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Supplementary information.

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1 Comparison between both models

Here we test our conceptual model against the far more comprehensive Earth system model CLIMBER-2. In order to compare both models we examine their response to a bi-sinusoidal forcing, given by

$$f(t) = A \cdot (\cos[\frac{2\pi t}{T_1} + \phi_1] + \cos[\frac{2\pi t}{T_2} + \phi_2]), \quad (1)$$

with T_2 = 1470/17 (≈86.5) years and ϕ_2 = 0. A, T_1 and ϕ_1 are varied throughout the analysis. Units of f are milli-Sverdrup (mSv, 1 mSv = 10^3 m^3/s), since f represents a freshwater flux anomaly. A summary of our findings is given at the end of this text.

1.1 Forcing amplitude A

In this test we chose ϕ_1 = 0 and T_1 = 1470/7 (=210) years, and we vary the amplitude A. Note that for this value of T_1 the forcing repeats with a period of 1470 years. The maximum of f is 2A, the minimum is -2A. We start the conceptual model in the cold state (which is also the initial state in CLIMBER-2). Note that in CLIMBER-2 the threshold character of the switches between both modes of deep water formation is a result of the underlying hydro-/thermodynamical processes. This means that a threshold function is explicitly not defined in that model. This implies that it is very difficult to start the conceptual model with similar initial conditions as in CLIMBER-2. For simplicity we thus begin our model runs with a threshold value of $A_0 = -27$ mSv.

Supplementary fig. 1 shows the response of the conceptual model for A = 4 mSv (a), A = 5 mSv (b) and A = 6 mSv (c). For A = 4 mSv the model remains in the cold state ($s = 0$) since the forcing never crosses the threshold (because $-2A \geq B_0$). For A = 5 mSv the model switches into the warm state ($s = 1$) since the forcing crosses the threshold at some time (because $-2A < B_0 = -9.7$ mSv). After this switch it remains in the warm state since the threshold is never crossed again (because $2A \leq B_1 = 11.2$ mSv). For A = 6 mSv the model oscillates between both states since the forcing repeatedly crosses the threshold (since $-2A < B_0$ and $2A > B_1$). Note that two critical values for the forcing amplitude thus exist, namely $B_0/2$ and $B_1/2$: When started in the cold state with $B_1 > -B_0$ (and $B_0 \leq 0$, $B_1 \geq 0$), the model stays in this state for A $\leq -B_0/2$, it ends up in the warm state for $-B_0/2 <$ A $\leq B_1/2$ and it oscillates between both states for A $> B_1/2$. In this case the simulated climate cycle is asymmetric (the extrema occur at the time of the switches between both states) and evolves on a different (i.e. on a much longer) time scale than the forcing.

The response of CLIMBER-2 to the forcing shows a very similar pattern (supplementary fig. 2): For A = 4 mSv (a), and also for smaller values, the model remains in the cold state. For A = 5 mSv (b) it switches into the warm state (within the very first years of the model run) and afterwards remains in this state. For A = 6 mSv (c), as well as for larger values, it oscillates between both states. This means that two critical values for the forcing amplitude also exist in CLIMBER-2. The fact that the location of the three regimes in the amplitude space is similar in both models shows that the choice of B_0 and B_1 in our conceptual model is consistent with CLIMBER-2.

Both models seem to disagree in the time relation between the forcing and the simulated climate events: In supplementary fig. 1b, for example, the switch into the warm state occurs after roughly 5880 years, i.e. after four periods of the forcing. In supplementary fig. 2b, in contrast, this switch already occurs within the first years of the simulation. A similar mismatch exists between supplementary fig. 1c and 2c. This mismatch, however, might be solely due to an inadequate choice of the initial conditions in the conceptual model: When the model is started from equilibrium conditions, i.e. with an initial threshold value of -9.7 mSv, a transition into the warm state is immediately triggered for A = 5 mSv (because the forcing takes its minimum value of -10 mSv at t = 0 years and thus crosses the threshold already at the start of the simulation). Another difference between both models is that in the conceptual model the time evolution of the state variable S is by construction very smooth and does not show variations on the same time scale as the forcing. In CLIMBER-2, however, century-scale temperature fluctuations exist, albeit of small magnitude (e.g. supplementary fig. 2c). According to our interpretation these represent a second, more linear component in the model response to the forcing. Since our focus is the timing of DO events, however, we do not intend to include a similar component in the conceptual model.

