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Turbulence in nearly incompressible fluids: density spectrum, flows, correlations and implication to the interstellar medium

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Abstract. Interstellar scintillation and angular radio wave broadening measurements show that interstellar and solar wind (electron) density fluctuations exhibit a Kolmogorov-like $k^{-5/3}$ power spectrum extending over many decades in wavenumber space. The ubiquity of the Kolmogorov-like interstellar medium (ISM) density spectrum led to an explanation based on coupling incompressible magnetohydrodynamic (MHD) fluctuations to density fluctuations through a "pseudosound" relation within the context of "nearly incompressible" (NI) hydrodynamics (HD) and MHD models. The NI theory provides a fundamentally different explanation for the observed ISM density spectrum in that the density fluctuations can be a consequence of passive scalar convection due to background incompressible fluctuations. The theory further predicts generation of long-scale structures and various correlations between the density, temperature and the (magneto) acoustic as well as convective pressure fluctuations in the compressible ISM fluids in different thermal regimes that are determined purely by the thermal fluctuation level. In this paper, we present the results of our two dimensional nonlinear fluid simulations, exploring various nonlinear aspects that lead to inertial range ISM turbulence within the context of a NI hydrodymanics model. In qualitative agreement with the NI predictions and the in-situ observations, we find that i) the density fluctuations exhibit a Kolmogorov-like spectrum via a passive convection in the field of the background incompressible fluctuations, ii) the compressible ISM fluctuations form long scale flows and structures, and iii) the density and the temperature fluctuations are anti-correlated.

1 Introduction

An outstanding, as yet unexplained, observation is that density fluctuations in the interstellar medium (ISM) exhibit a Kolmogorov-like spectrum over an extraordinary range of scales (from an outer scale of a few parsecs to scales of 200 km or less) with a spectral index close to −5/3 (Armstrong et al., 1981). These fluctuations are detected with great sensitivity by Very Long Baseline Interferometer (VLBI) phase scintillation measurements (Armstrong et al., 1995; Spangler, 2001). In interstellar plasma turbulence, the plasma density fluctuates randomly in time and space. As the radio refractive index is proportional to the plasma density, there will be corresponding variations in the refractive index. The angular broadening measurements also reveal, more precisely, a Kolmogorov-like power spectrum for the density fluctuations in the interstellar medium with a spectral exponent slightly steeper than −5/3 (Mutel et al, 1998; Spangler, 1999). Regardless of the exact spectral index, the density irregularities exhibit a definite power-law spectrum that is essentially characteristic of a fully developed isotropic and statistically homogeneous incompressible fluid turbulence, described by Kolmogorov (1941) for hydrodynamic and Kraichnan (1965) for magnetohydrodynamic fluids. This means that turbulence, manifested by the interstellar plasma fluid motions, plays a major role in the evolution of the ISM plasma density, velocity, magnetic fields, and the pressure. Radio wave scintillation data indicates that the rms fluctuations in the ISM and interplanetary medium density, of possibly turbulent origin and exhibiting Kolmogorov-like behaviour, are only about 10% of the mean density (Matthaeus et al., 1991; Spangler, 2001). This suggests that ISM density fluctuations are only weakly compressible. Despite the weak compression in the ISM density fluctuations, they nevertheless admit a Kolmogorov-like power law, an ambiguity that is not yet completely resolved by any fluid/kinetic theory or computer simulations. That the Kolmogorov-like turbulent spectrum stems from purely incompressible fluid theories (Kolmogorov, 1941; Kraichnan, 1965) of hydrodynamics and magnetohydrodynamics offers the simplest possible turbulence description in an isotropic and statistically homogeneous fluid. However, since the observed electron density fluctuations in the ISM possess a weak degree of compression, the direct application of such simplistic turbulence

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models to understanding the ISM density spectrum is not entirely obvious. Moreover, the ISM is not a purely incompressible medium and can possess many instabilities because of gradients in the fluid velocity, density, magnetic field etc. where incompressibility, inhomogeneity and even isotropy are certainly not good assumptions. A fully self-consistent description of ISM fluid, one that couples the incompressible modes with the weakly compressible modes and deals with the strong nonlinear interactions amongst the ISM density, temperature, velocity and the magnetic field, is, therefore highly desirable.

With this motivation, we develop a theory of weakly compressible fluid within the perspective of ISM observations that relate incompressible fluid motions to compressible ones. Since the fluid models of ISM plasmas have been surprisingly efficient in describing a number of complicated nonlinear interactions, we have adopted a fluid approach to develop the underlying model. Our fluid model unifies the multi-scale distinct modes (weakly compressible and purely incompressible) in terms of a perturbative expansion series in which the leading order fluctuations describe the incompressible vortical modes, while the higher orders represent the weakly compressible ISM fluctuations. The two solutions together form a nearly incompressible (NI) solution. The chronological development and the underlying theoretical assumptions of NI hydrodynamics as well as magnetohydrodynamic fluid models, in the context of solar wind and the ISM observations, are described in Sect. 2. The subsequent sections (i.e. 3, 4, and 5) explore nonlinear features such as spectral cascades, coherent structures and correlations of the NI fluid model that are important to the understanding of a fully developed Kolmogorov-like density spectrum in the ISM turbulence. Finally, Sect. 6 contains some conclusion.

