

## Recent progress in nonlinear kinetic Alfvén waves

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**Abstract.** This paper presents a review of recent progress in nonlinear kinetic Alfvén wave (KAW hereafter). We start with the two-fluid theory of KAWs and show how the difference between the motions of electrons and ions in small-scale fields of KAWs modifies the Alfvén wave properties. Then, we focus on nonlinear solitary structures of KAWs. A general criterion of the existence for solitary KAW (SKAW hereafter) and its exact analytical solution in a low- $\beta$  plasma ( $\beta < m_e/m_i$ ) are presented, where the electron drift velocity along the background magnetic field is larger than the thermal speed within a SKAW, and hence can excite, for instance, ion acoustic turbulence as showed by in situ observations of satellites in space plasmas. In consequence, the turbulence results in kinetic dissipation of the SKAW and dynamical evolution in its structure. We further discuss the structure of the dissipated SKAW (DSKAW hereafter) that evolves from the SKAW due to the dissipation. The result shows that the DSKAW has a local shock-like structure in its density profile and a net electric potential drop over the shock-like structure. In particular, the electric potential drop of the DSKAW can be expected to accelerate electrons efficiently to the order of the local Alfvén speed. The application of the DSKAW acceleration mechanism to the auroral electron acceleration is also discussed. Finally, a few perspectives of KAW studies in future are presented.

### 1 Introduction

It is well-known that the mechanical effects of a magnetic field  $\mathbf{B}_0$  in a conducting fluid are equivalent to an isotropic magnetic pressure  $P_B = B_0^2/2\mu_0$ , combined with a magnetic tension  $T_B = B_0^2/\mu_0$  along the magnetic field lines. Analogy with the theory of stretched strings suggests that this tension

may lead to a transverse wave propagating along the field lines, with the Alfvén velocity given by

$$v_A = \sqrt{T_B/\rho} = B_0/\sqrt{\mu_0\rho} \quad (1)$$

and with the dispersion relation

$$\left(\frac{\omega}{k}\right)^2 = v_A^2 \cos^2 \theta, \quad (2)$$

where  $\rho$  is the mass density of the fluid. This transverse wave was first predicted to exist by Hannes Alfvén (1942), and was first experimentally studied in the laboratory by Lundquist (1949). Now, we call it the Alfvén wave, or the shear Alfvén wave for the sake of distinguishing from the compressional Alfvén wave driven by the magnetic pressure and discovered by Herlofson (1950). The most striking property of the shear Alfvén wave is that it propagates in one direction only (i.e. along the background magnetic field lines), and therefore suffers no geometric attenuation with distance. The fact that the shear Alfvén wave does not propagate across steady magnetic field lines has important implications for energy transporting and transferring in cosmic plasma fluids.

No surprising, our knowledge of the Alfvén wave has expanded enormously in the intervening a half century period. Maybe, of the many types of waves that exist in plasmas, the Alfvén wave is probably the best studied. The main development of the theory of the Alfvén wave has been focused on two aspects, the one is various modified effects of the ideal Alfvén wave due to plasma processes which lead to the MHD approach of plasmas to be invalid and the Alfvén wave to become dispersive one. The other one is various nonlinear processes and structures relevant to the Alfvén wave.

Kinetic Alfvén wave (KAW hereafter) is the shear Alfvén wave modified by the short wavelength effects, and it can be created when an obliquely propagating shear Alfvén wave is affected by the electron temperature ( $k\rho_s \sim 1$  for  $\alpha > 1$ ) and inertia ( $k\lambda_e \sim 1$  for  $\alpha < 1$ ) such that a non-zero parallel electric field arises within the wave itself due to the local charge separation (Stefant, 1970; Hasegawa, 1976; Goertz and Boswell, 1979), where  $k = 2\pi/\lambda$  is the wavenumber,

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$\alpha = v_{Te}/v_A$  is the thermal speed of electrons  $v_{Te}$  to the Alfvén speed  $v_A$ ,  $\rho_s = \sqrt{Q}v_{Te}/\omega_{ci}$  is the ion acoustic gyroradius,  $\lambda_e = \sqrt{Q}v_A/\omega_{ci}$  is the electron inertial length,  $Q = m_e/m_i$  is the mass ratio of electron to ion, and  $\omega_{ci}$  and  $n_i$  are the ion gyrofrequency and density, respectively. The dispersion relation and polarization of the perturbed electric fields for KAWs can be expressed as follows (Streltsov and Lotko, 1995; Lysak and Lotko, 1996):

$$\omega^2 = \frac{1+k_{\perp}^2\rho_s^2}{1+k_{\perp}^2\lambda_e^2}\omega_A^2, \quad (3)$$

and

$$\frac{E_{\parallel}}{E_{\perp}} = \frac{(\lambda_e^2 - \rho_s^2)k_{\perp}k_{\parallel}}{1 + \lambda_e^2 k_{\perp}^2}, \quad (4)$$

respectively, where the subscripts “ $\parallel$ ” and “ $\perp$ ” both are to the background magnetic field, and

$$\omega_A = v_A k_{\parallel} = v_A k \cos \theta \quad (5)$$

is the local Alfvén frequency, and  $\theta$  is the propagating angle of the wave. It is because its parallel perturbed electric field component  $E_{\parallel}$  can play an important role in accelerating and heating plasma particles that KAW has been a very interesting topic for discussion extensively in the fields of laboratory, space, and astrophysical plasmas since the pioneering theoretical work of Hasegawa and Chen (1975).

The dispersion of KAWs, combined with the nonlinear steepening, may lead to the formation of solitary waves called the solitary KAW (SKAW hereafter). Hasegawa and Mima (1976) first studied the existence of SKAWs for the case of  $1 \gg \beta \gg Q$ , where

$$\beta \equiv \frac{nT}{B_0^2/2\mu_0} \quad (6)$$

is the pressure ratio of the thermal to magnetic. They found that the dynamical behaviour of SKAWs can be controlled by a nonlinear equation called the Sagdeev equation in the following form

$$\frac{1}{2} \left( \frac{dn}{d\eta} \right)^2 + K(n) = 0, \quad (7)$$

where  $n$  is the local plasma density,  $\eta$  is coordinates along the direction of wave propagation in the traveling wave frame, and  $K(n)$  is the so-called Sagdeev potential. This Sagdeev equation of SKAWs is analogous to the motion equation of a classical particle in the Sagdeev potential well  $K(n)$ , and a SKAW's solution corresponds to a bound state of the particle motion. The results show that the SKAW has a density soliton with a hump and a perturbed electric field with a kink due to the appearance of a nonzero parallel electric field  $E_{\parallel}$ . The correction effect of a finite  $\beta$  was considered by Yu and Shukla (1978). A similar case, but for  $\beta \ll Q \ll 1$ , was investigated by Shukla et al. (1982). They also showed the existence of SKAWs, but accompanied by a dip density soliton. The correction effect of ion inertial motion along the

magnetic field lines was later discussed by Kalita and Kalita (1986). For both above cases, three-dimensional stabilities of SKAWs were studied by Das et al. (1989) and Ghosh and Das (1994), respectively.

In recent decade, some great progresses of SKAWs have been made in both theory and observation motivated by satellite in situ measurements in space plasmas. Some recent studies of observations by space satellites (e.g. FREJA and FAST) showed that the physical nature of strong electric spikes in the auroral ionosphere and magnetosphere, which are characterized by the perturbed electric and magnetic fields of  $\Delta E \sim$  hundreds of mV and  $\Delta B \sim$  tens of nT, the spatial scale of  $\Delta x \sim$  hundreds of m, and the time duration of  $\Delta t \sim$  tens of ms, can be explained in terms of SKAW because (1) the ratio of the perturbed transversal electric to magnetic fields  $\Delta E/\Delta B \sim v_A$ ; (2) the perpendicular scale size  $\lambda_{\perp} \sim$  several  $\lambda_e$ ; and (3) they are frequently accompanied by strong density fluctuations ( $\Delta n/n \sim 50\%$ ) (Louarn et al., 1994; Volwerk et al., 1996; Wu et al., 1996a; Wu et al., 1997; Huang et al., 1997; Chaston, et al., 1999). Owing to these observations motivating, some new processes in theoretical studies of SKAWs have also been made by many authors. Wu et al. (1995) first derived an exactly analytical solution of SKAWs with an arbitrary amplitude in a low- $\beta$  plasma with  $\beta/2 \ll Q$ . Taking the cold ion approximation of  $T_i \ll T_e$ , Wu et al. (1996b) extended the works of Hasegawa and Mima (1976) (for  $\alpha \gg 1$ ) and Shukla et al. (1982) (for  $\alpha \ll 1$ ) to more general ranges for the parameter  $\alpha$  and took account of both the electron temperature and inertia terms simultaneously. They obtained a generalized Sagdeev equation and found a general criterion for the existence of SKAWs. Wu and Wang (1996) also studied SKAWs on the ion acoustic velocity branch. In recent, the correction effect of finite ion temperature was considered by Roychoudhury and Chatterjee (1998) and Wang et al. (1998). Cheng et al. (2000) and Roychoudhury (2002) studied SKAWs in a dust plasma and in a non-thermal plasma, respectively. The comparison of SKAWs with the observations in detail has also been presented by some authors (Wu et al., 1996a, b, 1997; Seyler et al., 1998; Seyler and Wu, 2001).

Some further analyses of data, however, revealed clearly that electron collisional dissipation could considerably affect the structure and evolution of SKAWs. For example, on the basis of the analysis for a large number SKAW events observed by FREJA, Wahlund et al. (1994a) found that these events could be classified as three different observational phases, which were possibly responsible for three different stages in the dynamical evolution of SKAWs due to the dissipative effect that is caused by the collision of electrons and turbulent ion acoustic waves with the effective collisional frequency  $\nu_{\text{eff}} = \omega_{pe} (W_T/n_0 T_e)$  (Hasegawa, 1975), where  $\omega_{pe}$  is the electron plasma frequency,  $W_T$  is the energy density of the turbulent ion acoustic waves, and  $n_0$  and  $T_e$  are the background electron density and temperature, respectively. In a most recent work, Wu (2003a, b) generalized the equation set governing the dynamics of SKAWs to include dissipative effects due to the electron collision, and presented

a new model of nonlinear KAWs, called dissipated SKAW (DSKAW hereafter). The results showed that the DSKAW could produce a local shock-like structure in its density profile and a net parallel electric potential drop over the shock-like structure. In particular, it was demonstrated that the electric potential drop of the DSKAW could accelerate electrons efficiently to orders of the local Alfvén speed. This suggests that DSKAWs can provide an efficient acceleration mechanism for energetic electrons, which can frequently be encountered in various space and cosmic plasma environments. The application of the DSKAW acceleration mechanism in the auroral electron acceleration has been proposed by Wu and Chao (2003, 2004).