We now compare the output of both models for A $\geq$ 6 mSv (supplementary figures 3 and 4). After the start of the simulations both models require some time (several centuries/millennia) until the output settles in a periodic / quasi-periodic pattern. Since this settling time is not necessarily the same in both models we only consider the periodic part of the model output, but not the first years. In order to account for the difference in the settling time we introduce a very simple time delay in the conceptual model: Instead of starting the model in the year t = 0 we allow a different starting time Δ (in years). We then use an optimal value for the parameter Δ in order to maximise the agreement between both model outputs. Note that a modification of Δ does not change the period of the simulated events. It can, however, change their timing (supplementary fig. 3): Instead of being triggered by the first order minima of the forcing (i.e. the ones at t = 0 years, 1470 years, 2940 years, etc.; supplementary figures 3a and 3c), the events can also be triggered by the second order minima (i.e. the ones at t = $\pm$ 430 years, 1470 $\pm$ 430 years, 2940 $\pm$ 430 years, etc.; supplementary fig. 3b) when Δ is varied. To some degree, an adjustment of this parameter can thus compensate for the difference in the settling time of both models. In the following we use an optimal value Δ = 1000 years.

Supplementary fig. 4 shows the comparison between both model outputs for this choice of Δ. In that figure the forcing amplitude A is varied from 6 mSv to 12 mSv. Since the threshold function in the conceptual model is repeatedly crossed by the forcing, DO events occur in that model for all of these values of A. Like the forcing, the simulated events are strictly periodic (after a settling time of at most a few millennia). With increasing forcing amplitude the conceptual model oscillates faster between both states, since the forcing crosses the threshold function more frequently. The duration of the simulated events and their spacing thus decreases as A increases. Depending on A, the characteristic spacing of the events can be 1470 years (4e, 4f), smaller than that value (4g) or larger (4a-d). In particular, spacings of integer multiples of 1470 years are also possible (4a, 4c). The output of CLIMBER-2, in contrast, is not always strictly periodic. However, it is possible to find time intervals of tens of thousands of years during which the output is almost periodic. In these intervals the events in CLIMBER-2 agree surprisingly well with the ones in the conceptual model: The conceptual model correctly reproduces the period of the events in CLIMBER-2 (apart from 4g and, to some degree, 4d) and in most cases also the absolute timing of the switches between both states. For A = 12 mSv (and also for larger values), however, the events occur far too often in the conceptual model: In supplementary fig. 4g, the average spacing between the events in the conceptual model is 735 years, but it is several centuries larger in CLIMBER-2. The reason for this mismatch is probably that on the century-scale additional processes of the ocean circulation are important (e.g. mixing processes between the surface and intermediate depths), which are not included in our conceptual model.

To summarise this part of the tests, for a larger range of forcing-amplitude values A the response of our conceptual model to the forcing agrees very well (both qualitatively and quantitatively) with the response of CLIMBER-2. A mismatch between both models exists when the simulated events have a period of much less than 1470 years. In that case the period of the events in the conceptual model is systematically too small.

1.2 Forcing frequency

In this test we chose ϕ_1 = 0 and A = 10 mSv, and we vary the period T_1 of the first forcing cycle from 206 to 214 years. We start the conceptual model in the cold state with Δ = 1000 years and a corresponding threshold value of -27 mSv. We run both models over 60.000 years and calculate histograms (supplementary fig. 5) in order to compare the spacing of the events as simulated by both models. We omit the very first event in each model run, since its timing is strongly influenced by the initial conditions and by the different settling time of both models.