2 Chronological development of nearly incompressible fluid model

Fluid models, describing the turbulent motion of a compressible ISM fluid, have been based mostly on isothermal and adiabatic assumptions, due largely to their tractability in terms of mathematical and numerical analysis. Unfortunately, such models cannot describe the complex nonlinear dynamical interactions amongst ISM fluctuations self-consistently. For instance, density fluctuations, in the context of related solar wind work, were thought to have originated from nonlinear Alfvén modes (Spangler, 1987). A simple direct relationship of density variations with Alfvénic fluctuations is not entirely obvious as the latter are not fully self-consistent and are incompressible by nature thereby ignoring effects due to magnetoacoustic perturbations for example. On the other hand, fully compressible nonlinear MHD solutions, for both high- and low-cases, show that Alfvén and slow modes exhibit a $k^{-5/3}$ spectrum, while fast modes follow a $k^{-3/2}$ spectrum (Cho and Vishniac, 2000; Cho and Lazarian, 2003). The kinetic as well as magnetic energy spectra for the fast or slow modes nevertheless do not relate to a Kolmogorov-like density spectrum. The latter modes have been suggested as candidates for generating density fluctuations (Lithwick and Goldreich , 2001) in the interstellar medium. Alternate explanations are that density structures (anisotropic) in the ISM emerge from pressure-balance stationary modes of MHD (also called Pressure Balance Structures, PBS) (Higdon, 1986), or from inhomogenities in the large-scale magnetic field via the four-field model of Bhattacharjee et al. (1998). These descriptions are inadequate for a general class of ISM problems. The PBSs form a special class of MHD solutions and are valid only under certain situations when the magnetic and the pressure fluctuations exert equal forces in the stationary state. These structures, limited in their scope to the general ISM conditions, nevertheless do not offer an entirely self-consistent explanation to the observed density spectrum. Similarly, an inertial range turbulent cascade associated with the low turbulent Mach number four field MHD model is not yet known. Moreover this isothermal inhomogeneous fluid model is valid only for a class of MHD solutions and yields a linear Mach number (M) scaling, $\mathcal{O}(M)$, amongst the various fluctuations (Bhattacharjee et al., 1998).

One of the earlier attempts to understand the ISM density fluctuations, and relate it to an incompressible fluid turbulence model dates back to a paper by Montgomery et al who used an assumed equation of state to relate ISM density fluctuations to incompressible MHD (Montgomery et al., 1987). This approach, called a pseudosound approximation, assumes that density fluctuations are proportional to the pressure fluctuations through the square of sound speed. The density perturbations in their model are therefore "slaved" to the incompressible magnetic field and the velocity fluctuations. This hypothesis was further contrasted by Bayly et al. (1992) on the basis of their 2D compressible hydrodynamic simulations by demonstrating that a spectrum for density fluctuations can arise purely as a result of abondoning a barotropic equation of state without even requiring a magnetic field. The pseudosound fluid description of compressibility, justifying the Montgomery et al. approach to the density-pressure relationship, was further extended by Matthaeus and Brown (1988) in the context of a compressible magnetofluid (MHD) plasma with a polytropic equation of state in the limit of a low plasma acoustic Mach number (Matthaeus and Brown, 1988). The theory, originally describing the generation of acoustic density fluctuations by incompressible hydrodynamics (Lighthill, 1952), is based on a generalization of Klainerman and Majda's work (Klainerman and Majda, 1981, 1982; Majda, 1984) and accounts for fluctuations associated with a low turbulent Mach number fluid, unlike purely incompressible MHD. Such a nontrivial finite departure from the incompressibility state is termed a "nearly incompressible" fluid description. The primary motivation behind NI fluid theory was to develop an understanding and explanation of the interstellar scintillation observations of weakly compressible ISM density fluctuations that exhibit a Kolmogorov-like power law. The NI theory is, essentially, an expansion of the compressible fluid or MHD equations in terms of weak fluctuations about a background of strong incompressible fluctuations. The expansion parameter is the turbulent Mach number. The leading order expansion satisfies the background incompressible hydrodynamic or magnetohydrodynamic equations (and therefore fully nonlinear) derived on the basis of Kreiss principle (Kreiss, 1982), while the higher order yields a high frequency weakly compressible set of nonlinear fluid equations that describe low turbulent Mach number compressive HD as well as MHD effects. Zank and Mathaeus derived the unified self-consistent theory of nearly incompressible fluid dynamics for non-magnetized hydrodynamics as well as magnetofluids, with the inclusion of the thermal conduction and energy effects, thereby identifying different and distinct routes to incompressibility (Zank and Matthaeus, 1990, 1991, 1993).

In the NI theory, the weakly perturbed compressive fluctuations (denoted by suberscript 1) are expanded about the incompressible modes (denoted by superscript ∞) for velocity and pressure variables as $\mathbf{U}{=}\mathbf{U}^{\infty}{+}\epsilon\mathbf{U}_1$, $p{=}1{+}\epsilon^2(p^{\infty}{+}p^*)$ respectively. Here ϵ is a small parameter associated with the turbulent fluid Mach number M_s through the relation $\epsilon^2{=}\gamma M_s^2$, and $M_s{=}u_0/C_s$, γ is the ratio of the specific heats, u_0 is the characteristic speed of the turbulent fluid, and C_s is the acoustic speed associated with sound waves, $C_s^2{=}\gamma p_0/\rho_0$. The p_0 and ρ_0 are typical amplitudes of the fluid pressure and density fluctuations. Due to a lack of uniqueness in the representation of the fluid density and temperature fields, either of the choices $\rho{=}1{+}\epsilon\rho_1$, $T{=}T_0{+}\epsilon T_1$ or $\rho{=}1{+}\epsilon^2\rho_1$, $T{=}T_0{+}\epsilon^2 T_1$ is consistent. The first choice corresponds to a state where temperature fluctuations dominate both the incompressible (p^{∞}) as well as compressible pressures (p^*) and is referred to as the heat fluctuation dominated (HFD) regime. On the other hand the second choice in which all the variables are of similar order represents the heat fluctuation modified (HFM) regime. Since the thermal fluctuations in HFD regime appear at an order $\mathcal{O}(\epsilon)$ as compared with the pressure $\sim\mathcal{O}(\epsilon^2)$, they dominate the NI ordering. By contrast, the thermal fluctuations have the same ordering with respect to the other fluctuations (density, pressure etc) in a HFM regime. The NI theory introduces a further fundamentally different explanation for the observed Kolmogorov-type density spectrum in that the ISM density fluctuations can be a consequence of passive scalar convection due to background incompressible fluctuations as well as a generalized pseudo-sound theory. The theory further predicts various correlations between the density, temperature and the acoustic as well as convective pressure fluctuations (Zank and Matthaeus, 1990, 1993). However, the validity and nonlinear aspects of the NI model, within the context of the interstellar medium, remain largely unexplored.