Stasiewicz et al. (2000a) presented a comprehensive review of KAWs in the auroral plasma, including their identification in satellite in situ observations and their relationship with the auroral plasma dynamics. In this paper, we review recent progress in theoretical study of KAWs, especially in their nonlinear coherent structures. After introducing the two-fluid theory of KAWs and their basic features in Sect. 2, we focus on their nonlinear solitary structures. A general criterion for the existence of SKAWs and an exact analytical solution of SKAWs are presented in Sect. 3. The kinetic dissipation of SKAWs due to the ion-acoustic turbulence, the physics of DSKAWs that evolve from SKAWs due to the dissipation and their role in the electron acceleration are reviewed in Sect. 4. Then the application of the DSKAW acceleration mechanism in the auroral electron acceleration is discussed in Sect. 5. Finally, Sect. 6 devotes to a short perspective of KAWs.

## 2 Two-fluid theory of KAW

In a plasma magnetized by a homogeneous background magnetic field  $\mathbf{B}_0$  along the  $z$  direction, the complete two-fluid equation set can be written as follows:

$$(\partial_t + \mathbf{v}_e \cdot \nabla) n_e + n_e \nabla \cdot \mathbf{v}_e = 0, \quad (8)$$

$$(\partial_t + \mathbf{v}_i \cdot \nabla) n_i + n_i \nabla \cdot \mathbf{v}_i = 0, \quad (9)$$

$$(\partial_t + \mathbf{v}_e \cdot \nabla) \mathbf{v}_e = -\omega_{ce} \mathbf{v}_e \times \mathbf{e}_z - \frac{e}{m_e} (\mathbf{E} + \mathbf{v}_e \times \mathbf{B}) - \frac{1}{m_e n_e} \nabla p_e, \quad (10)$$

$$(\partial_t + \mathbf{v}_i \cdot \nabla) \mathbf{v}_i = \omega_{ci} \mathbf{v}_i \times \mathbf{e}_z + \frac{e}{m_i} (\mathbf{E} + \mathbf{v}_i \times \mathbf{B}) - \frac{1}{m_i n_i} \nabla p_i, \quad (11)$$

$$\nabla \times \mathbf{E} = -\partial_t \mathbf{B}, \quad (12)$$

$$\nabla \times \mathbf{B} = \mu_0 e (n_i \mathbf{v}_i - n_e \mathbf{v}_e) + \epsilon_0 \mu_0 \partial_t \mathbf{E}, \quad (13)$$

$$(\partial_t + \mathbf{v}_e \cdot \nabla) (p_e n_e^{-\gamma_e}) = 0, \quad (14)$$

$$(\partial_t + \mathbf{v}_i \cdot \nabla) (p_i n_i^{-\gamma_i}) = 0, \quad (15)$$

where  $e$ ,  $\mu_0$ , and  $\epsilon_0$  are the elementary charge, the permittivity, and the permeability in the free space, respectively,  $m_{e(i)}$ ,

$n_{e(i)}$ ,  $\omega_{ce(i)}$ ,  $\mathbf{v}_{e(i)}$ , and  $p_{e(i)}$  are the electron (ion) mass, density, gyrofrequency, velocity, and thermal pressure, respectively,  $\mathbf{e}_z$  is the unit of vector along the  $z$  axis,  $\mathbf{E}$  and  $\mathbf{B}$  are the perturbed electric and magnetic fields, respectively,  $\partial_t$  is the partial derivative with respect to time  $t$ ,  $\nabla$  is the 3-dimension spatial gradient operator, and  $\gamma_{e(i)}$  is the polyindex of the electron (ion) state equation.

For one dimensional plane wave with the wave vector  $(k_x, 0, k_z)$  and the frequency  $\omega$  in which all perturbed physical quantities vary in the following form:

$$\delta f \propto e^{-i(\omega t - k_x x - k_z z)}, \quad (16)$$

after linearizing Eqs. (8)–(15) one can obtain a general dispersion equation as follows (Wu, 2003c):

$$\Lambda_{xx} \Lambda_{yy} \Lambda_{zz} + \Lambda_{xx} \Lambda_{yz}^2 + \Lambda_{zz} \Lambda_{xy}^2 - \Lambda_{yy} \Lambda_{xz}^2 + 2\Lambda_{xy} \Lambda_{xz} \Lambda_{yz} = 0, \quad (17)$$

where

$$\Lambda_{xx} = \frac{M_z^2 - \gamma_i \beta_i / 2}{\Delta_i} - \frac{1}{M_z^2} + Q \frac{Q M_z^2 - \gamma_e \beta_e / 2}{\Delta_e} + G, \quad (18)$$

$$\Lambda_{xy} = -\Lambda_{yx} = \frac{i}{\delta \omega} \left( \frac{M_z^2 - \gamma_i \beta_i / 2}{\Delta_i} - \frac{Q M_z^2 - \gamma_e \beta_e / 2}{\Delta_e} \right), \quad (19)$$

$$\Lambda_{xz} = \Lambda_{zx} = \left( \frac{1}{M_z^2} + \frac{\gamma_i \beta_i / 2}{\Delta_i} + Q \frac{\gamma_e \beta_e / 2}{\Delta_e} \right) \tan \theta, \quad (20)$$

$$\Lambda_{yy} = -\frac{\delta k^2}{\delta \omega^2} + \frac{\delta \omega^2 - (\gamma_i \beta_i / 2) \delta k^2}{\Delta_i \delta k_{\parallel}^2} + Q \frac{Q \delta \omega^2 - (\gamma_e \beta_e / 2) \delta k^2}{\Delta_e \delta k_{\parallel}^2} + G, \quad (21)$$

$$\Lambda_{yz} = -\Lambda_{zy} = -\frac{i}{\delta \omega} \left( \frac{\gamma_i \beta_i / 2}{\Delta_i} - \frac{\gamma_e \beta_e / 2}{\Delta_e} \right) \tan \theta, \quad (22)$$

$$\Lambda_{zz} = -\frac{\tan^2 \theta}{M_z^2} - \frac{1 - [\delta \omega^2 - (\gamma_i \beta_i / 2) \delta k_{\perp}^2]}{\Delta_i \delta k_{\parallel}^2} - \frac{1 - Q [Q \delta \omega^2 - (\gamma_e \beta_e / 2) \delta k_{\perp}^2]}{\Delta_e \delta k_{\parallel}^2} + G, \quad (23)$$

where  $\delta \omega = \omega / \omega_{ci}$ ,  $\delta k = \lambda_i k = 2\pi \lambda_i / \lambda$ ,  $\delta k_{\perp} = \delta k_x = \delta k \sin \theta$ ,  $\delta k_{\parallel} = \delta k_z = \delta k \cos \theta$ ,  $G = v_A^2 / c^2$ , and  $c$  is the light speed. In the above expressions,  $i$  is the unit imaginary number,  $M_z = \omega / (k_{\parallel} v_A) = \omega / \omega_A$  is the parallel phase speed in  $v_A$ ,  $\beta_{e(i)}$  is the ratio of the electron (ion) thermal to magnetic pressures,  $\theta$  is the angle between the wave vector  $\mathbf{k}$  and the background magnetic field  $\mathbf{B}_0$  (i.e. the  $z$  axes), and  $\Delta_i$  and  $\Delta_e$  are, respectively,

$$\Delta_i = M_z^2 - \frac{\gamma_i \beta_i}{2} - M_z^2 \left( \delta \omega^2 - \frac{\gamma_i \beta_i}{2} \delta k^2 \right) \quad (24)$$

and

$$\Delta_e = Q M_z^2 - \frac{\gamma_e \beta_e}{2} - Q^2 M_z^2 \left( Q \delta \omega^2 - \frac{\gamma_e \beta_e}{2} \delta k^2 \right). \quad (25)$$

For Alfvénic waves with frequency  $\omega \sim \omega_A$ , one has  $M_z = \omega / \omega_A \sim 1$ . In the long wavelength limit of  $\lambda / \lambda_i \gg 1$ , hence  $\delta k^2 = 4\pi^2 \lambda_i^2 / \lambda^2 \ll 1$  and  $\delta \omega^2 = M_z^2 \delta k^2 \cos^2 \theta \ll 1$ , the first two terms on the left hand side of the dispersion equation (17), which are proportional to the factor  $1 / \delta \omega^2 \gg 1$ , are much larger than the other three terms. Therefore the resulting dispersion equation can be simplified as

$$\Lambda_{xx} \left( \Lambda_{yy} \Lambda_{zz} + \Lambda_{yz}^2 \right) = 0. \quad (26)$$

Neglecting the small quantities of  $Q$  and  $G$ , as approximated usually in the ideal MHD model, the first factor of Eq. (26),  $\Lambda_{xx} = 0$ , gives the dispersion relation for the shear Alfvén wave  $\omega = v_A k_{\parallel} = \omega_A$ , and the second factor,  $\Lambda_{yy} \Lambda_{zz} + \Lambda_{yz}^2 = 0$ , gives the dispersion relations for the other two ideal MHD modes (i.e. the fast and slow magnetosonic waves)

$$M^2 v_A^2 \equiv \omega^2 / k^2 = (1/2) \left( v_A^2 + v_s^2 \right) \left( 1 \pm \sqrt{1 - 4v_A^2 v_s^2 \cos^2 \theta / (v_A^2 + v_s^2)^2} \right),$$

where  $v_s = \sqrt{(\gamma_i \beta_i + \gamma_e \beta_e) / 2} v_A$  is the sound speed, and  $M = \omega / (k v_A)$  is the phase speed of the waves in  $v_A$ .