As shown before (supplementary fig. 4e), for T_1 = 210 years the simulated events have a period of 1470 years, and correspondingly one single peak exists in that histogram (supplementary fig. 5c). For other values of T_1 additional peaks occur, all of which correspond to a spacing of integer multiples of T_2 (e.g. 1643 years $\approx 19 \cdot T_2$, supplementary fig. 5a). In the context of the conceptual model this is not surprising since the simulated events can only be triggered by pronounced minima in the forcing, and a necessary condition for the existence of such a minimum is that the second forcing cycle is close to a minimum. The simulated events are thus expected to occur with a spacing close to multiples of T_2. Note that for all shown values of T_1 a 1470-year peak occurs, and that this peak is most pronounced when T_1 is close to 210 years. The agreement between both models is surprisingly good: The conceptual model reproduces the position, but also the width and the approximate magnitude of all peaks as obtained with CLIMBER-2. The only exemption is the 1297-year peak in supplementary fig. 5d: This peak only occurs in the output of CLIMBER-2, but not in the conceptual model.

To summarise this part of our tests, the conceptual model also agrees well (both qualitatively and quantitatively) with CLIMBER-2 when the frequency of one forcing cycle is modified (and, what we did not demonstrate here, also when both cycles are simultaneously changed).

1.3 Forcing phase

In this test we use A = 10 mSv and T_1 = 1470/7 (=210) years, which implies that the forcing repeats with a period of 1470 years. Forty different values from 0 to 2π are considered for the phase ϕ_1. Again, we start the conceptual model in the cold state with Δ = 1000 years and a corresponding threshold value of -27 mSv. Note that for ϕ_1 = 0 both model outputs agree in the absolute timing of the simulated events (supplementary fig. 4e). We now investigate if both models still agree in the timing of the events for other values of ϕ_1.

The period of the simulated climate cycle in both models is 1470 years for all values of ϕ_1. A modification of this phase, however, can alter the absolute timing of the events in such a way that the 1470-year cycle is shifted in time by integer multiples of $T_2 \approx 86.5$ years. The reason for this behaviour is that the minima of the first forcing cycle are shifted in time when ϕ_1 is changed. As a consequence, the position of the most pronounced minima of the total forcing (which occur when both cycles have a minimum at about the same time) also changes, so that these minima can now occur at a different minimum of the second forcing cycle. Since the simulated DO events are only triggered by such pronounced minima it is plausible that the start of the simulated events can be shifted by multiples of T_2 when ϕ_1 is changed.

Supplementary fig. 6 compares both model outputs. In that figure the absolute timing of the start of the simulated DO events is shown. The comparison is restricted to the periodic part of the simulated time series (i.e., we omit the first few 1470-year intervals before the output settles in a period pattern). Since the output of both models has a period of 1470 years, the absolute timing of the n-th event in each simulation is given by $t_n(\phi_1) = t_0(\phi_1) + n \cdot 1470$ years (n = 1, 2, 3, etc.). In the figure t_0 is shown as a function of ϕ_1. Note that t_0 always takes integer values of 1470/17 ($\approx$86.5) years, corresponding to multiples of T_2. The pattern obtained with CLIMBER-2 (6a) is very well reproduced by the conceptual model, with Δ = 1000 years (6b). Note that Δ was only optimised for a single value of ϕ_1 (i.e. for $\phi_1 = 0$). Nevertheless, the conceptual model also agrees with CLIMBER-2 for almost every other value of ϕ_1, apart from ϕ_1 = 0.125, 0.275 and 0.425 (in 2π). When the conceptual modelual model is started with Δ = 0 years, in contrast, both models agree only for 10 values (6c).

The surprisingly good agreement between both models outputs in supplementary fig. 6b shows that once the difference in the settling time and in the initial conditions of both models is accounted for (by an appropriate choice of the parameter Δ), the conceptual model provides a reliable tool to reproduce the timing of DO events in CLIMBER-2: 37 out of 40 predictions for the absolute timing of the 1470-year cycle are correct, although that timing can take 17 different values (corresponding to 17 minima of the second forcing cycle). According to the Binomial distribution, a random guess of the timing, with a probability of 1/17 for each of the 17 possible values, yields a probability of the order of 10^{-40} for a random agreement in that many elements.

To summarise this part of our tests, our conceptual model also shows a very good qualitative and quantitativ agreement with CLIMBER-2 when the phase of one forcing cycle is changed (and the agreement is also good when both phases are simultaneously varied).