The theory of nearly incompressible (NI) fluid, developed by Matthaeus, Zank and Brown, based on a perturbative expansion technique is, perhaps the first rigorous theoretical attempt to understand the origin of weakly compressible density fluctuations in the interstellar medium, and one that provides formally a complete fluid description of ISM turbulence with the inclusion of thermal fluctuations and the full energy equation self-consistently, unlike the previous models described above (Zank and Matthaeus, 1990, 1991, 1993; Matthaeus and Brown, 1988). Owing to its broad perspective and its wide range of applicability for interstellar medium problems, we, in this paper, use a nearly incompressible description of fluids to investigate interstellar turbulence with a view to explaining the observed Kolmogorov-like ISM density spectrum. A central tenant of NI theory is that the ISM density fluctuations are of higher order, of higher frequency and possess smaller length-scales than their incompressible counterparts to which they are coupled through passive convection and the low frequency generation of sound. Thus, the NI fluid models, unlike fully incompressible or compressible fluid descriptions, allows us to address weakly compressible effects directly in a quasi-neutral ISM fluid. Furthermore, NI theory has enjoyed notable successes in describing fluctuations and turbulence in the supersonic solar wind. The NI model, nevertheless, has not yet been explored thoroughly against a number of established nonlinear turbulent hypotheses and our paper concentrates primarily on such issues with the objective of understanding the Kolmogorov-like density spectrum in the ISM.

It is worth noting here that the results, to be presented subsequently, concentrate specifically on a high plasma-β limit (where the plasma beta is defined as ratio of plasma particle pressure energy and magnetic energy) and therefore are based essentially on our NI hydrodynamics model. The high plasma-β assumption has largely been utilized in studies of the ISM. Such an assumption can also be justified in the solar wind, since, within $\sim$6AU, the solar wind plasma beta $\sim$1, but beyond the ionization cavity where pickup ions dominate, β (the plasma beta) easily exceeds 1 (see Zank (1999) for a comprehensive review and the references therein). In a hot ISM region, the typical local interstellar parameters are; plasma density $n_0{\approx}0.5{-}1cc$, plasma temperature $T{\approx}10^{6\circ}$ K, and interstellar magnetic field $B_0{\approx}0.1\,\mu$G. This essentially yields a very high value of plasma $\beta{=}4{\times}10^5$ where thermal pressure in an interstellar gas dominates the magnetic pressure. Moreover, we restrict ourselves to a two-dimensional (2D) NI model due primarily to its relative affordibility over 3D computations. Although turbulence in 2D and 3D possesses distinct spectral features characterized essentially by the number of the inviscid quadratic invariants, our two-dimensional NI hydrodynamics simulations offers a plausible approach to the understanding of the origin of ISM density spectrum.

The validity of our model, in 2D NI simulations, is shown numerically in Fig. 1 which demonstrates that the root mean square (rms) density fluctuations are associated with the subsonic turbulent motion, i.e. $\delta\rho{\propto}M_s^2$, such that their ratio remains constant as described in the NI theory.

3 ISM density spectrum

Numerical simulations of a 2D NI hydrodynamics model, in the HFD and HFM regimes, are performed to understand the

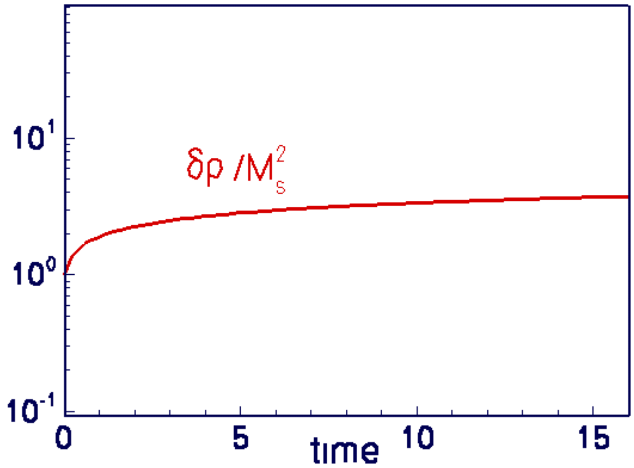

Fig. 1. A validity of NI hydrodynamic simulation. The ratio of root mean square (rms) density fluctuations ($\delta\rho$) and the squared of turbulent subsonic Mach number (M_s^2) remains constant time asymptotically.

inertial range turbulent cascades of the density fluctuations that exhibit a $k^{-5/3}$ spectrum in the ISM. The model equations, normalizations are described in a series of papers by us (Dastgeer and Zank, 2004a, b, c, 2005a[1]). The nonlinear evolution is investigated using a $2/3$ de-aliased Fourier spectral code (our Heat Fluctuation Dominated Hydro, and Heat Fluctuation Modified Hydro; HFD and HFM codes) with 256^2 or 512^2 Fourier modes in a two-dimensional box of size $10\pi\times10\pi$ with periodic boundary conditions along the x and the y-directions. The time integration uses a second order predictor-corrector method. All fluctuations in the simulations are initialized with a Gaussian random number generator to ensure that the Fourier modes are all spatially uncorrelated and randomly phased. The initially normalized energy spectrum, peaked at $k_{\rm min}$, is chosen to lie within the wavenumber band $k_{\rm min}<k<k_{\rm max}/2$.