When the wavelength becomes so short to approach the electron inertial length  $\lambda_e$ , or the effective ion gyroradius  $(1 + \gamma_i T_i / \gamma_e T_e) \rho_s$ , one has

$$\delta k^2 = \frac{1}{Q} \lambda_e^2 k^2 \sim \frac{1}{Q} \gg 1, \quad (27)$$

or

$$\delta k^2 = \frac{2}{\beta_{e(i)}} \rho_{s(i)}^2 k^2 \sim \frac{2}{\beta_{e(i)}} \gg 1 \quad (28)$$

for the cases in a low- $\beta$  plasma. Meanwhile, one also has

$$\frac{1}{\cos^2 \theta} = \frac{M_z^2}{\delta \omega^2} \delta k^2 = \left( \frac{\omega c_i}{\omega_A} \right)^2 \delta k^2 > \delta k^2 \gg 1. \quad (29)$$

This implies that the short wavelength ( $\lambda < \lambda_i$ ) and low frequency ( $\omega < \omega_{ci}$ ) Alfvénic waves can propagate only at the direction quasi-perpendicular to the background magnetic field. For the short wavelength case of  $\delta k^2 \gg 1$ , the magnitude comparison in the dispersion equation (17) shows that the first and fourth terms with the order of  $\delta k^4$  are much larger than the other three terms. Therefore the dispersion equation can be simplified as

$$\Lambda_{yy} \left( \Lambda_{xx} \Lambda_{zz} - \Lambda_{xz}^2 \right) = 0 \implies \Lambda_{xx} \Lambda_{zz} - \Lambda_{xz}^2 = 0. \quad (30)$$

To reserve only the terms of the  $\delta k^4$  order, it leads to the dispersion equation as follows:

$$\left( \frac{1}{1 - \delta \omega^2 + (\gamma_i \beta_i / 2) \delta k^2} - \frac{1}{M_z^2} \right) \left( \frac{\tan^2 \theta}{M_z^2} + \frac{M_z^2}{Q M_z^2 - \gamma_e \beta_e / 2} \frac{1}{\delta \omega^2} \right) + \frac{\tan^2 \theta}{M_z^4} = 0. \quad (31)$$

The corresponding dispersion relation is (Wu, 2003c):

$$M_z^2 = \frac{1 + (\gamma_i \beta_i / 2) \delta k^2 + (\gamma_e \beta_e^2 / 2) \delta k_{\perp}^2}{1 + Q \delta k_{\perp}^2 + \delta k_{\parallel}^2}, \quad (32)$$

to restore the dimension, that is,

$$\omega^2 = \frac{1 + \rho_s^2 k_{\perp}^2 + \rho_i^2 k_{\perp}^2}{1 + \lambda_e^2 k_{\perp}^2 + \lambda_i^2 k_{\parallel}^2} \omega_A^2, \quad (33)$$

where the polyindex  $\gamma_{e(i)} = 1$  has been used, the small quantity  $\rho_i^2 k_{\parallel}^2$  has been neglected, and  $k_{\perp} = k \sin \theta$  and  $k_{\parallel} = k \cos \theta$  are the perpendicular and parallel wave numbers, respectively.

In the KAW dispersion relation of Eq. (33), the terms  $\rho_{s(i)}^2 k_{\perp}^2$  in the numerator and  $\lambda_e^2 k_{\perp}^2$  ( $\lambda_i^2 k_{\parallel}^2$ ) in the denominator represent the effect modified by the electron (ion) temperature and inertia on the shear Alfvén wave, respectively. Neglecting the ion contributions of them one obtains the dispersion relation of KAWs in Eq. (3). When neglecting further the electron contributions the dispersion relation of the ideal shear Alfvén wave  $\omega = \omega_A$  can be recovered as expected.

There are a few points to be worth noticing in the dispersion equation (30). The one is that the dispersive modification of KAWs can be attributed to the coupling of the shear Alfvén wave with electrostatic fluctuations, which are caused by the local charge separation when electrons and ions moving in small-scale wave fields. The second is that the low  $\beta$  condition of  $\beta \ll 1$  is one of necessary conditions to obtain the KAW dispersion relation of Eq. (33) (or Eq. 3). In fact, in a high  $\beta$  plasma of  $\beta_{i(e)} \sim 1$  the ion (or ion-acoustic) gyroradius,  $\rho_{i(s)}$  is comparable to the ion inertial length  $\lambda_i$ , and therefore the term  $\Lambda_{xx} \Lambda_{yz}^2$  in the general dispersion equation (17), which causes the magnetosonic waves (i.e. the compressive Alfvén waves), can no longer be neglected, and the result is the coupling of KAW with the magnetosonic waves. The third is that KAW is a quasi-perpendicular propagating mode such that its phase speed  $v_p = \omega / k \simeq v_{p\perp} \ll v_A$ , although the parallel phase speed  $v_{p\parallel} \sim v_A$ . This implies that KAWs appear to be almost nonpropagation in comparison with the local Alfvén speed and the result can just explain why the observed ratio of the wavelength ( $\sim$  hundreds of m) to the period ( $\sim$  tens of ms) of KAWs is always much less than the local Alfvén speed (Louarn et al., 1994; Volwerk et al., 1996; Chaston et al., 1999).

The most important property of KAWs is that their perturbed electric field has a non-zero parallel component  $E_z$ . The polarization of KAWs can be described in the following forms

$$\frac{E_z}{E_x} = \frac{\Lambda_{xx}}{\Lambda_{xz}} = \frac{(x^2 + \alpha^2 T_i / T_e) - \alpha^2 (x^2 + \alpha_I^2 - 1)}{(x^2 + \alpha^2 T_i / T_e) (x^2 + 1) - \alpha^2 (\alpha_I^2 - 1)} \cot \theta \quad (34)$$

and

$$\frac{E_x}{B_y} = \frac{\omega \Lambda_{xz}}{k_x \Lambda_{xx} + k_z \Lambda_{xz}} = \frac{(x^2 + \alpha^2 T_i / T_e) (x^2 + 1) - \alpha^2 (\alpha_I^2 - 1)}{x^2 (x^2 + \alpha^2 + \alpha^2 T_i / T_e)} M_z v_A, \quad (35)$$

where  $x=1/(\lambda_e k_\perp)=\lambda_\perp/(2\pi\lambda_e)$  is the perpendicular wavelength in units of  $2\pi\lambda_e$ , and the dimensionless parameter  $\alpha_I$  is given by

$$\alpha_I=\sqrt{1+Q^{-1}\cot^2\theta}. \quad (36)$$

Neglecting the effects of the ion inertia (i.e.  $\alpha_I=1$ ) and temperature (i.e.  $T_i/T_e=0$ ), Eqs. (34) and (35) lead to

$$\frac{E_z}{E_x}=\frac{1-\alpha^2}{1+x^2}\cot\theta \quad (37)$$

and

$$\frac{E_x}{B_y}=\frac{x^2+1}{x^2+\alpha^2}M_z v_A=\frac{1}{M_z}v_A, \quad (38)$$

respectively. For a low- $\beta$  ( $\alpha<1$ ) plasma with warm ions ( $T_i/T_e>1$ ) that is the case of the auroral plasma, Eqs. (34) and (35) give (Stasiewicz et al., 2000b)

$$\frac{E_z}{E_x}=\frac{1}{1+x^2}\cot\theta=\frac{k_\parallel k_\perp \lambda_e^2}{1+k_\perp^2 \lambda_e^2} \quad (39)$$

and

$$\frac{E_x}{B_y}=\frac{x^2+1}{x^2}M_z v_A=\sqrt{(1+k_\perp^2 \lambda_e^2)(1+k_\perp^2 \rho_i^2)}v_A, \quad (40)$$

respectively, where the dimension has been restored in the last expressions.

### 3 SKAW: general criterion for existence and analytical solution

#### 3.1 Sagdeev equation for SKAW

The dispersion of KAWs, combined with the nonlinear steepening, may lead to the formation of SKAWs. In the low frequency approximation of  $\omega\ll\omega_{ci}$ , the two-fluid equation set of Eqs. (8)–(15) can be greatly simplified through decoupling of the drift motion perpendicular to  $\mathbf{B}_0$  from the free motion parallel to  $\mathbf{B}_0$ :

$$\partial_t n_e + \partial_z (n_e v_{ez}) + \nabla_\perp \cdot (n_e \mathbf{v}_{e\perp}) = 0, \quad (41)$$

$$\partial_t n_i + \partial_z (n_i v_{iz}) + \nabla_\perp \cdot (n_i \mathbf{v}_{i\perp}) = 0, \quad (42)$$

$$(\partial_t + \mathbf{v}_e \cdot \nabla) v_{ez} = -\frac{e}{m_e} E_z - v_{Te}^2 \partial_z \ln n_e, \quad (43)$$

$$(\partial_t + \mathbf{v}_i \cdot \nabla) v_{iz} = \frac{e}{m_i} E_z - v_{Ti}^2 \partial_z \ln n_i, \quad (44)$$

$$\mathbf{v}_{i\perp} \simeq \frac{\mathbf{E} \times \mathbf{e}_z}{B_0} + \frac{\partial_t \mathbf{E}_\perp}{\omega_{ci} B_0} \quad (45)$$

$$\mathbf{v}_{e\perp} \simeq \frac{\mathbf{E} \times \mathbf{e}_z}{B_0} \quad (46)$$

$$\nabla \times \mathbf{E} = -\partial_t \mathbf{B}_\perp, \quad (47)$$

$$\nabla \times \mathbf{B}_\perp \simeq \mu_0 e (n_i \mathbf{v}_{i\perp} - n_e v_{ez}), \quad (48)$$

$$n_i \simeq n_e = n, \quad (49)$$

where the subscript “ $\perp$ ” represents the vector perpendicular to the background magnetic field  $\mathbf{B}_0$ . In the above equation set of Eqs. (41)–(49), the constant temperature assumption of  $\gamma_{i(e)}=1$  has been used for the state equations (14) and (15), and Eq. (49) represents the charge neutrality condition which is valid when  $v_A \ll c$ .

For one-dimensional plane wave propagating at the direction of  $(k_x, 0, k_z)$ , we look for steady nonlinear structures of the wave in the traveling-wave frame with co-moving coordinates

$$\eta = k_x x + k_z z - \omega t. \quad (50)$$

The above two-fluid equation set of Eqs. (41)–(49), then, can be rewritten as follows:

$$d_\eta [n (k_z v_{ez} - M)] = 0, \quad (51)$$

$$d_\eta [n (k_z v_{iz} - k_x M d_\eta E_x - M)] = 0, \quad (52)$$

$$(k_z v_{ez} - M) d_\eta v_{ez} = - (E_z + \alpha^2 k_z d_\eta \ln n), \quad (53)$$

$$(k_z v_{iz} - k_x M d_\eta E_x v_{ix} - M) d_\eta v_{iz} = Q E_z, \quad (54)$$

$$k_x d_\eta (k_z E_x - k_x E_z) = -M n v_{ez}, \quad (55)$$

where space  $x, y, z$ , time  $t$ , plasma number density  $n_{e(i)}$ , velocity  $\mathbf{v}_{e(i)}$ , and perturbed electric and magnetic fields  $\mathbf{E}$  and  $\mathbf{B}_\perp$  are normalized by  $\lambda_e$ ,  $\lambda_e/v_A = \sqrt{Q}/\omega_{ci}$ ,  $n_0$ ,  $v_A$ ,  $\sqrt{Q}v_A B_0$  and  $\sqrt{Q}B_0$ , respectively,  $M$  is the phase speed in  $v_A$ ,  $n_0$  in the unperturbed density, and  $d_\eta$  denotes the derivative with respect to  $\eta$ . In the derivation of the above equation set of Eqs. (51)–(55), the cool ion assumption of  $T_i \ll T_e$  and the charge neutrality condition of  $n_e = n_i = n$  have been used.