2 Summary

In addition to the above tests we also performed additional experiments in order to test the ability of the conceptual model to mimic DO events. The detailed agreement in the output of both models is in fact robust and strongly suggests that our simple model successfully captures the main dynamical features of DO events in CLIMBER-2. Here we summarise our main results:

1. Three different regimes exist in the response of the conceptual model to the prescribed forcing (supplementary fig. 1): For small forcing amplitudes ($A \leq -B_0/2$ and $A \leq B_1/2$) the model stays in its initial state. For intermediate amplitudes ($\min[-B_0/2, B_1/2] < A \leq \max[-B_0/2,$

$B_1/2$]) it ends up in one state (for our choice of B_0 and B_1 in the warm one). This regime vanishes for $B_0 = B_1$. For large amplitudes (A $>$ max[$-B_0/2$, $B_1/2$]) the model oscillates between both states and never reaches equilibrium (i.e. the model shows a new form of non-equilibrium oscillations which we call *overshooting relaxation oscillation*. In this regime millennial-scale DO events are triggered by the century-scale forcing. The same pattern exists in the response of CLIMBER-2 to the forcing (supplementary fig. 2). The parameters B_0 and B_1 of the conceptual model can be adjusted such that both model agree in the location of the three regimes in the forcing-amplitude space.

2. The conceptual model very well reproduces DO events as seen in CLIMBER-2 (and in particular their timing), as long as the events have a spacing of more than about 1000 years (supplementary fig. 4). For events with a smaller spacing the conceptual model has a systematic error: In that case the events occur too often, and their spacing is too small compared with CLIMBER-2 (supplementary fig. 4g).

3. The conceptual model successfully reproduces many key features of DO events as simulated with CLIMBER-2, in particular their asymmetry (i.e. the saw-tooth shape) and their three-phase evolution (an initial abrupt warming, a subsequent gradual cooling, and an abrupt drop back to pre-event conditions at the end of the events).

4. By construction the conceptual model only reproduces the non-linear response of CLIMBER-2 to the forcing (i.e. the abrupt shifts and the gradual trends). The linear response, which manifests itself as small wiggles that are superimposed on the events in CLIMBER-2 (supplementary fig. 4), is not explicitly reproduced.

5. The analogy between both models is very stable with respect to changes of the forcing parameters: The amplitudes, frequencies and phases of the forcing as used here can be varied over a large range without loosing the detailed agreement between the models (supplementary figures 5 and 6).

Figure Captions

Fig. 1. Three regimes in the response of the conceptual model to the forcing. Shown are the forcing f (black), the model threshold T (red), the simulated state variable S (green) and the model state s (grey). The forcing amplitude A is 4 mSv (a), 5 mSv (b), 6 mSv (c). 1 mSv $= 10^3\,\mathrm{m}^3/\mathrm{s}$. In a the model remains in the cold state ($s = 0$), in b it ends up in the warm state ($s = 1$), and in c repeated oscillations between both states occur.

Fig. 2. Three regimes in the response of CLIMBER-2 to the forcing. The figure shows the forcing (black) and the simulated annual mean temperature anomaly in the model box comprising Greenland (green). The forcing amplitude A is: 4 mSv (a), 5 mSv (b) and 6 mSv (c). In a the model remains in the cold state, in b it ends up in the warm state and in c it repeatedly oscillates between both states.

Fig. 3. Sensitivity of the simulated climate cycle with respect to changes of the parameter Δ. Shown is the time evolution of the state variable S as simulated by the conceptual model. The forcing is shown in black. The forcing amplitude A is 6 mSv. The parameter Δ is 0 years (a), 735 years (b) and 1470 years (c). The dashed lines are spaced by 1470 years (i.e. by the period of the forcing). A change of Δ can only cause a shift in the timing of the climate cycle: The first event of the simulated cycle starts at t ≈ 2940 years (a), t ≈ 3370 years (b), t ≈ 4410 years (c). All subsequent events follow with a period of 4·1470 years.