In two-dimensional turbulence, the background incompressible (hydrodynamic) fluid admits two invariants (constants of motion), namely the energy and the mean squared vorticity (i.e. irrotational velocity field). The two invariants, under the action of an external forcing, cascade simultaneously in turbulence, thereby leading to a dual cascade phenomena commonly observed in a numerous 2D turbulence systems. In these processes, the energy cascades towards longer length-scales, while the fluid vorticity transfers spectral power towards shorter length-scales. Usually a dual cascade is observed in a driven turbulence simulation, in which certain modes are excited externally through random turbulent forces in spectral space. The randomly excited Fourier modes transfer the spectral energy by conserving the constants of motion in k space. On the other hand, in freely decaying turbulence, the energy contained in the large-scale eddies is transferred to the smaller scales leading to a statistically stationary inertial regime associated with the cascade of one of the invariants. Decaying turbulence often leads to the formation of coherent structures as the turbulence relaxes, thus making the nonlinear interactions rather inefficient when they are saturated. Theoretical turbulence models further suggest that any physical quantity, convecting passively in the field of the background turbulence velocity fluctuations, tends to develop eventually a similar energy spectrum (Lesieur, 1990). This characteristic behavior of the passive scalars provides useful information pertaining to its evolution and has been used as a prime diagnostic in our investigations since our NI model possesses a passive scalar-like density equation. We therefore follow the spectrum of the density fluctuations together with the background velocity in our nonlinear NI fluid simulations. On the basis of the HFM as well as the HFD models, we see that the weakly compressible density spectrum in a 2D simulation follows the IN velocity fluctuation spectrum and yields a $k^{-5/3}$ Kolmogorov-like spectrum in the driven case (see, for example, Fig. 2 in HFM regime) and a k^{-3} spectrum in the decaying case, shown in Fig. 3 for a HFM case (Dastgeer and Zank, 2004a). Similar characteristic spectra are observed for the temperature, density and background incompressible velocity fluctuations in the HFD regime of decaying NI turbulence Dastgeer and Zank (2005b)[2]. Thus, in agreement with the NI prediction that a passive scalar explanation emerges as natural and self-consistent description of the weakly compressible density fluctuation spectrum in NI fluid theory, our NI fluid simulation results demonstrate such characteristics in both the forward and the inverse cascade regimes.

Despite the complexity and richness of 2D dynamics due to co-existence of a dual-cascades phenomenon, an important outcome of our simulations is that the spectral law is a consequence of the passive convection of density fluctuations in the field of background incompressible velocity fluctuations, rather than a simple pseudosound correlation between density fluctuations and incompressible pressure fluctuations through the square of the sound speed. By virtue of the nature and nonlinear dynamical characteristics of the NI model, and in anology with our 2D simulation results, 3D turbulent motions of the ISM fluids admiting only a forward cascade of energy could very well exhibit an omnidirectional $k^{-5/3}$ spectrum in kinetic energy and so too could the convective density fluctuations. However, at this point this remains speculative and will be investigated in future. The passive scalar density spectrum, obtained in our simulations, is in a qualitative agreement with Lithwick and Goldreich (2001) hypothesis that entropy fluctuations account for the spectrum of density. The latter, however, considers spectrum of a passive scalar mixed by the Alfvén wave cascade.

[1]Dastgeer, S. and Zank, G. P.: Nonlinear structures and correlations in interstellar fluid, Astron. Astrophys., submitted, 2005a.

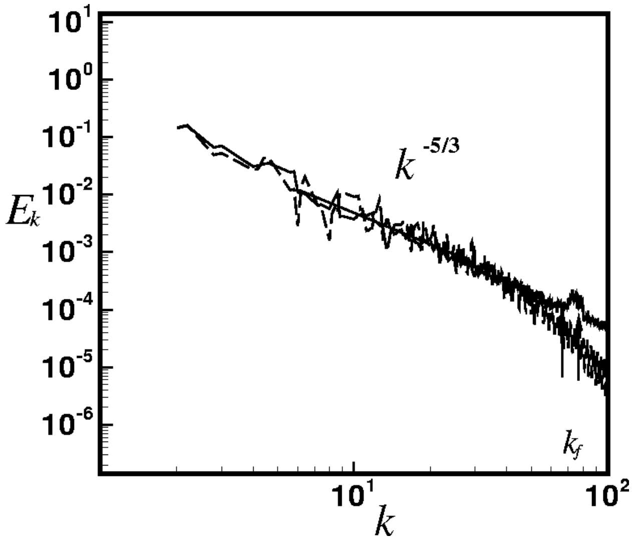

Fig. 2. A driven turbulence NI hydrodynamic simulation yields a Kolmogorov-like spectrum close to $\sim k^{-5/3}$ in the inverse (or energy) cascade regime for incompressible velocity fluctuations, shown by the solid curve. The energy injection wavenumber is indicated by k_f in Fourier space where the turbulence is driven by a random forcing in space and time. The compressible density fluctuations (dashed curve) follows the incompressible velocity spectrum closely in the inertial regime of turbulence.

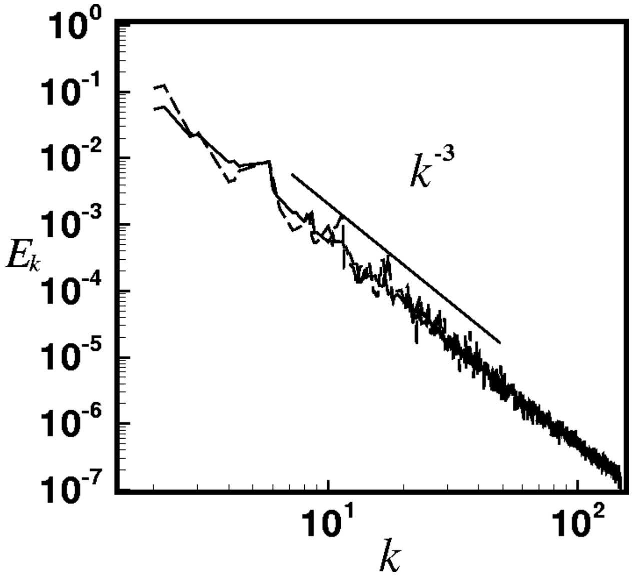

Fig. 3. Density (dashed curve) power spectrum from a decaying NI hydrodynamics simulation. The incompressible velocity fluctuations (solid curve) follow a Kolmogorov spectrum close to $\sim k^{-3}$ in a forward (or enstrophy) cascade regime of decaying turbulence. It is clear that density fluctuations are passively convected by the incompressible velocity fluctuations and exhibit nearly the same spectrum.