Integrating Eqs. (51)–(55) by use of the localized boundary conditions for solitary wave solutions:  $n=1$ ,  $v_{ez}=v_{iz}=d_\eta=0$  for  $|\eta| \rightarrow \infty$ , we can obtain the nonlinear Sagdeev equation governing SKAWs as follows

$$\frac{1}{2} \left( \frac{dn}{d\eta} \right)^2 + K(n; M_z, k_x) = 0, \quad (56)$$

where the Sagdeev potential  $K(n; M_z, k_x)$  is given by (Wu et al., 1996b)

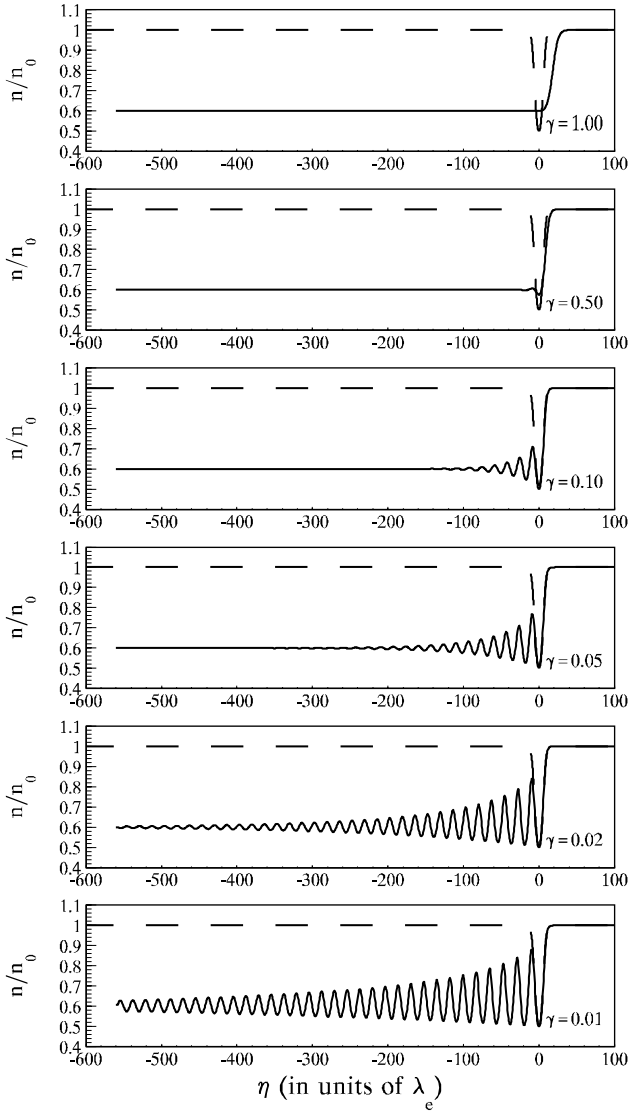
$$K(n; M_z, k_x) = -\frac{1+Q}{M_z^2 k_x^2} \frac{n^4}{(1-\alpha M_z^{-2} n^2)^2} \left[ \Phi_1 + \frac{\alpha}{M_z^2} \Phi_2 + Q \frac{\alpha}{M_z^2} \left( \Phi_3 + \frac{\alpha}{M_z^2} \Phi_4 \right) \right] \quad (57)$$

with

$$\Phi_1 = (n-1)^2 (3M_z^2 - 1 - 2n^{-1})/6, \quad (58)$$

$$\Phi_2 = -n(n-1)(M_z^2 n + 1) + (M_z^2 + 1)n^2 \ln n, \quad (59)$$

$$\Phi_3 = -(n^2 - 1)/2 + (M_z^2 + 1)(n-1)n - M_z^2 n^2 \ln n, \quad (60)$$



**Fig. 1.** Density distributions of DSKAWs (normalized to  $n_0$ ) with  $\theta=89^\circ$ ,  $M_z=1.29$ , and  $\gamma=1, 0.5, 0.1, 0.05, 0.02$ , and  $0.01$ , from the top down. Dashed lines represent the density of SKAWs with the same  $\theta$  and  $M_z$  as DSKAWs but  $\gamma=0$ .

$$\Phi_4 = M_z^2 n^2 (n^2 - 1) / 2 - (1 + M_z^2) (n - 1) n^2 + n^2 \ln n, \quad (61)$$

and  $M_z \equiv M/k_z$  is the parallel phase speed in  $v_A$ .

Neglecting the terms proportional to  $Q$ , namely  $\Phi_3$  and  $\Phi_4$ , in Eq. (57), the Sagdeev potentials obtained by Hasegawa and Mima (1976) and Shukla et al. (1982) can be recovered from Eq. (57) in the limits of  $\alpha \gg 1$  and  $\alpha \ll 1$ , respectively.

### 3.2 General criterion for existence of SKAW

In consideration of analogy between the Sagdeev equation (56) and the motion equation of a classical “particle” in the Sagdeev potential well  $K(n; M_z, k_x)$  of Eq. (57), where  $\eta$  and  $n$  represent “time” and “space”, respectively, SKAW solution of Eq. (56) indicates that the “particle” reciprocates

between  $n=1$  and  $n=n_m$  but with an infinite period, where  $n_m$  is the maximum (or minimum) of the SKAW density profile determined by the root of  $K(n; M_z, k_x)$  nearest  $n=1$

$$K(n; M_z, k_x)|_{n=n_m \neq 1} = 0 \implies n_m = n_m(M_z, \alpha). \quad (62)$$

In consequence, the necessary conditions for the existence of SKAW solutions of Eq. (56) are

- (i)  $K(n) < 0$ , for  $n$  between  $n=1$  and  $n=n_m$ ,
- (ii)  $d_\eta n = 0$ , at  $n=1$  and  $n=n_m$ ,
- (iii)  $d_\eta^2 n = 0$ , at  $n=1$ , and  $(n_m - 1)d_\eta^2 n < 0$ , at  $n=n_m$ .

The condition (i) means that the “particle” moves in a binding potential well. The condition (ii) implies that the “motion velocity”  $d_\eta n$  of the “particle” reaches zero at the potential well boundaries of  $n=1$  and  $n=n_m$  (otherwise the “particle” escapes from the boundaries). And the condition (iii) connotes the “particle” is reflected back at the boundary  $n=n_m$ , that is, the “acceleration” (at  $n=n_m$ )  $d_\eta^2 n|_{n=n_m} \neq 0$  and points at the other boundary at  $n=1$ , where the “acceleration”  $d_\eta^2 n|_{n=1} = 0$  (otherwise the “particle” reciprocates with a finite period). The combination of these three conditions gives the general criterion for the SKAW existence.

In the small amplitude limit of  $|N| \leq |N_m| \ll 1$ , where  $N=1-n$  and  $N_m=1-n_m$  to be the density amplitude of SKAWs, the Sagdeev potential  $K(n; M_z, k_x)$  of Eq. (57) becomes, at the lowest order,

$$K(N; \Delta_M, k_x) = -\frac{(1+Q)(1-Q\alpha)}{3k_x^2(1-\alpha)} \left( \frac{3}{2} \Delta_M - N \right) N^2, \quad (63)$$

where  $\Delta_M = M_z^2 - 1$ . The density amplitude  $N_m$  can be obtained from  $K(N; \Delta_M, k_x) = 0$  as follows

$$N_m = 3\Delta_M/2. \quad (64)$$

And the Sagdeev equation (56) may be written as:

$$\left( \frac{dN}{d\eta} \right)^2 = \frac{4}{D^2} N^2 \left( 1 - \frac{N}{N_m} \right), \quad (65)$$

where

$$D = k_x \sqrt{\frac{6(1-\alpha)/N_m}{(1+Q)(1-Q\alpha)}}. \quad (66)$$

This is the standard form of the KdV equation in the traveling-wave frame, which has a soliton solution

$$N = N_m \operatorname{sech}^2(\eta/D) \quad (67)$$

with the characteristic width  $D$ .

### 3.3 Exact analytical solution of SKAW

For the case of  $\alpha \ll 1$ , the Sagdeev equation (56) can be rewritten as

$$\frac{1}{2} \left( \frac{dN}{d\eta} \right)^2 + K(N; N_m, k_x) = 0, \quad (68)$$

where  $N=1-n$  and the amplitude  $N_m=1-n_m$  is given by

$$N_m = \frac{M_z^2 - 1}{M_z^2 - 1/3} \implies M_z^2 = \frac{1 - N_m/3}{1 - N_m}. \quad (69)$$

The Sagdeev potential may be simplified as (Wu et al., 1995)

$$K(N) = -R^2(N)P(N), \quad (70)$$

where

$$R(N) = \frac{\sqrt{2}}{k_x D} (1-N)N, \quad (71)$$

and

$$P(N) = (1-N/N_m)(1-N), \quad (72)$$

with

$$D = \sqrt{\frac{6}{1+Q} \frac{1-N_m/3}{N_m}}. \quad (73)$$

In general, when  $R(N)$  is a rational function of  $N$  the solution of Eq. (68) may be expressed by an elliptic (elementary) function if  $P(N)$  is a polynomial lower than the fifth (third) power of  $N$  (see the appendix in Wu et al., 1995). Here,  $P(N)$  is a polynomial with the second power of  $N$ . Hence, the solution of Eq. (68) may be expressed by an elementary function as follows (Wu et al., 1995)

$$\eta = \pm k_x D \left( \frac{N_m}{1-N_m} \sqrt{\frac{1-N/N_m}{1-N}} + \ln \sqrt{\frac{N_m}{N} \frac{\sqrt{1-N/N_m} + \sqrt{1-N}}{\sqrt{1-N_m}}} \right), \quad (74)$$

or i.e.