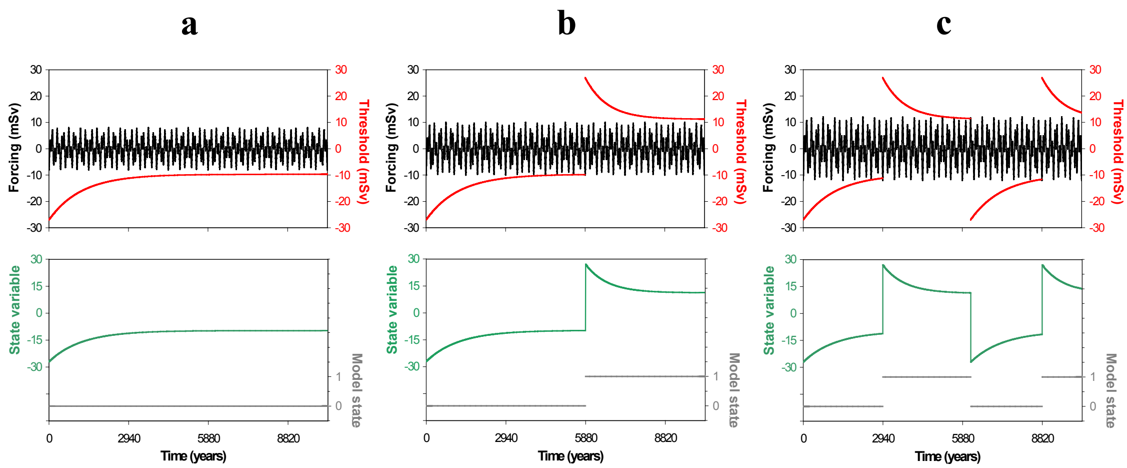
Fig. 4. Response of both models to the forcing. Shown are Greenland temperature anomalies ΔT as simulated by CLIMBER-2 (black) and the time evolution of the (normalised) state variable S in the simple conceptual model (green). The forcing amplitude A is 6 mSv (a), 7 mSv (b), 8 mSv (c), 9 mSv (d), 10 mSv (e), 11 mSv (f), 12 mSv (g). The dashed lines are spaced by 1470 years (i.e. by the period of the forcing). Note that the conceptual model is started at time Δ = 1000 years and that in (a)-(d) the temperature curves obtained with CLIMBER-2 are shifted by integer multiples of 1470 years. Because of its 1470-year period, the forcing is invariant under these shifts (i.e. only the absolute timing of the simulated events is changed under this transformation, but not their timing relative to the forcing).

Fig. 5. Spacing of the simulated DO events, depending on the period of the first forcing cycle. The histograms show the distribution of the spacing Δt between successive events as simulated with both models (left column: CLIMBER-2, right column: conceptual model). The period T_1 is 206 years (a), 208 years (b), 210 years (c), 212 years (d) and 214 years (e). The binning is five years.

Fig. 6. Absolute timing of the 1470-year cycle in both model outputs, as a function of the phase ϕ_1 of the first forcing cycle. Results are shown after several forcing periods (i.e. several multiples of 1470 years), when the model output is periodic (period: 1470 years). 40 different values (0 to 2π) are used for ϕ_1. The output of CLIMBER-2 (i.e. the timing of the start of the simulated DO events) is shown in a, the output of the conceptual

model in b (for Δ = 1000 years). Figure c shows the output of the conceptual model for Δ = 0 years. Horizontal lines are spaced by 1470/17 ($\approx$86.5) years, i.e. by the period of the second forcing cycle. In b and c green dots indicate agreement between both models, grey dots indicate disagreement.

