and large bed slopes an opposite trend is visible. Furthermore, it appears that for an offshore wave height of about 1.8 m the migration speed exhibits a maximum: the migration is slower for both smaller and larger offshore waves. The orientation of the bottom pattern with respect to the shoreline does not depend on the offshore wave height.

3 Sensitivity to wave period and inner shelf slope

As is shown in Fig. 3, the wave orbital velocity is larger for low-frequency waves. The onshore increase in the wave orbital velocity is similar for low- and high-frequency waves.

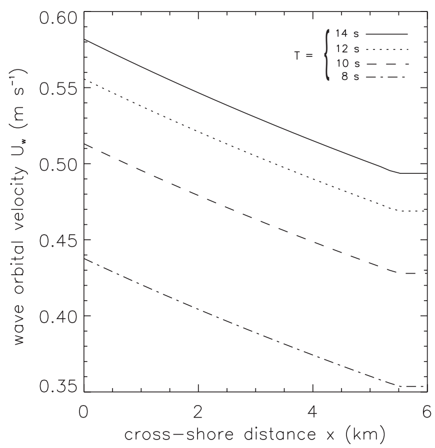

Figure 3: Cross-shore profiles of the wave orbital velocity in the basic state for different wave periods.

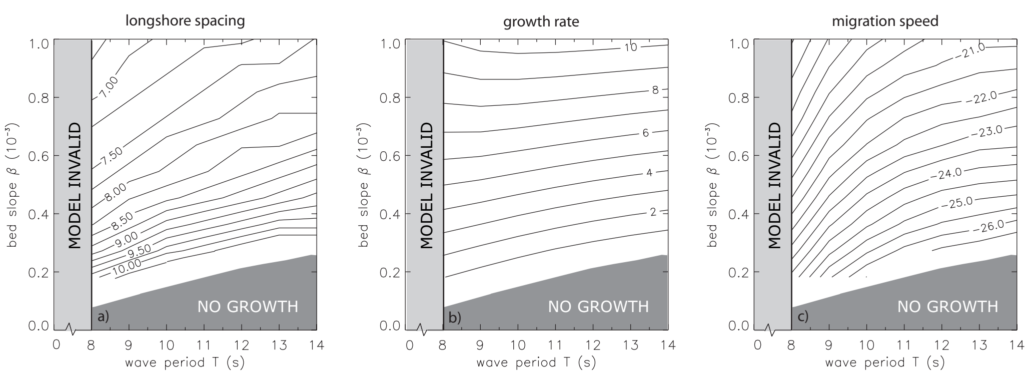

Figure 4: As Fig. 2, but in the $T-\beta$ plane.

The dependence of the longshore spacing, maximum growth rate and migration speed on the transverse bottom slope β and the wave period T are shown in Fig. 4. For a wave period smaller than 8 s the model assumption that a weak current limit is considered is no longer satisfied. The longshore spacing varies between about 7 km and 10 km and is larger for smaller transverse bottom slopes and low-frequency waves. In general, the initially most preferred mode evolves slower and migrates faster when the wave period is increased. Again, a critical bed slope has to be exceeded before growing bedforms are obtained. The bedforms cover the entire width of the inner shelf under all conditions and are up-current oriented. The angle between crest axis and coastline is $\varphi \sim 30°$ for both low- and high-frequency waves.

4 Physics of the instability mechanism

The equation governing the evolution of the bed perturbations is derived from the linearised version of the bed evolution equation, the linearised continuity equation and the linearised formulations for $\vec{q}_b$ and $\vec{q}_s$, respectively. The detailed expressions can be found in Eq. (13) and in Appendix A of the main paper. If settling lag effects are neglected, the result is

$$\overbrace{(1-p)\frac{\partial h'}{\partial t}}^{T0} + \overbrace{\frac{\frac{3}{2}\nu_b U_w^2 V}{H}\frac{\partial h'}{\partial y}}^{T1} - \overbrace{\vec{\nabla}\cdot(\lambda_{bs}\vec{\nabla}h')}^{T2} = - \overbrace{\frac{d}{dx}\left(\frac{3}{2}\frac{\nu_b U_w^2}{H}\right)Hu'}^{T3} - \overbrace{\frac{d}{dx}\left(\frac{C}{H}\right)Hu'}^{T4}, \quad (1)$$

where $\lambda_{bs} = (3/2)\nu_b\lambda_b U_w^3 + \lambda_s U_w^5$. In this equation, term T0 represents the growth or decay of bedforms. Bedforms grow (decay) if $\partial h'/\partial t > 0$ ($\partial h'/\partial t < 0$) above the crests. Term T1 describes the alongshore migration of the bed perturbations due to bedload transport of sediment, whereas term T2 is a consequence of the downslope sediment transport and causes diffusion of bedforms.