$$N = N_m \frac{1 - \tanh^2 Y}{1 - N_m \tanh^2 Y}, \quad (75)$$

where

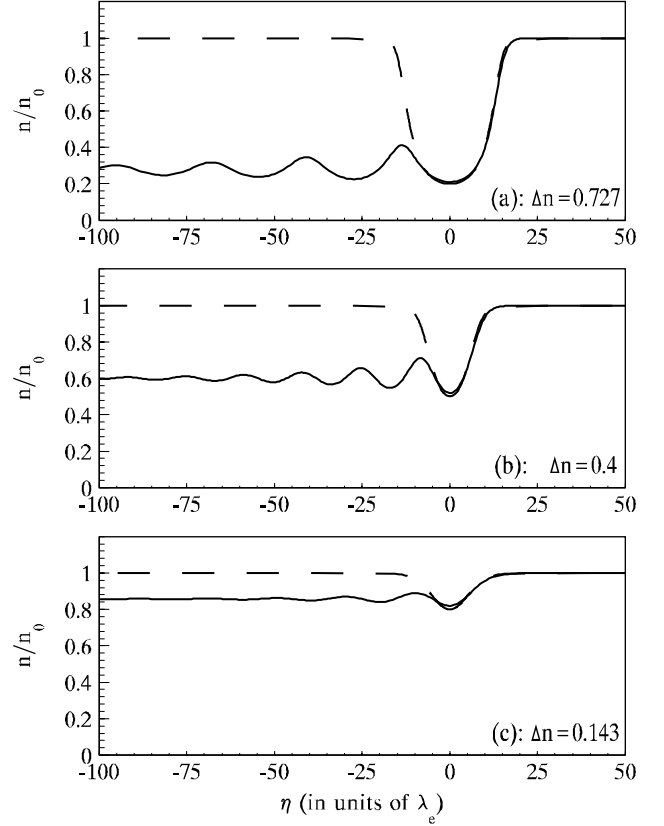
$$Y \equiv \frac{|\eta|}{k_x D} - \frac{N_m}{1-N_m} \sqrt{\frac{1-N/N_m}{1-N}}, \quad (76)$$

and the boundary condition  $N = N_m$  at  $\eta = 0$  has been used without loss of generality. This solution describes a dip density soliton with an amplitude  $N_m$  and a symmetrical center at  $\eta = 0$ . The profiles of density solitons for SKAWs are showed by dashed lines in Figs. 1 and 2, and they have the characteristic width of  $\sim 10\lambda_e$ . Their perturbed electromagnetic fields can be expressed as follows (Wu et al., 1995)

$$E_z = \pm \frac{(1+Q)D}{3} \left( \frac{N_m}{1-N_m} \right)^2 (1 - \tanh^2 Y) \tanh Y \cot \theta, \quad (77)$$

$$E_x = \pm (1+Q)D \frac{N_m}{(1-N_m)^2} \left( 1 - \frac{N_m}{3} \tanh^2 Y \right) \tanh Y, \quad (78)$$

$$B_y = \pm (1+Q)D \frac{N_m}{1-N_m} \sqrt{\frac{1-N_m/3}{1-N_m}} \tanh Y. \quad (79)$$



**Fig. 2.** Density distributions of DSKAWs (normalized to  $n_0$ ) with  $\gamma = 0.1$ ,  $\theta = 89^\circ$ , and  $M_z = 1.91$  (a), 1.29 (b), and 1.08 (c), where  $\Delta n$  is the density jump of DSKAWs. Dashed lines represent the density of SKAWs with the same  $\theta$  and  $M_z$  as DSKAWs but  $\gamma = 0$ .

## 4 Kinetic dissipation of SKAW: DSKAW and electron acceleration

### 4.1 Dissipative Sagdeev equation and DSKAW

The steady solution of SKAWs presented in the last section is an ideal case without dissipation. In fact, electrons within a SKAW have parallel drift velocities larger than their thermal speed in a low- $\beta$  plasma of  $\beta/2 < Q$  such that the plasma turbulence can easily be excited. In consequence, the turbulence will probably cause the SKAW to be dissipated and to evolve dynamically in the structure. Indeed, some further analyses of the data has also clearly revealed that the observed SKAWs are often correlated with ion acoustic turbulent activities, and that the ion acoustic turbulence can cause their kinetic dissipation and structure evolution. For example, on the basis of analysis of a large number of SKAW events observed by FREJA, Wahlund et al. (1994a) found that these events could be classified into three observational phases. The first is ordinary SKAWs, the second is SKAWs associated with a large amplitude broadband electrostatic component in the ion acoustic wave mode, and the third is SKAWs that have been transformed into electrostatic-like structures.

We propose that these three observational phases of

SKAWs are responsible for three stages in the dynamical evolution of SKAWs due to the ion acoustic turbulent dissipation. In a recent work, Wu (2003a, b) generalized the Sagdeev equation of SKAWs to include the dissipative effect. The “dissipative” Sagdeev equation results in that the DSKAW, which evolves from the SKAW, has a local shock-like density structure.

The effect of the kinetic dissipation caused by the plasma turbulence can be described by the effective collision damping  $-v_{\text{eff}}v_{ez}$  in the electron parallel momentum equation (43), that is,

$$(\partial_t + v_{ez}\partial_z)v_{ez} = -(e/m_e)E_z - v_{\text{eff}}v_{ez} \quad (80)$$

for the low- $\beta$  case of  $\beta \ll Q$ . And the effective collision frequency  $v_{\text{eff}}$  can be estimated by the formula (Hasegawa, 1975)

$$v_{\text{eff}} = \frac{W_T}{n_0 T_e} \omega_{pe} \quad (81)$$

for the ion acoustic turbulence, where  $W_T = \epsilon_0 \delta E_T^2 / 2$  is the wave energy density of the ion acoustic turbulence and  $\delta E_T$  the turbulent electric field strength in the ion acoustic mode.

To include this dissipative effect, the Sagdeev equation of SKAW without dissipation is modified into the following “dissipative” Sagdeev equation (Wu, 2003a)

$$d_\eta \left[ \frac{1}{2} \left( d_\eta \frac{2}{n^2} \right)^2 + K(n) \right] = \gamma \left( d_\eta \frac{2}{n^2} \right)^2 \quad (82)$$

with the Sagdeev potential

$$K(n) = -\frac{3M_z^2 - 1}{6k_x^2 M_z^2} \left( 1 - \frac{1}{n} \right)^2 \left( 1 - \frac{2}{3M_z^2 - 1} \frac{1}{n} \right), \quad (83)$$

and the damping coefficient

$$\gamma \equiv \frac{\sqrt{Q} v_{\text{eff}}}{M \omega_{ci}} = \frac{W_T}{n_0 T_e} \frac{kc}{\omega} = \frac{1}{2} \frac{\epsilon_0 \delta E_T^2}{n_0 T_e} \frac{kc}{\omega}. \quad (84)$$

This “dissipative” Sagdeev equation describes the density structure of DSKAWs. It can be found that when neglecting the dissipative effect (i.e.  $\gamma=0$ ) the integral of Eq. (82) gives the well-known Sagdeev equation (68) for low- $\beta$  SKAWs.

#### 4.2 Shock-like structure of DSKAW

Similar with the Sagdeev equation, the “dissipative” Sagdeev equation (82) also describes the motion of a classical “particle” in the “potential well”  $K(n)$ , but in the present of the damping with the coefficient  $\gamma$ , where “time”  $t = -\eta$ , “space”  $x = 2/n^2$ , and hence “velocity”  $d_t x = -d_\eta (2/n^2)$ . The “particle” can conserve its energy when  $\gamma=0$ . But when  $\gamma>0$ , the “resistance”  $-\gamma (d_t x)^2 = -\gamma [d_\eta (2/n^2)]^2$  will make the “particle” lose gradually its “energy” and stay ultimately at the bottom of the “potential well”  $K(n)$  at  $n = n_d = M_z^{-2}$  when  $t = -\eta \rightarrow +\infty$ .

Figure 1 illustrates the density behaviors of DSKAWs for the damping coefficient  $\gamma=1.00, 0.50, 0.10, 0.05, 0.02$ , and

0.01 from the top down, where the parameters  $k_x = \sin 89^\circ$  (i.e. the quasi-perpendicular propagating angle  $\theta=89^\circ$ ),  $M_z=1.29$  [i.e. the typical density amplitude  $N_m \simeq 0.5$  for low- $\beta$  SKAWs (Wu et al., 1995)] have been used. In comparison with SKAWs, the dashed lines in Fig. 1 show the solutions of SKAWs with the same parameters ( $M_z=1.29$  and  $k_x = \sin 89^\circ$ ) as DSKAWs but  $\gamma=0$ . From Fig. 1, one can find that the density of DSKAW behaves itself like an ordinary “shock” with a density jump  $\Delta n \simeq 0.4$  in the strong dissipative regime of  $\gamma \sim 1$ . In the weakly dissipative regime of  $\gamma \ll 1$ , however, the waveform of DSKAW appears to be a train of oscillatory waves of decreasing amplitude in the downstream of  $\eta < 0$  and converges upon an ultimate downstream density  $n_d \simeq 0.6$  when  $\eta \rightarrow -\infty$  and, as a result, a local shock-like structure with the same density jump  $\Delta n = 1 - n_d \simeq 0.4$  as that in the strong dissipative regime is produced. It is worth mentioning that the ultimate density  $n_d$  is actually independent of special values of  $\gamma$ . In fact, the magnitude of  $\gamma$  affects only the oscillating strength and the converging speed in the manner that the lower  $\gamma$  leads to the stronger oscillation and the lower convergence.

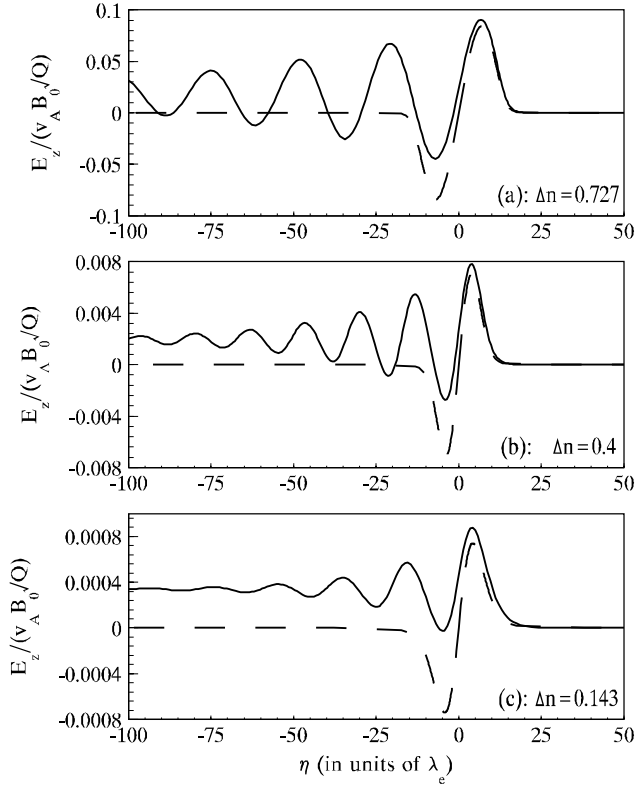
It is also worthy of note that Eq. (82) can recover newly the exactly analytical solution of low- $\beta$  SKAWs when  $\gamma=0$  as expected (Wu et al., 1995). Similar to the case in the ion acoustic shock and soliton (Infeld and Rowlands, 1990), however, the solution of Eq. (82) can not continuously recover SKAW from DSKAW when  $\gamma \rightarrow 0$ . This indicates that SKAW is an ideal critical state without dissipation. Once the dissipation occurs the dynamical system governed by Eq. (82) jumps from the “soliton” structure of SKAW to the “shock-like” structure of DSKAW.