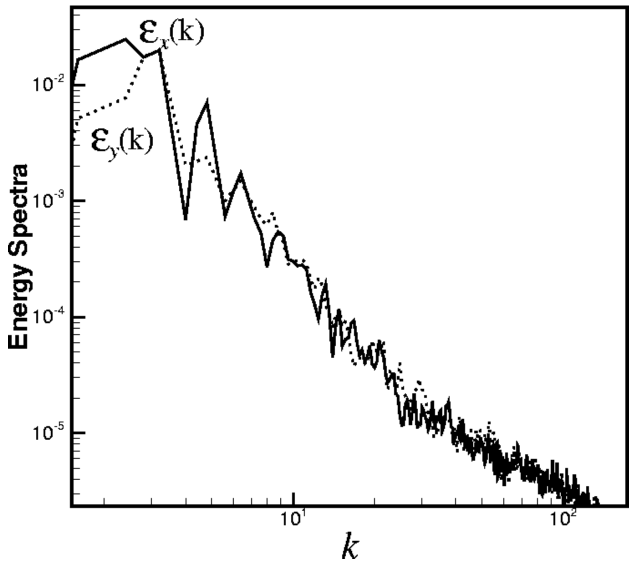

Fig. 4. Inertial range velocity spectra showing $\mathcal{E}_x(k)$ and $\mathcal{E}_y(k)$ components of energies in the compressible fluctuations. Notice the spectral disparity at the lower k-modes. This indicates that larger scales show a greater propensity for anisotropization. The two spectra are isotropic for larger k (i.e. $k \geq 50$).

4 Anisotropic turbulence

While density fluctuations follow a global Kolmogorov-like spectrum in the ISM, there is some observational and theoretical evidence to suggest that they are anisotropic in local spectral space (Armstrong et al., 1995; Higdon, 1984). The existence of an anisotropic density spectrum has generally been ascribed to the presence of a strong directed magnetic field (Higdon, 1986; Zank and Matthaeus, 1992, 1991). However, the ISM need not always be permeated by a strong magnetic field and the local ISM surrounding the heliosphere is a particularly important example (Zank, 1999). Motivated by these issues, we quantitatively distinguish local spectral transfer of the density fluctuations in Fourier space by decomposing the total spectrum into its x- and y-components. Our 2D simulation results demonstrate unequal spectral cascades explicitly, i.e. anisotropy, in the energy spectrum associated with the weakly compressible velocity fluctuations. The turbulent anisotropic cascades occurs primarily at the lower Fourier modes (see Fig. 4). This means large-scale density fluctuations are anisotropic in fully developed ISM turbulence. As the weakly divergent velocity field is the main source of compressional density fluctuations in the NI model, any change in its evolution will progressively be reflected in the density spectrum. Thus the anisotropic cascades in the compressive velocity fluctuations lead to disparity in spectral transfer of turbulent density fluctuations. The spectrally averaged density spectrum over all the Fourier modes ex-

[2]Dastgeer, S. and Zank, G. P.: Density and Temperature spectra in the interstellar medium, Manuscript under preparation, 2005b.

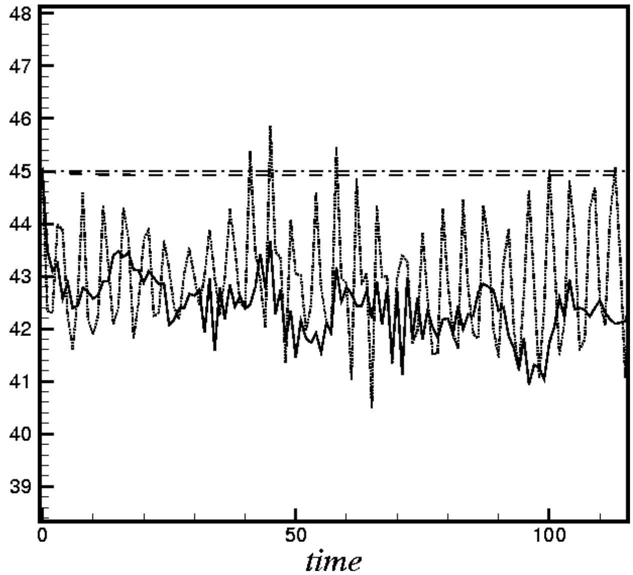

Fig. 5. We measure the symmetry in the spectral cascade by the quantity $\tan\theta_f = \langle k_y \rangle / \langle k_x \rangle$ (Shebalin et al., 1983) ($f = \tilde{f}(\mathbf{k})$ being an evolution variable in Fourier space), and θ is therefore the degree of anisotropy. The angle brackets denote spectral averages. The anistropy angle θ_f is plotted along the y-axis. It essentially measures the disparity in the spectral cascades, if any, when either $\theta > 45°$ or $\theta < 45°$. The figure shows development of the angular anisotropy θ in an initially isotropic spectrum of density (solid curve) and pressure (dotted curve) fluctuations in NI turbulence. The short-scale vorticities in NI (dashed curve) and incompressible (dash-dot curve) exhibit almost isotropy.

cited in turbulent fluctuations demonstrates clearly the progressive development of anisotropy in Fig. 5. This result appears to be consistent with in-situ measurements (Armstrong et al., 1995; Dastgeer and Zank, 2004b). The small length-scales, for instance the vorticity modes, on the other hand show isotropic cascades.