The different sources of instability are given by the two terms on the right-hand side of the bed evolution equation. First consider term T3 in the specific case of $(3/2)\nu_b U_w^2 \rightarrow K_{stir}$ = constant, which is the case discussed by Trowbridge (1995). Clearly, this term $\neq 0$ if the transverse bottom slope $dH/dx = \beta \neq 0$. For $\beta > 0$ sfcr grow if u' and h' are positively correlated, i.e., the current should exhibit an offshore deflection over the crests. As u' decreases seaward, due to the increase in water depth, the current is convergent. As here sediment transport is linearly related to the current, deposition of sediment occurs on top of the crests and sfcr will grow. An obliquely oriented ridge can induce a deflection of the longshore current as a consequence of water mass continuity, which causes an increase in the cross-bank component of the flow over the crest. Only for up-current oriented sfcr the deflection in the current is directed offshore above its crest, thus only up-current rotated sfcr will grow. The offshore current deflection over sfcr is indeed reproduced by the model (results not shown here).

A more general case is considered by Calvete et al. (2001), who consider term T3 and T4 in equation (1), where U_w now depends explicitly on H and $C \sim HU_w^3$ (see Eq. (11) of the main paper). They argue that stirring of sediment by the waves increases towards the coast, which is an additional source of sediment deposition over the crests and thus growth is enhanced. More generally, sfcr grow if the cross-shore gradient of the depth-averaged volumetric sediment concentration in the basic state is negative. The latter quantity is defined as $\frac{d(C/H)}{dx}$, thus U_w^3 should decrease with increasing distance from the coast. It appears that term T4 is dominant over term T3. A sketch of the Trowbridge and Calvete mechanisms is given in Fig. 5. The offshore deflection of the longshore current over an up-current oriented ridge is illustrated in the top view. The cross-section through the ridge shows (1) a convergence of the flow as it enters deeper water and (2) a decrease in the wave orbital velocity into deeper water. As a consequence sediment transport is convergent and sediment is deposited on the crests. The convergence is most effective on the downstream side of the ridge again due to flow convergence, which explains the downstream migration of the bedforms.

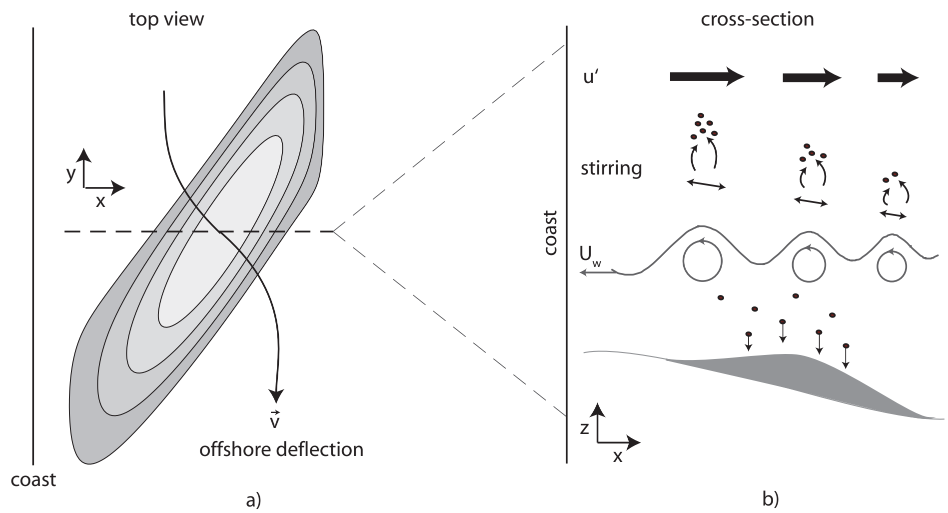

Figure 5: Schematic view of the Trowbridge and Calvete mechanisms. (a) An up-current rotated ridge causes an offshore deflection of the current. (b) Flow converges when it enters deeper water, which causes sediment convergence over the ridge. This effect is enhanced by nonuniform wave stirring: stirring of sediment by the waves is stronger in shallow water compared to deep water and thus also causes convergence of sediment over the ridge.

Sensitivity of the model results (longshore spacing, growth rate and migration speed) to offshore wave characteristics can be understood by considering the magnitude of the different terms in equation (1). Whether bedforms have the tendency to grow or decay depends on the competition between advective terms (T3 and T4, of which T4 is dominant) and the diffusion term (T2) which render unstable and stable bottom perturbations, respectively. If advective terms are larger than the diffusion term then bedforms can grow, otherwise they decay. Variation in parameter values leads to changes in the growth rate if the absolute change in the magnitude of the advective terms is different from that of the diffusive term. Furthermore, changes in the longshore spacing of bedforms occur if there is a change in the magnitude of the ratio of advective and diffusive terms. Term T2 is proportional to U_w^5, whereas term T4 is proportional to $\frac{d}{dx}\left(\frac{C}{H}\right)\frac{u'}{H} \sim \frac{d}{dx}\left(\frac{C}{H}\right)\frac{V}{H^2}h'$. The latter estimate follows from water mass continuity. Note that V itself is inversely proportional to U_w due to frictional effects. The magnitude of the bedform migration is determined by term T1, which is proportional to $U_w^2 V/H \sim U_w/H$.