For a typical plasma environment explored by FREJA, one has  $n_0 \sim 10^3 \text{ cm}^{-3}$ ,  $T_e \sim 1 \text{ eV}$ , and the ion acoustic fluctuation field  $\delta E_T \sim 10 \text{ mV/m}$  (Wahlund et al, 1994a, b), and hence  $v_{\text{eff}} \sim 1 \text{ Hz}$ , which is around one tenth of the SKAW’s characteristic frequency (10–20 Hz). In result, the damping coefficient can be estimated as  $\gamma \sim 0.1$ . Three panels of Fig. 2 show the density distributions of DSKAWs with the same  $\gamma=0.1$  and  $\theta=89^\circ$  but  $M_z=1.91, 1.29$ , and  $1.08$  for Figs. 2a, 2b, and 2c, respectively, where the dashed lines represent the solutions of SKAWs with the same  $\theta$  and  $M_z$  as those of DSKAWs but  $\gamma=0$ . From Fig. 2 one can find that the density amplitude of DSKAWs (i.e. the density jump over the shock-like structure)  $\Delta n$  ( $\simeq 0.727, 0.4$ , and  $0.143$  for Figs. 2a, 2b, and 2c, respectively) is always less than the amplitude of the corresponding SKAWs  $N_m = (M_z^2 - 1) / (M_z^2 - 1/3)$  ( $\simeq 0.8, 0.5$ , and  $0.2$  for Figs. 2a, 2b, and 2c, respectively) (Wu et al., 1995).

In fact, from Eqs. (82) and (83) the ultimate density of DSKAWs  $n_d$  when  $\eta \rightarrow -\infty$  can be determined by the value of  $n$  at the bottom of the “potential well”  $K(n)$ , that is,  $n_d \equiv n|_{\eta \rightarrow -\infty} = 1/M_z^2$ . In result, one has

$$\Delta n \equiv 1 - n_d = 1 - M_z^{-2} = N_m - \left( M_z^2 - 1 \right) \left( M_z^2 + 1/3 \right) \left( M_z^2 - 1/3 \right), \quad (85)$$





**Fig. 3.** Distributions of the parallel electric field  $E_z$  of DSKAWs (normalized to  $\sqrt{Q}v_A B_0$ ) with the same  $\gamma$ ,  $\theta$  and  $M_z$  as Fig. 2. Dashed lines represent the parallel electric field of SKAWs with the same  $\theta$  and  $M_z$  as DSKAWs but  $\gamma=0$ .

which is always less than  $N_m$  because the parallel phase speed of low- $\beta$  DSKAWs  $M_z > 1$  (Wu et al., 1995). In addition, from Fig. 2 it is also found that the higher  $M_z$  leads to the stronger oscillation, the slower convergence, and the larger jump.

### 4.3 Electron acceleration by DSKAW

The most attractive property of DSKAW is that it can accelerate electrons efficiently to the order of the local Alfvén speed along the background magnetic field. The distribution of the parallel velocity of electrons in DSKAW can be obtained by integrating the electron continuity equation (51) as follows

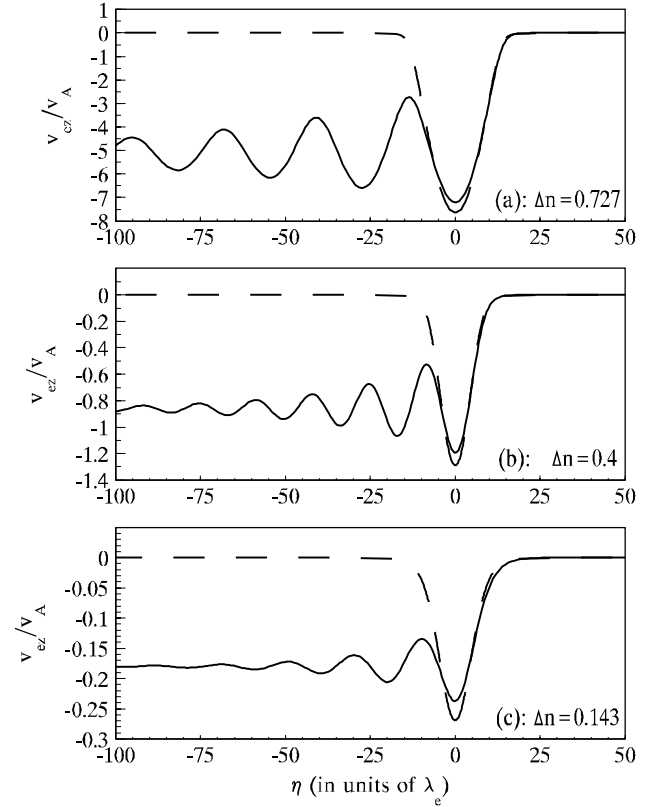
$$v_{ez} = -M_z (n^{-1} - 1) v_A. \quad (86)$$

In the ultimate downstream of DSKAWs, from Eq. (85) the electrons have an escaping velocity along the magnetic field  $v_{ed}$  to be

$$v_{ed} \equiv v_{ez}|_{n=n_d} = -\left(M_z^2 - 1\right) M_z v_A = -\Delta n (1 - \Delta n)^{-3/2} v_A. \quad (87)$$

On the other hand, the parallel electric field in the ultimate downstream of DSKAWs  $E_{zd}$  can be obtained from the parallel electron momentum equation (80) as follows

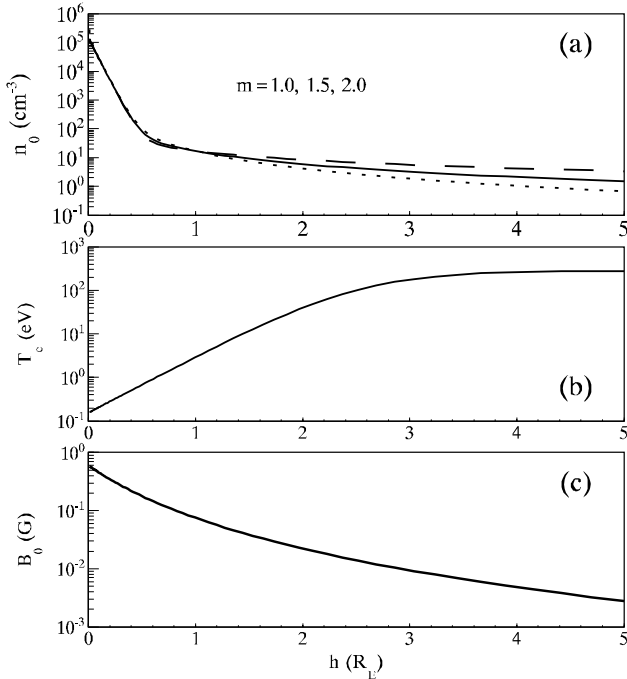
$$E_{zd} \equiv E_z|_{n=n_d} = \gamma k_z M_z^2 \left(M_z^2 - 1\right) \implies (e/m_e) E_{zd} = -v_{\text{eff}} v_{ed}. \quad (88)$$



**Fig. 4.** Distributions of the parallel velocity of electrons  $v_{ez}$  of DSKAWs (normalized to  $v_A$ ) with the same  $\theta$  and  $M_z$  as Fig. 2. Dashed lines represent the electron parallel velocity of SKAWs with the same  $\theta$  and  $M_z$  as DSKAWs but  $\gamma=0$ .

This indicates that the electron collisional “resistance”  $-v_{\text{eff}} v_{ed}$  can be balanced by the parallel electric field force  $-e E_{zd}$  at the downstream of DSKAWs. In other words, it is that this parallel electric field drive the electrons to get over the collisional resistance at the downstream and finally to escape from the downstream with the escaping velocity  $v_{ed}$ .

With the same parameters  $\theta$ ,  $\gamma$ , and  $M_z$  as those in Fig. 2, Figs. 3 and 4 show the distributions of the parallel electric field  $E_z$  (in units of  $\sqrt{Q}v_A B_0$ ) and electron velocity  $v_{ez}$  (in units of  $v_A$ ) in DSKAWs, respectively, where the dashed lines are the solutions of SKAWs similar to Fig. 2. To compare DSKAWs (the solid lines) with SKAWs (the dashed lines) in Figs. 2–4 one can find that their local structures have a similar feature width  $\Delta x \sim 10\lambda_e$  in the direction perpendicular to the background magnetic field. Unlike SKAWs, however, the symmetry in DSKAWs has been destroyed due to the dissipative effect, and the waveform of DSKAWs in the downstream region of  $\eta < 0$  appears to be a train of oscillatory waves of decreasing amplitude and converges upon a new ultimate state when  $\eta \rightarrow -\infty$ . In consequence, a local shock-like structure is formed in DSKAWs. In particular, this shock-like structure of DSKAWs can provide an effective mechanism to accelerate electrons to the “escaping” velocity  $v_{ed}$  of the  $v_A$  order. And the corresponding “escaping” energy  $U_{ed}$  is



**Fig. 5.** Background parameter distributions with altitude  $h$ : (a) the plasma number density  $n_0$  in  $\text{cm}^{-3}$  (where dashed, solid, and dotted lines indicate  $m=1.0, 1.5, 2.0$ , respectively), (b) the electron temperature  $T_e$  in eV, and (c) the Earth magnetic field  $B_0$  in G.

$$U_{ed} \equiv (1/2) m_e v_{ed}^2 = \left( M_z^2 - 1 \right)^2 M_z^2 \epsilon_A = \Delta n^2 (1 - \Delta n)^{-3} \epsilon_A, \quad (89)$$

where  $\epsilon_A \equiv m_e v_A^2 / 2$  is the electron energy with the Alfvén speed. The typical electron escaping energy ranges from one tenth to tens of  $\epsilon_A$ .

## 5 Auroral electron acceleration by DSKAW

### 5.1 Aurora plasma parameters

It has long been known that the auroral phenomena are associated with energetic electrons with energies of 1–10 keV, which impact the ionosphere (McIlwain, 1960; Mozer et al., 1980). The energy distribution of these auroral electrons is characterized by a precipitous decrease above 10 keV, without a rapid increase towards energies below 4 keV (Bryant, 1981). The tendency for the electron intensities to be higher at the smaller pitch angles clearly indicates that the acceleration of the auroral electrons is field-aligned one and that the auroral acceleration region is located at altitudes of  $\sim 5000$ – $12\,000$  km (i.e.  $0.78$ – $1.88 R_E$ ) above the auroral ionosphere (Reiff et al., 1988). Although many plasma processes, e.g. the “anomalous resistivity” (Papadopoulos, 1977), the “weak double layer” (Temerin et al., 1982; Boström et al., 1988; Møllki et al., 1993), and the lower-hybrid waves (Bingham et al., 1984), have been invoked in the auroral plasma physics, our understanding of the microphysics of the au-

roral, especially of the auroral electron acceleration are still far from complete.