Figures



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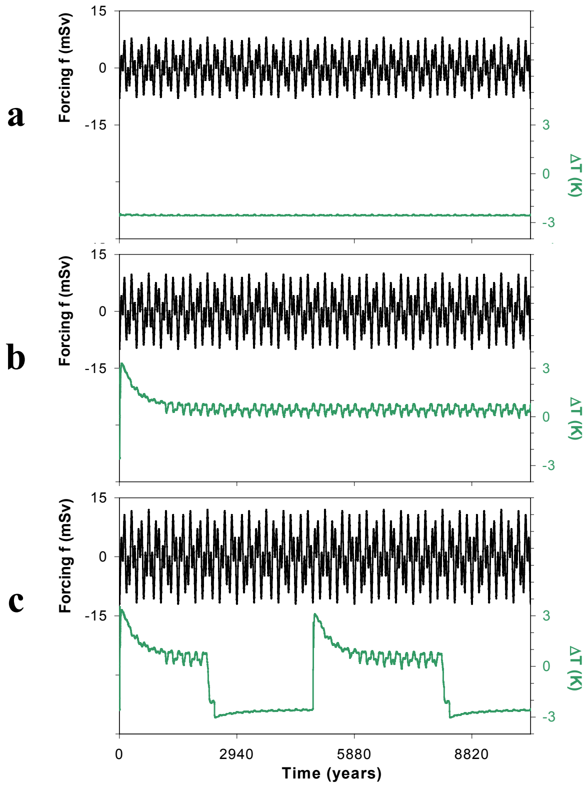

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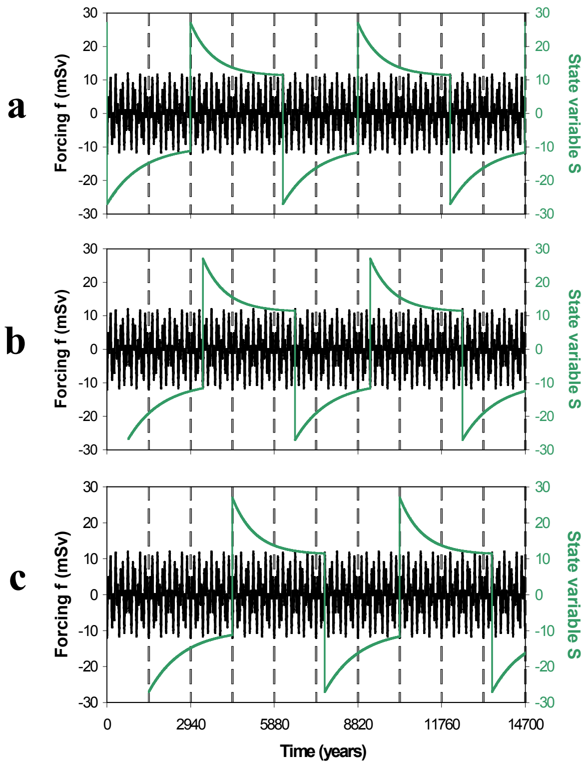

