**Nonlinear Processes
in Geophysics**

Comment on "Obliquely propagating large amplitude solitary waves in charge neutral plasmas" by F. Verheest, Nonlin. Processes Geophys., 14, 49–57, 2007

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In a recent paper on nonlinear stationary waves in a magnetized plasma, Verheest (2007) imposed the constraint of "strict" charge neutrality by which is meant that simultaneously with $n_i=n_e$ the longitudinal electric field E_x is strictly zero. Armed with this constraint Verheest (2007) goes on to show that nonlinear whistler waves, oscillitons and obliquely propagating Hall-MHD solitons do not exist. This is a case of throwing the baby out with the bathwater.

There are circumstances in which quasi-charge neutrality holds with $n_i \approx n_e$ but $\mathrm{div}\boldsymbol{E} \neq 0$. The nonrelativistic whistler wave is one such circumstance. In this case we may write Poisson's equation in the form

$$\frac{1}{\omega_{pe}^2}\frac{d}{dx}\left(\frac{eE_x}{m_e}\right) = \frac{\delta n}{n} \tag{1}$$

in which ω_{pe} is the electron plasma frequency and $\delta n/n$ is the ratio of excess of ions over electrons to the plasma density. The whistler oscilliton propagates at approximately $V_{Ae}/2$ speed (where V_{Ae} is the Alfven speed based on the electron mass density) with a characteristic length scale V_{Ae}/Ω_e (where Ω_e is the electron gyrofrequency) and an electron acceleration $V_{Ae}\Omega_e/4$ (see e.g. Sauer et al., 2002; Dubinin et al., 2003; McKenzie et al., 2004; Mace et al., 2007). Equation (1) then shows that

$$\frac{\delta n}{n} \sim \frac{\Omega_e^2}{4\omega_{pe}^2} = \frac{V_{Ae}^2}{4c^2} \tag{2}$$

Hence quasi-charge neutrality $n_i \approx n_e$ and $E_x \neq 0$ is a good approximation if the wave speed $V_{Ae}/2$ is not relativistic. According to Chen (1984) "the novice finds this property difficult to understand". On the other hand, if V_{Ae} is of the order of c then we must not only use Poisson's equation but also the equations of motion must be made relativistic as discussed by McKenzie et al. (2005).

A similar argument can be made for Hall-MHD solitons in which the acceleration of the protons is $0(V_A\Omega_p)$, the length scale is V_A/Ω_p yielding the fractional charge separation $\delta n/n \sim \Omega_p^2/\omega_{pi}^2 = V_A^2/c^2$, where V_A and Ω_p are the Alfven speed based on the proton mass density and the proton gyrofrequency, respectively, demanding that $\delta n/n$ is relativistically small and $\mathrm{div}\boldsymbol{E} \neq 0$. This is the reason why the standard Hall-MHD equations do not require Poisson's equation and do not impose any constraint on $\mathrm{div}\boldsymbol{E}$. The electric fields are derived from Faraday's law and the equations of the motion together with the condition $\mathrm{div}\boldsymbol{j}=0$. Poisson's equation provides us only with an estimate of the charge separation ratio $\delta n/n$ as was shown above.

In the classic nonlinear wave propagating transverse to the magnetic field (Adlam-Allen, 1958; Sagdeev, 1966) the circumstances are not quasi-charge neutral. In fact, such an assumption would violate conservation of longitudinal momentum requiring a balance between dynamic pressure and magnetic pressure. It is interesting that in this case the electric stresses are again relativistically small compared with magnetic stresses (pressure) but Poisson's equation is not required even with $n_i \neq n_e$ (McKenzie et al., 2001).

By setting $E_x=0$ simultaneously with $n_i=n_e(u_{ix}=u_{ex})$ Verheest (2007) has taken a wrong turning and got a paradoxical result even for the linear (evanescent) waves. In conclusion, note that a question of using Poisson's equation versus the quasineutrality condition is not a new one. It was discussed and clarified 40 years ago in the classical review paper by Braginskii (1965).

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