The anisotropic cascades in our simulations are observed to be due to propagating compressional acoustic modes that hinder spectral transfer in the local Fourier space. These modes in turbulence could be excited either by a large-scale or ambient velocity component of the background hydrodynamic turbulence. The latter are also believed to be responsible for breaking the symmetry of the underlying dynamical HFM NI equations (Dastgeer and Zank, 2004b). It is to be noted down that the anisotropy cascades of the large-scale density and the pressure fluctuations are global with respect to the entire Fourier modes allowed in the simulations. Furthermore, the description of the anisotropic turbulence, motivated under a high plasma-β limit, may appear far more complex within the paradigm of an NI MHD model because of subtle nonlinear interactions amongst different magnetoacousitc and Alfvénic turbulent fluctuations.

5 Large-scale flows and structures

The NI model, although motivated originally to understand the spectral cascades that lead to a Kolmogorov-like density fluctuation spectrum in the ISM turbulence, possesses much more complex and rich nonlinear dynamics that remain to be explored in the context of common turbulent hypotheses. Such investigations are not only relevant to the understanding of the Kolmogorov-like density power spectrum, but also shed light on a number of other nonlinear dynamical ISM phenomena. For instance, coherent wave interaction in a HFM regime leads to the formation of asymmetric long-scale flows, as shown in Fig. 6. The high plasma beta simulations of turbulence in this regime demonstrate the formation of nonlinearly saturated asymmetric flows that vary along the x-direction only (Dastgeer and Zank, 2004c). The weakly compressible flow component possesses very long length-scale structures and is excited by nonlinear instabilities that are driven by background incompressible fluid fluctuations (Dastgeer and Zank, 2004c). The response and interaction of acoustic modes in a fluid to and with incompressible turbulence leads to the generation of periodic nonlinear flows, driven by effective Reynolds stresses. Our weak turbulence calculations further show that the Reynolds forces are operative only on the longer length-scales, while the shorter length-scales are damped by turbulent viscosity. An intriguing question arises as to how the flow and turbulence achieve stationarity simultaneously in the steady state. An explanation that emerges from our simulations is as follows. The flow is excited by the Reynolds stresses and eventually quenches the turbulence. Nonlinear instabilities then dissipate the flow and tend to trigger the turbulence. This results in a steady state in which the flow and the turbulence regulate each other periodically.

On the other hand, nonlinear turbulent interactions excited amongst various randomly phased Fourier modes in the HFD regime also show the formation of large-scale compressible coherent structures of the size of computational box (see Fig. 7). During its evolution, the turbulence eventually decays through vortex-merging in which like-signed smaller length-scale fluctuations merge to form relatively large-scale fluctuations. The process continues until all merging has occurred to finally form the largest scale coherent vortex dominated by the minimum allowed k in the simulation. The formation of large scale structure through nonlinear interactions can be attributed to an inverse cascade phenomenon. In this process, small scale turbulent fluctuations, in an inertial range Fourier space, transfer their spectral energy primarily amongst neighboring Fourier modes. While the energy cascade towards the smaller scales in the simulations is terminated essentially by viscous damping, which is efficient at the smaller scales, the largest allowed scale is determined typically by the $k_{\min}$ Fourier mode. Various stages in the simulations are shown in Fig. 7 with the final stage being a large-scale (comparable to computational box) coherent vortex. These structures are formed as a result of turbulence relaxation through a selective decay phenomenon in which

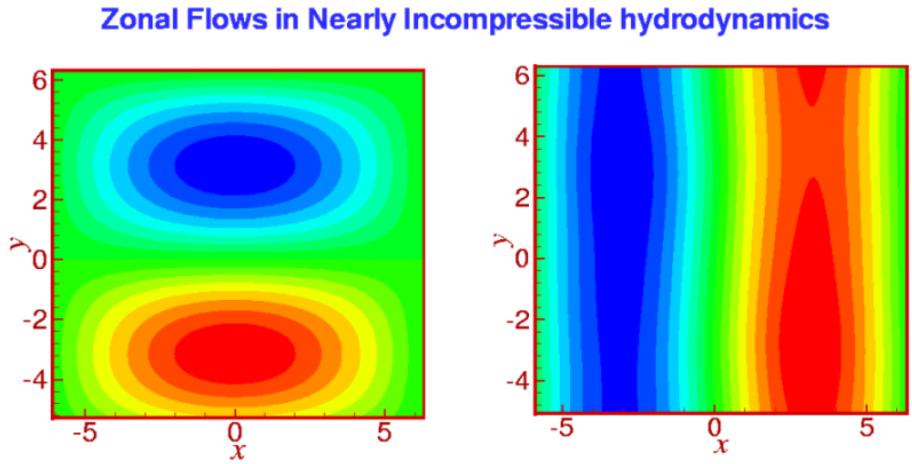

Fig. 6. Evolution of waves in NI fluid. Shown here are the constant contours of the compressible component of the fluid velocity. **(a)** Initial states for both IN and NI fluid modes consist of large scale coherent waves. These waves merge with each other due to nonlinear interactions and form nonlinear flow modes as in **(b)**.

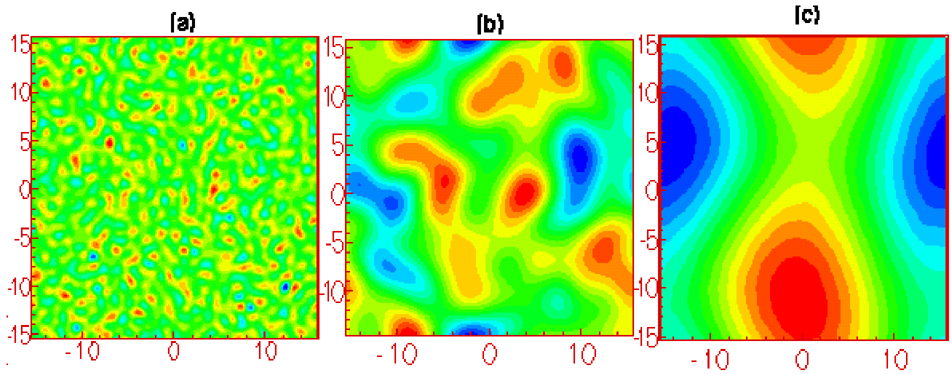

Fig. 7. Decaying turbulence in a NI fluid coupled to an IN fluid through nonlinear interactions. **(a)** Initial condition (t=0) specified on the compressive component of the NI velocity field shows random fluctuations in a two dimensional box **(b)**, **(c)** and (d) show fields at t=15, 40, respectively. The horizontal and the vertical axes correspond to the x- and the y-axes.