In fact, the field-aligned current carried by the auroral electrons is a part of the magnetospheric substorm current system, and therefore the auroral electron acceleration is related to the storage and release of energy during geomagnetic activities. In the complex chain of processes leading from geomagnetic activities to the aurora there must be some mechanism that transfers energy into the auroral electrons. In most recent work, Wu and Chao (2003, 2004) suggested that the DSKAW acceleration for electrons could be a such mechanism accelerating electrons efficiently to the auroral energies of keV in the observed auroral acceleration region. Because of having a parallel electric field it has been proposed by some authors that KAW can probably play an important role in the magnetosphere-ionosphere coupling, especially in the auroral electron acceleration (Hasegawa, 1976; Goertz and Boswell, 1979; Goertz, 1981; Hui and Seyler, 1992; Kletzing, 1994; Thompson and Lysak, 1996). More detailed review of these proposals can be found in Stasiewicz et al. (2000a). Here, we will give a brief review for the application of the DSKAW acceleration mechanism presented in the last section in the aurora electron acceleration (Wu and Chao, 2003, 2004).

The physics of the DSKAW acceleration mechanism is closely related to the background plasma parameters such as the plasma density  $n_0$ , electron temperature  $T_e$ , and the magnetic field  $B_0$ . Therefore, before discussing the auroral electron acceleration by the DSKAW mechanism we review briefly the altitude distribution of the auroral plasma parameters in the auroral zone.

There are two ways to calculate the plasma density, the one is to calculate the ion density and the other is to calculate the electron density. The ion components in auroral plasma consist of mainly oxygen  $\text{O}^+$  originating from the ionosphere and hydrogen  $\text{H}^+$  originating in the magnetosphere. They behave at lower altitudes according to the barometric exponent law and the exospheric power law in the following forms, respectively, (Thompson and Lysak, 1996)

$$n_{\text{O}} = n_{\text{IO}} e^{-h/h_0}, \quad (90)$$

$$n_{\text{H}} = n_{\text{IH}} (1+h)^{-m}, \quad (91)$$

where  $n_{\text{IO(H)}}$  is the oxygen (hydrogen) density at the ionosphere,  $h$  is altitude in units of  $R_E$ ,  $h_0$  is the scale height (usually a few hundredths of  $R_E$ ), and  $m$  is typically between 1 and 3. Indeed, the electron density measurements of the auroral plasma can also be well fitted by the combination of the functional forms of Eqs. (90) and (91) (Lysak and Hudson, 1987; Kletzing and Torbert, 1994; Wu and Chao, 2004) as

$$n_0 = 1.38 \times 10^5 e^{-h/0.06} + 17 h^{-m} \quad (\text{cm}^{-3}), \quad (92)$$

where  $m$  is a free parameter that is used to fit the magnetospheric plasma density. Figure 5a plots the background plasma density versus altitudes  $h$  above the ionosphere for  $m=1.0$  (dashed line),  $1.5$  (solid line), and  $2.0$  (dotted line), respectively.

Although there is little published data to be available for the background electron temperature, it is generally believed that the magnetospheric temperature remarkably differs from that in the ionosphere and that the plasma temperature changes from below 1 eV in the ionosphere up to a few hundreds of eV in the magnetosphere at altitudes of  $h \sim 3\text{--}4R_E$  where the plasma is dominated by the plasma sheet with temperature of  $\sim 300\text{--}600$  eV. In recent works, the analysis based on measurements of temperature up to the highest altitudes measured by S3-3 (Kletzing et al., 1998, 2003) shows that the temperature does not exceed 10 eV at altitudes below 10 000 km ( $h \sim 1.5R_E$ ). The majority of the measurements give a temperature lower than 5 eV at altitudes below  $1R_E$ . Taking the magnetospheric temperature as  $\sim 300$  eV, Wu and Chao (2004) proposed the following formula to model the temperature profile described by Kletzing et al. (1998, 2003)

$$T_e = 100 [1 + \tanh(h - 2.5)]^{3/2} \text{ eV}, \quad (93)$$

which is shown in Fig. 5b.

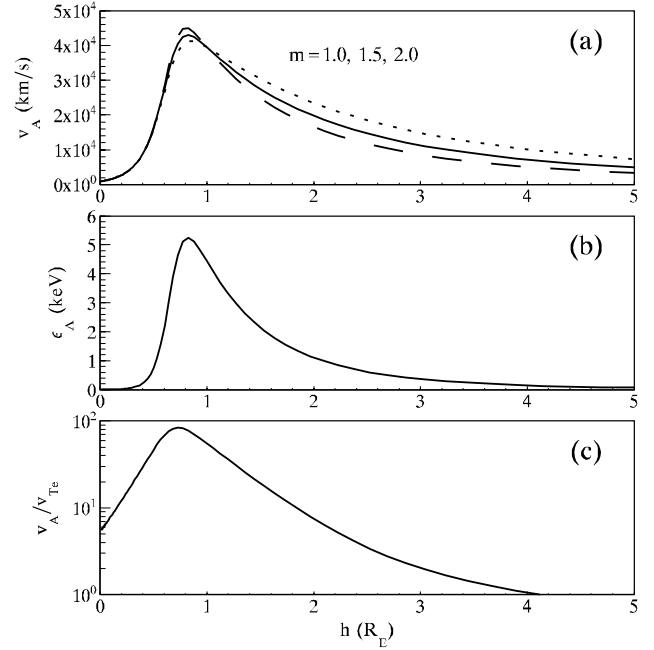
In contrast to the density and temperature cases, the background magnetic field of the auroral plasma is relatively simple and can be modeled, at least within a few Earths radii, by the variation of the Earth dipolar field in the following form:

$$B_0 = 0.6(1+h)^{-3}, \quad (94)$$

where  $B_0$  is in units of G. Fig. 5c shows the variation of the dipolar background field with altitude.

## 5.2 Auroral electron acceleration by DSKAW

In the physics of the DSKAW acceleration mechanism the electron escaping velocity  $v_{ed}$  in Eq. (87) and energy  $U_{ed}$  in Eq. (89) can be determined by the density amplitude  $\Delta n$  and the Alfvén velocity  $v_A$  (for  $v_{ed}$ ) or by the electron Alfvén energy  $\epsilon_A$  (for  $U_{ed}$ ), respectively. Based on the models of the auroral plasma parameters presented in Fig. 5, Figs. 6a and 6b plot  $v_A$  (in units of km/s) and  $\epsilon_A$  (in units of keV) versus  $h$ , respectively, where the dashed, solid, and dotted lines in Fig. 6a indicate  $m=1.0, 1.5,$  and  $2.0$ , respectively. From Fig. 6 it can be found that both  $v_A$  and  $\epsilon_A$  have higher values in the observed acceleration region between  $0.78$  and  $1.88 R_E$ , and reach their maximal values at  $\sim 0.8 R_E$ , which is close to the bottom of the observed acceleration region. In particular,  $\epsilon_A$  is higher than  $0.5$  keV in the altitude range from  $0.5$  to  $2.5 R_E$ , which well covers the observed acceleration region between  $0.78$  and  $1.88 R_E$ , and reaches its maximal value of  $5.25$  keV at the altitude of  $0.82 R_E$ . Since strong DSKAW events with amplitude  $\Delta n > 50\%$  are rather rare in the observations the highest energy that the auroral electrons accelerated by DSKAW can reach is  $\sim 10.5$  keV and they come from the bottom of the acceleration region at  $h \sim 0.8 R_E$ . We et al. (2004) suggested that this could explain the observed energy distribution of the auroral electrons, which is characterized by a precipitous decrease above  $10$  keV (Bryant, 1981), as well as the location of the acceleration occurring between  $0.78$  and  $1.88 R_E$ .



**Fig. 6.** Plots of the Alfvén speed  $v_A$  in km/s (a), energy  $\epsilon_A$  in keV (b), and the speed ratio  $v_A/v_{Te}$  (c) vs altitudes  $h$ . In (a) the dashed, solid, and dotted lines indicate  $m=1.0, 1.5,$  and  $2.0$ , respectively.

On the other hand, the plasma pressure parameter  $\beta \ll 2Q$  (or i.e.  $v_A \gg v_{Te}$ ) is one of necessary conditions for the formation of DSKAW (Wu, 2003a), especially the higher speed ratio  $v_A/v_{Te}$  will lead to the higher accelerating efficiency for electrons. The distribution  $v_A/v_{Te}$  with  $h$  is shown by Fig. 6c for  $m=1.5$ . From Fig. 6c one can see that in the observed acceleration region the DSKAW acceleration mechanism not only produces the higher acceleration energy but also has the higher acceleration efficiency.

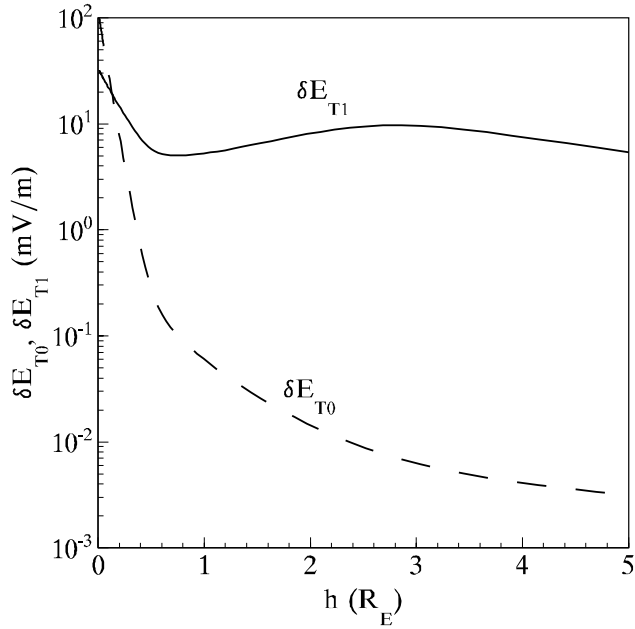
Ion acoustic turbulence  $\delta E_T$  also plays an important role in the formation of DSKAW (Wu, 2003a; Wahlund et al., 1994a). To produce efficient “collisional damping”, it is required that the strength of the turbulent wave  $\delta E_T$  is considerably higher than its thermal noise level  $\delta E_{T0}$  (Hasegawa, 1975)

$$\delta E_{T0} \simeq 0.94 \times 10^{-2} T_e^{-1/4} n_0^{3/4} \text{ mV/m}, \quad (95)$$

where  $T_e$  in eV and  $n_0$  in  $\text{cm}^{-3}$ . From Eq. (84), to produce the damping coefficient of  $\gamma=0.1$  the requisite ion acoustic turbulent strength  $\delta E_{T1}$  can be evaluated by

$$\delta E_{T1} = \sqrt{\gamma \frac{\omega}{kc} \frac{2n_0 T_e}{\epsilon_0}} \simeq 5.53 n_0^{1/4} T_e^{1/2} B_0^{1/2} \text{ mV/m}, \quad (96)$$

where  $k \simeq 2\pi/(10\lambda_e)$  and  $\omega \simeq (1/30)\omega_{ci}$  have been assumed for the typical values of KAW wavenumber and frequency in the auroral plasma. Figure 7 shows the variation of  $\delta E_{T0}$  (dashed) and  $\delta E_{T1}$  (solid) with altitudes. The former indicates the turbulent wave strength that produces an effective collisional frequency  $\nu_{\text{eff}}$  equal to the electron Coulomb collisional frequency  $\nu_{ec}$ , and the latter does the strength that