Let us now consider the case of increasing the bed slope β. Here, an increase in β corresponds with an increase in H_s and thus a larger water depth at the outer shelf. Across the major part of the inner shelf, an increase in β results in a smaller wave

Table 1: Sensitivity of model results to the transverse bottom slope and offshore wave characteristics. For explanation see the text.

	T1 $\sim \frac{U_w^2 V}{H}$	T2 $\sim U_w^5$	T4 $\sim \frac{d}{dx}\left(\frac{C}{H}\right)\frac{V}{H^2}$	σ_r	λ_p	V_m
$\beta \uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$
$H_{rms,s} \uparrow$	$\uparrow$	$\uparrow\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$
$\|\Theta_s\| \uparrow$	$\simeq$	$\downarrow$	$\downarrow\downarrow$	$\downarrow$	$\uparrow$	$\simeq$
$T \uparrow$	$\uparrow$	$\uparrow\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$

orbital velocity and thus T2 $\sim U_w^5$ decreases. On the other hand, considering Fig. 6, the cross-shore gradient in the depth-averaged sediment concentration increases for a larger slope. Together with an increase in the magnitude of V, due to reduced friction, term T4 clearly increases. Hence, the growth rate of bedforms becomes larger for larger β. The relative increase of T4/T2 with increasing β results in a preferred wavelength which is smaller on steeper inner shelf slopes. As the migration is determined by T1$\sim U_w/H$, it explains the decrease in migration speed with increasing β.

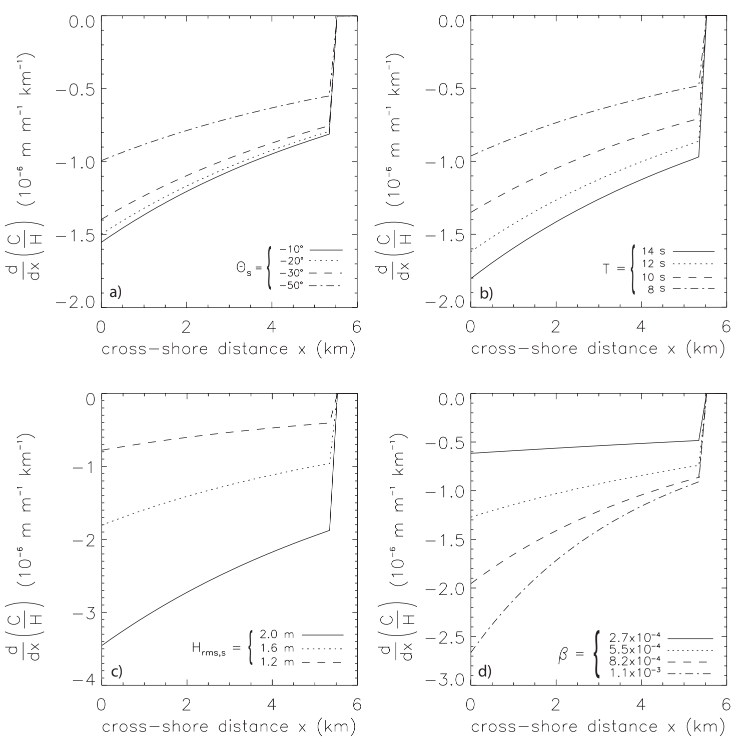

Figure 6: Cross-shore profiles of the cross-shore gradient of the depth-averaged volume concentration in the basic state for (a) different offshore angles of wave incidence (b) different offshore root-mean-square wave heights (c) different wave periods and (d) different values for the transverse bottom slope of the inner shelf. Other parameters have their default values.

In a similar way the dependence of growth rate, migration speed and longshore spacing of bedforms on the magnitude of the offshore wave height can be understood. The wave orbital velocity is proportional to $H_{rms,s}$ and therefore T2 is larger for higher offshore waves. The dominant advection term T4 also increases (see Fig. 6), but not as fast as T2. The absolute increase in T2 is so large compared to the absolute increase in T4 that bedforms grow slower in case of larger offshore wave heights. As the ratio T4/T2 also increases, bedforms have longer spacings for higher offshore waves. In general, bedform migration increases with the presence of higher offshore waves, which corresponds with the increase in the magnitude of U_w. Note that under conditions that term T3 and T4 become large (e.g. for high offshore waves), their imaginary parts will no longer be small compared to term T1 and will therefore also contribute to the migration speed. This might explain the decrease in migration speed when waves become very high (see Fig. 2). The sensitivity of model results for a variation in the wave period and the angle of wave incidence can be explained using similar arguments. A summary is given in Table 1.

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