one of invariants decays faster than the other (Dastgeer and Zank, 2005a)[3]. In our 2D simulations, the enstrophy decays faster than the energy thereby leading to a faster decay of the shorter scales in turbulence evolution. The larger-scales thus persist in the form of a coherent vortical structures and terminate nonlinear spectral cascades in the compressive turbulent fluctuations.

Thus, quite remarkably, we find that the NI model predicts nonlinearly excited flows and (coherent) structures in low turbulent Mach number compressible ISM fluids. The interaction and the influence of these structures on an inertial range turbulent spectrum remain to be addressed though. The feedback of these compressive fluctuations in the NI hydrodynamics appears to have a weaker effect on the background incompressible fluctuations, as they remain turbulent and exhibit Kolmogorov-like characteristic spectra in the HFD regime Dastgeer and Zank (2005b)[4]. The large-scale structures emerge in our fluid simulations are attributed to an inverse cascade phenomenon in a 2D NI hydrodynamics turbulence. These structures cannot survive in a 3D NI hydrodynamics turbulence due to vortex stretching effect that tears appart the large-scale structures and forbids the inverse cascade phenomena.

6 Turbulent correlations

The NI turbulence theory predicts various correlations between the density, temperature and the acoustic as well as convective pressure fluctuations (Zank and Matthaeus, 1990), and further suggests that compressible ISM fluids be classified as either Type I or II depending upon which fluctuations dominate. However a non-unique decomposition (temperature can either be greater than or equal to pressure,

[3]Dastgeer, S. and Zank, G. P.: Nonlinear structures and correlations in interstellar fluid, Astron. Astrophys., submitted, 2005a.

[4]Dastgeer, S. and Zank, G. P.: Density and Temperature spectra in the interstellar medium, Manuscript under preparation, 2005b.

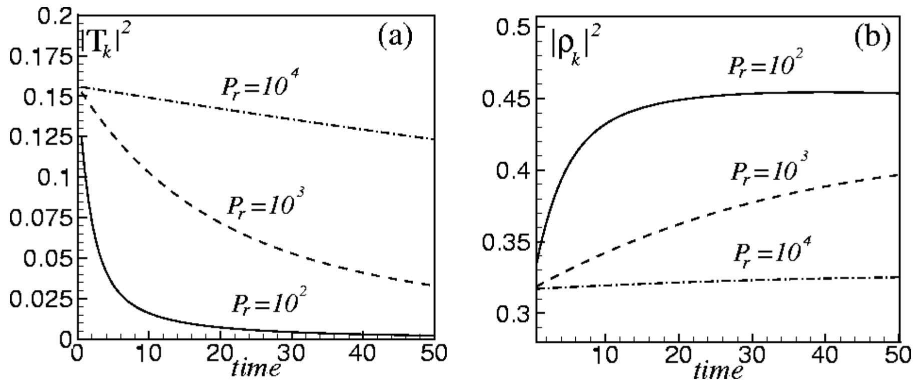

Fig. 8. Energy associated with temperature and density fluctuations in NI turbulence are shown respectively in **(a)** and **(b)**. Shown here are solid (P_r=10^2), dashed(P_r=10^3), and dashed-dot(P_r=10^4) curves. The decay rates depends critically upon the Prandtl number P_r (defined as the ratio of fluid viscosity to thermal conductivity). The density and the temperature fluctuations are clearly anti-correlated in agreement with the prediction of Zank and Matthaeus (1990).

density and velocity fluctuations) of these fluctuations may lead to entirely different correlations. What correlations are observed in the ISM plasma turbulence is therefore an intriguing question that remains to be explored. For instance, in the Type I case, the temperature fluctuations dominate the pressure and the density thereby leading to a density-temperature anti-correlation (such an anti-correlation is also classified above under the HFD regime). On the other hand, when all the fluctuations are of equal magnitude, a Type II or HFM fluid results in a generalized pseudosound relationship. The two types (I and II) predict that quite different physical processes govern the ISM turbulence. In one case, density fluctuations correspond to a modified pseudo-sound description (i.e. the Type I) whereas in the other (Type II) case, density fluctuations respond as a passive scalar (Zank and Matthaeus, 1990, 1991, 1993). The latter is demonstrated by Figs. 2 and 3 in which a passive scalar density spectrum follows the incompressible velocity fluctuations to yield $k^{-5/3}$ and k^{-3} spectra respectively in the inverse and forward cascade regimes.

As mentioned above, correlations in the NI fluid model form self-consistent closures and relate various fluctuating quantities in a rather specific manner. For example, the density and temperature fluctuation anti-correlation, predicted by the HFD NI model, is observed qualitatively in our simulations (see Fig. 8). This anti-correlation indicates that density fluctuations grow as rapidly as thermal fluctuations dissipate to maintain the anti correlation. The density-temperature anti-correlations further suggest a plausible physical mechanism for thermal instabilities in that the adiabatic fluid increases the density that corresponds to a cooler ISM fluid variation. This means that a large fraction of the ISM fluid is not in a thermodynamic equilibrium. This could lead to thermal instabilities that are believed to be one possible source of ISM turbulence.

The density-temperature anti-correlations, in a general context, led to the prediction of, for example, a new class of pressure-balanced-structures (PBS), which were then discovered in the solar wind by Zank and Matthaeus (1990). This class of PBSs observed by Voyager 2 exhibits an anti-correlation in density and temperature.