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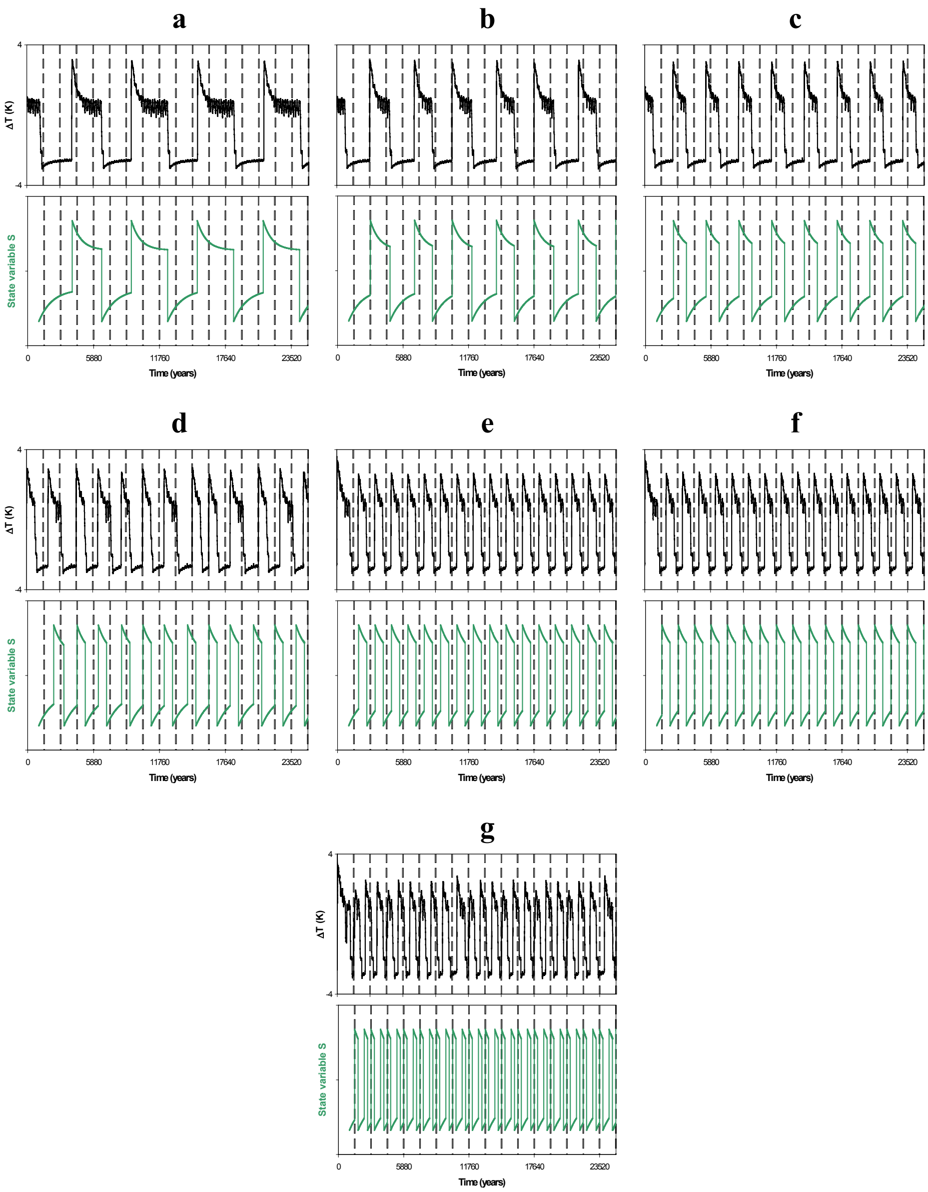

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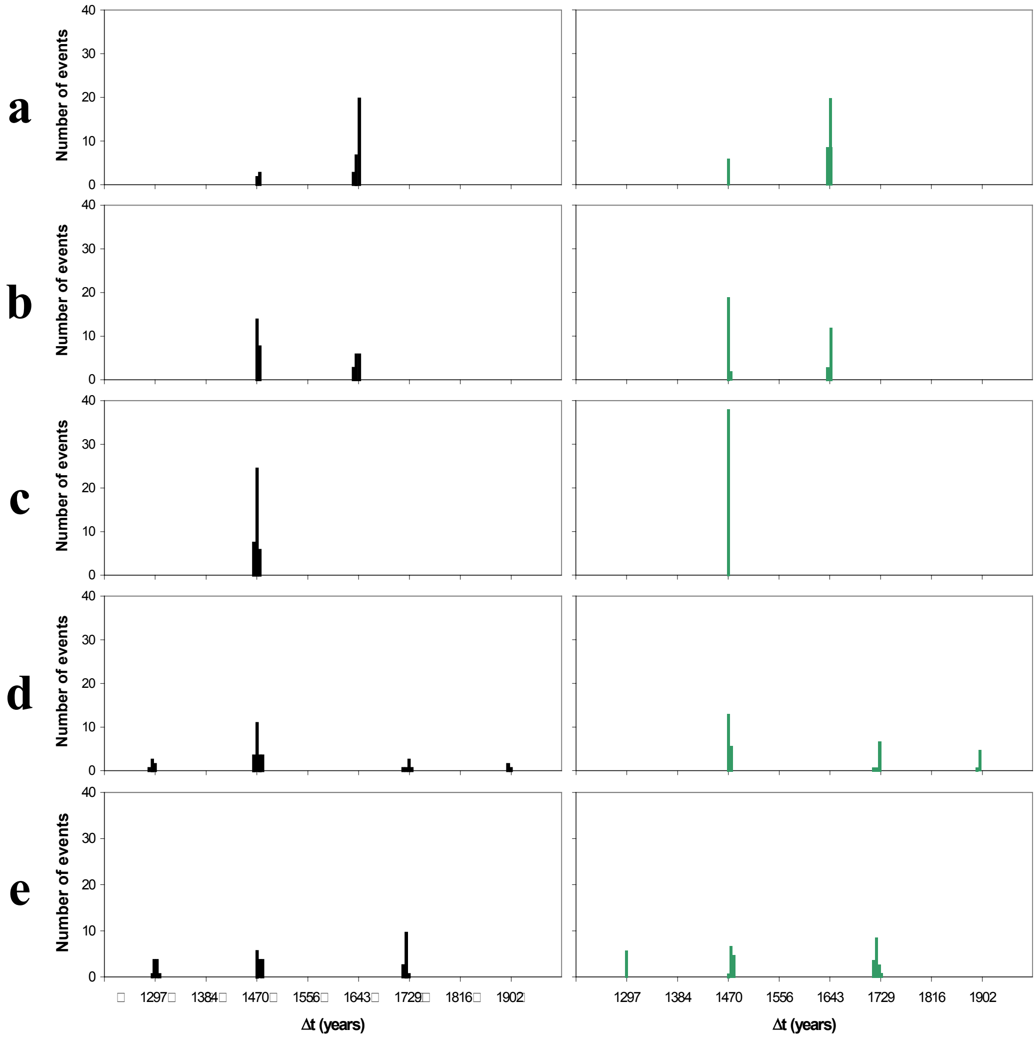