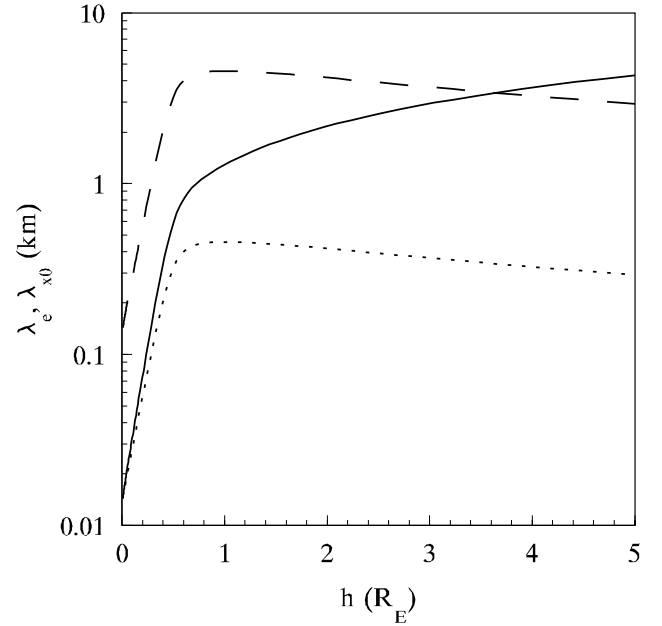
**Fig. 7.** Plots of the turbulent electric fields  $\delta E_{T0}$  (dashed) and  $\delta E_{T1}$  (solid) in mV/m vs altitudes  $h$ .

produces an effective collisional frequency  $\nu_{\text{eff}}$  to cause a damping coefficient  $\gamma$  of 0.1. From Fig. 7, one can see that  $\delta E_{T1} < \delta E_{T0}$  in the ionosphere below  $\sim 0.13 R_E$ . This implies that the Coulomb collision can cause a damping coefficient of  $\gamma > 0.1$  in the ionosphere below  $0.13 R_E$ . Above it, the effective collision by the turbulent wave  $\nu_{\text{eff}}$  will quickly dominate the Coulomb collision  $\nu_{ec}$  with the altitude increasing in the magnetosphere. For the auroral plasma above  $\sim 0.3 R_E$ , to produce damping coefficient of  $\gamma = 0.1$  the requisite turbulent wave strength  $\delta E_{T1}$  is well between 5 and 10 mV/m, which is just the range of the observed turbulent electric fields in the auroral plasma (Wahlund et al., 1994a, b, 1998; Vaivads et al., 1998). In particular, Fig. 7 shows that the requisite strength  $\delta E_{T1}$  reaches its minimal value ( $\sim 5$  mV/m) at the bottom of the observed acceleration region. This indicates that it is this location where the DSKAW structure can most easily form.

In the DSKAW acceleration mechanism of the auroral electrons, the widths of the produced auroral arcs can be estimated by the field-aligned projection of the characteristic width of DSKAW in the acceleration region on the ionosphere because the auroral electrons precipitate field-aligned into the ionosphere. Both theory (Wu, 2003a) and observation (Chaston et al., 1999) show that the DSKAW width has a typical value of several of  $\lambda_e$ . Figure 8 shows the variation of  $\lambda_e$  with altitudes  $h$  above the ionosphere (solid line), where the dotted line indicates the field-aligned projection of  $\lambda_e(h)$  on the ionosphere  $\lambda_{x0}$  in the following form

$$\lambda_{x0} = (1+h)^{-3/2} \lambda_e(h) \quad (97)$$

and the dashed line represents  $10\lambda_{x0}$ . The observed auroral arc widths can be expected to be between  $\lambda_{x0}$  and  $10\lambda_{x0}$ ,



**Fig. 8.** Plots of the electron inertial length  $\lambda_e$  (solid line), its field-aligned projection on the ionosphere  $\lambda_{x0}$  (dotted line), and  $10\lambda_{x0}$  (dashed line) in km vs  $h$ .

which is typically in the order of  $\sim$  km.

In summary, the above analyses suggest the following scenario for the auroral electron acceleration and the small-scale auroral arc formation. Nonlinear KAWs, which are produced by some generators (for instance, shear flows or pressure gradients in the plasma sheet, Borovsky, 1993), propagate into the auroral zone along the geomagnetic field lines. Then they dynamically evolve into a train of DSKAWs due to the kinetic dissipation in the auroral acceleration region where DSKAWs can most easily form. And these DSKAWs can be trapped in the auroral electron acceleration region due to the reflection by the ionosphere or by gradients of the Alfvén velocity (Vogt and Haerendel, 1998). When propagating downwards they accelerate electrons upwards and produce upgoing energetic electron flows (Boehm et al., 1995) (i.e. field-aligned currents downwards), and when propagating upwards they accelerate electrons but downwards and produce downgoing energetic electron flows (i.e. field-aligned currents upwards). It is the latter that leads to aurora.

## 6 Perspectives

The KAW is an extensively interesting topic in space physics and plasma astrophysics and has been investigated by many authors from theory, observation, to application. In this brief review we summarized only our recent work on KAWs, especially on the theory of their solitary nonlinear structures, which is not included in the review by Stasiewicz et al. (2000a). The energization of plasma particles is a common phenomenon in cosmic plasmas, such as auroral energetic electrons in geomagnetic substorms, flare energetic

particles in solar activities, relativistic energetic particles in active galactic nuclei and other compact objects, and energetic cosmic ray particles permeating through the vast cosmic space. Both theory and observation imply that the release of magnetic energy is probably an initial engine driving these energizing processes of cosmic plasma particles. On the other hand, the Alfvén wave is one of the most predominant electromagnetic fluctuations in these cosmic magnetoplasmas. In particular, in low- $\beta$  plasmas such as the auroral plasma, coronal plasma, accretion disk and jet plasmas KAW can probably play an important role in these particle energizing processes because its potential ability to heat (Wu and Fang, 1999, 2003) and accelerate (Wu and Chao, 2003, 2004) electrons along the magnetic field lines as well as to heat and accelerate ions cross the magnetic field lines (Voitenko and Goossens, 2004).

In recent decade KAW has been an increasingly interesting topic in space physics and astrophysics and has greatly progressed for motivating of satellite measurements of KAWs in the space plasma. FREJA and FAST satellites observations have clearly shown the existence of KAWs, especially the nonlinear coherent structures in relation to KAWs. The latter are consistent with strong electric spikes and are usually associated with broadband electrostatic activities at higher frequencies and shorter scales. In particular, these observations of KAWs in the auroral plasma and their probable relation with the auroral electron acceleration process provide us a “natural laboratory” to study in detail the role of KAWs in energization phenomena of plasma particles that can be encountered frequently in cosmic plasmas. Indeed, the experimental results from this “natural laboratory” have motivated theorists to formulate models to explain physical properties of linear and nonlinear KAWs including their excitation and generation mechanisms, steady structures, dissipative processes, and dynamical evolutions.

It has been noted that the dissipation of KAWs strongly depends on background plasma parameters, especially the plasma  $\beta$  parameter. When  $\beta \sim 1$  the mass ratio of electron to ion  $m_e/m_i$  KAW is strongly dissipated by the electron Landau damping due to the thermal electron resonance. And this leads to efficient electron heating (Wu and Fang, 1999, 2003). For the cases of  $\beta \ll m_e/m_i$  or  $\beta \gg m_e/m_i$ , nonlinear coherent structures of KAWs, such as SKAW and DSKAW, can probably be developed due to the combination of dispersion, nonlinear steepening, and weak dissipation. Both observations based on the in situ satellite measurements in the space plasma and theories based on the ion drift approximation have shown the existence of these nonlinear coherent structures.

We also noted some arguments on the existence of 1D SKAWs (Seyler and Lysak, 1999; Shukla and Stenflo, 2000), which is caused by that the inclusion of the nonlinear advection term in the ion polarization drift puts a question mark to the 1D SKAWs. This nonlinear advection term originates from the carrier acceleration in the fluid equation. It, however, is not proper to derive the ion drift velocity from the fluid equation because the fluid description is invalid when

wavelength is comparable with the microscopic kinematic scales of plasma particles, such as the electron inertial length or the ion acoustic gyroradius in the case of KAWs. Instead of the fluid equation, which is valid only for phenomena in scales much larger than the microscopic kinematic scales of particles, the ion drift velocity should be derived from the single particle equation. In the single particle description, the nonlinear advection term originating from the fluid carrier acceleration will not present in the ion polarization drift.

In an inhomogeneous plasma, the coupling of KAWs with the inhomogeneity of plasma density and temperature, flow velocity, and magnetic field can greatly enrich the physics of KAWs. This can produce many complex and interesting nonlinear coherent structures (for instance, monopoles, dipole vortices, and vortex streets) and cause their nonlinear dynamical evolution with their propagating through the inhomogeneous plasma. Our understanding of the physics of these more complex structures and their dynamical evolution is still in its infancy. We believe that the combination among more refined theoretical model, further data analysis, and power numerical simulation will help to improve our comprehension of them.

Besides the creation of localized nonlinear coherent structures, the interplay between dispersion and nonlinearity of KAWs can lead to spectral transport and development of turbulence in KAWs. Turbulent spectra of KAWs have been identified in the auroral zones by Freja (Stasiewicz et al., 2000b) and in the magnetopause boundary by Polar (Stasiewicz et al., 2001). Voitenko (1998) has investigated a nonlinear interaction among KAWs and generation of KAW turbulence induced by the three-wave resonant couplings. Another nonlinear KAW evolution has also been proposed by Onischenko et al. (2004), where the decay of KAWs into KAW sidebands and convective cells is considered. Both above studies suggest an inverse cascade in the KAW turbulence. The relationship between the developed KAW turbulence and the energizing activity of plasma particles and waves also is one of interesting topics to study in future.

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