7 Conclusions

While the interstellar medium possesses supersonic turbulent fluctuations characterized essentially by a higher turbulent Mach number (defined by the ratio of turbulent fluid characteristic speed to the sound acoustic speed) associated with the mean or bulk plasma speed, there can co-exist widely separated multi-scale subsonic fluctuations due to the weakly compressible ISM component. These weakly compressible subsonic fluctuations, within the context of our NI fluid models, describe a self-consistent approach to understanding the origin of the observed $k^{-5/3}$ density spectrum in the ISM. The NI fluid theory utilizes this low turbulent Mach number in a perturbative expansion to decompose explicitly the large-scale low frequency and the short-scale high-frequency motions. The small turbulent Mach number puts a stringent constraint in the NI theory. As Mach number increases, NI expansion diverges asymptotically thereby invalidating its physical application. A fully compressible fluid description is then required to describe the sonic or supersonic turbulent motions that deals with a higher Mach number associated with the higher frequency and the smaller length-scale acoustic phenomena. This component remains entirely absent in a usual incompressible fluid model. The latter describes a fluid motion associated with a relatively low-frequency and large-scale process thereby smearing off all the motions that vary on acoustic or sonic time scales. What is most remarkable in the NI theory is that it includes both the components, *viz*, low-frequency large-scale and the high-frequency small-scale through a low turbulent Mach number. The NI fluid model predominantly describes the large-scale incompressible fluid motion, alongwith a component that describes a small-scale motion associated with the acoustic disturbances.

Consequently, the NI fluid possesses multi-scales mediated by the acoustic motions that vary on a faster time scale as compared with the slow incompressible turbulent motion. The two distinct components associated with the disparate time scales however form a nonlinear parabolic coupled NI model with self-consistent closures that lead to certain types of correlations in the ISM. The NI fluid model, valid for low turbulent Mach number ($\mathcal{M}<1$) fluctuations, is fundamentally rich and complex, and explains qualitatively as well as quantitatively certain ISM observations and predictions.

The main conclusions of this work are as follows.

1. The most important result of this paper is the demonstration that density fluctuations couple advectively to the inompressible velocity field in weakly compressible hydrodynamic turbulence and can exhibit an omnidirectional Kolmogorov power law. Although the results presented in this paper correspond to two dimensional turbulence in a high plasma-beta limit, we expect a similar characteristic behaviour of the passive density field in 3D and speculate that compressible turbulence in three-dimensions will be described by a Kolmogorov-like power law for the density fluctuations. Thus, low frequency incompressible velocity fluctuations can drive high frequency compressible fluctuations[5], which when fully developed, behave like a driven passive scalar and consequently exhibit a Kolmogorov-like spectrum. This we suggest may be an origin of the ubiquitous $k^{-5/3}$ power law density spectrum observed in the diffuse ISM.

2. The pseudosound NI hydrodynamics fluctuations exhibit anisotropic, large scale compressible (density) fluctuations and structures. The anisotropic large-scale density fluctuations in NI turbulence may hint that field-aligned density fluctuations in the solar wind are mediated by a mean flow field such as mean velocity or magnetic field. These large scale or mean fields often excite basic propagating modes that lead to asymmetric spectral transfer in the modes in a local Fourier space.

3. Large amplitude wave-like structures, interacting under the influence of turbulence-driven Reynolds stresses, lead to the formation of large-scale laminar flows that possess only finite k_x variations. Turbulent dissipation does not damp such asymmetric nonlinearly saturated flows, for which $k_y \approx 0$. They are driven primarily by the background incompressible fields via subtle nonlinear interactions (Dastgeer and Zank, 2004c). Moreover, the preferential energy cascade to the larger scales in our simulations is known to be a 2D effect due primarily to a dual cascade nature of the underlying NI turbulence model.

4. Turbulent motions associated with a low Mach number, subsonic, compressible ISM fluid may lead to the formation of large-scale coherent vortical flow fields through an inverse cascade of energy in the decaying turbulence. These quasi-adiabatic compressible structures are driven by purely incompressible fluid fluctuations that vary on rapid time scales. Long-time scale slowly varying ISM flows emerge from the saturation of nonlinear interactions, which hinder further spectral transfer of the energy in an inertial range ISM turbulence. The ISM density and the temperature fluctuations, on the other hand, remain turbulent and convect passively.

5. The density-temperature anti-correlations, predicted by the analytic NI fluid theory and confirmed qualitatively by our nonlinear simulations, may provide useful insight into the formation of " cool-dense" ISM clouds. Such cloud formation is thought to stem from a thermal instability in that the underheated (or cool) ISM regions in the unstable regime are slightly denser and at lower pressure than the overheated (hot) regions (Elmegreen & Scalo (2004)). As shown by our simulations, the density and the temperature fluctuations are anti-correlated for the adiabatic fluid (i.e. $\rho_1 \propto -T_1$), suggesting that density increases correspond to cooler ISM fluid variation. In the ISM, not only may such density-temperature anti-correlations and PBSs play a vital role in understanding the interstellar density spectrum but they may also be important in understanding large-scale structure formation, such as cool-dense ISM cloud formation that is thought to be initiated by thermal gradient instabilities (Elmegreen & Scalo, 2004).

[5]As described elsewhere, the NI theory is based upon perturbative expansion of weakly compressible fluctuations associated with the high frequency fluctuations. The resulting equations, nonetheless, comprise the familiar incompressible hydrodynamical and MHD equations at leading order, together with a modified set of compressible hydrodynamical equations. The background incompressible fluid, describing essentially the low frequency motion in the expansion, can be described by the usual equations of incompressible hydrodynamics. The background (incompressible) motion couples with the NI fluctuations by means of a convective force and drive them through the finite solenoidal velocity component. The entire set of NI system thus appears as if the high frequency compressible motion is driven by the low frequency incompressible fluctuations.

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