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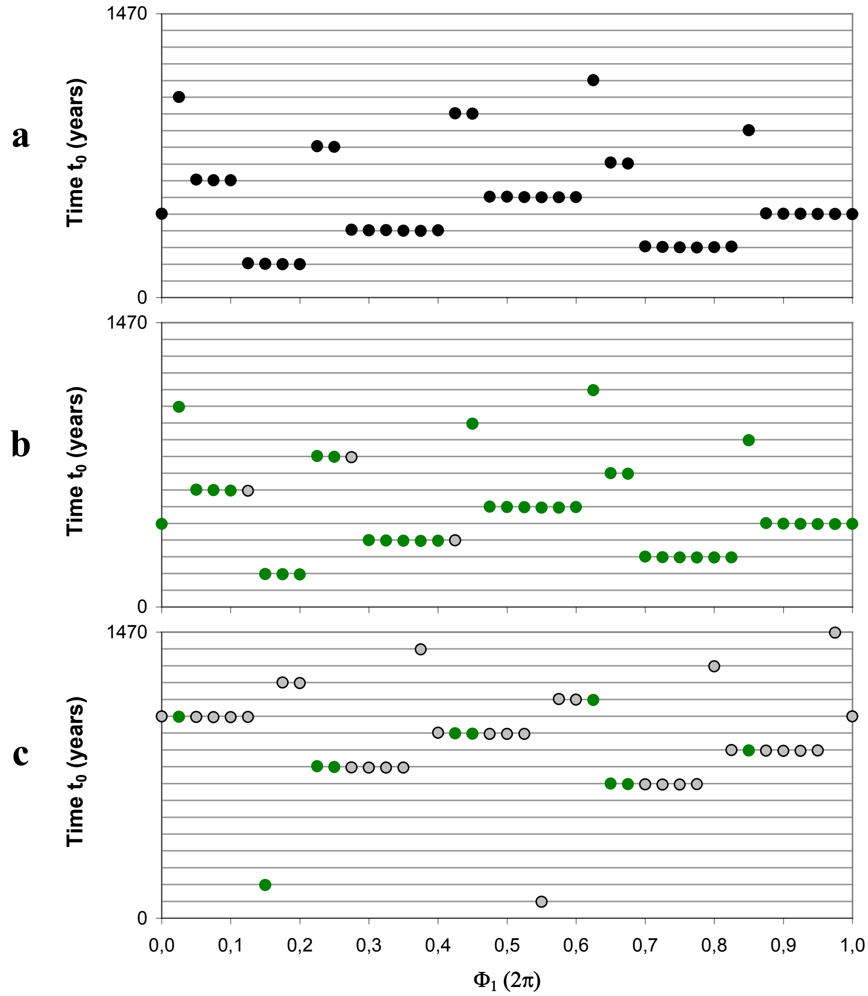

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